Int. J. Nonlinear Anal. Appl. 13 (2022) 1, 3191-3195
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.6067

Some results of weakly mapping in intuitionistic bi-topological spaces

Rasha A. Isewid[a,*], Samer R. Yaseen[b]

[a] College of Physical Education and sport science, Al-Qadisiyah University, Iraq
[b] Department of Mathematics, College of Pure Science, Tikrit University, Iraq

(Communicated by Madjid Eshaghi Gordji)

Abstract

We generalized the concept of a function poorly open to intuition raster spaces and we get some new descriptions of the opening up of the weak functions among the intuitive point areas. Furthermore, we are investigating some of the characteristics of these functions compared to the relevant functions.

Keywords: intuitionistic bi-topological spaces, weakly open function, weakly (i, j)-intuitionistic open functions.

1. Introduction

The idea of weak continuous functions is presented in [7] and [6]. The concept of weakly connected It is proved in [7]. In [8] and [9] Bubba and Rose introduced the notion of weakly which is a nature duality of the concept of continuity weakly. In this paper, we generalize the concept of weakly open function to intuitive bio-pathological spaces and obtain some descriptions of weakly mappings among intuitive apoptotic spaces. Further, we are looking into some characteristics of these jobs in comparison to related jobs.

2. Preliminaries

Definition 2.1. *[1] Let $X \neq \varnothing$. An intuitionistic set H is an object having the form $H = \langle x, H_1 , H_2 \rangle$, where H_1 and H_2 are subsets of X satisfying $H_1 \cap H_2 = \phi$. The set H_1 is called the set of member of H, while H_2 is called the set of nonmember of H.*

*Corresponding author
Email addresses: rasha.abbas@qu.edu.iq (Rasha A. Isewid), samer2017@tu.edu.iq (Samer R. Yaseen)

Received: November 2021 *Accepted:* January 2022

Definition 2.2. *Let $X \neq \varnothing$, $p \in X$ and let $H = \langle x, H_1, H_2 \rangle$, be an intuitionistic set (IS, for short). The IS $\dot{p}$ defined by $\dot{p} = \langle x, \{p\}, \{p\}^c \rangle$ is IP in X.*

Definition 2.3. *[3] Let $X \neq \varnothing$. An intuitionistic topology on X is a collection τ of IS in X which satisfying:*
(1) $\widetilde{\Phi}$, $X \in \tau$.
(2) *τ is locked under limited intersections.*
(3) *τ is locked under unlimited unions.*
Then (X, τ) is ITS, any element in τ is IOS. The complement of IOS in (X, τ) is ICS.

Definition 2.4. *[2] Let (X, τ) be an ITS and let $K = \langle x, K_1, K_2 \rangle \subseteq X$. The Interior ($IntA$, for short) and Closure (ClA, for short) of a set A of X are defined*

$$Int(H) = \cup\{H : H \subseteq K, K \in \tau\},$$
$$Cl(H) = \cap\{M : H \subseteq M, \overline{M} \in \tau\}.$$

3. Weakly open mappings in intuitionistic bi-topological spaces

Definition 3.1. *An intuitionistic bi-topological on $X \neq \varnothing$ is a two family τ_1, τ_2 of IS in X such that both (X, τ_1) and (X, τ_2) are ITS. The triple (X, τ_1, τ_2) is IBTS.*

Definition 3.2. *Let (X, τ_1, τ_2) be IBTS, and $H = \langle x, H_1, H_2 \rangle$ be IS in X. The interior ($IntH$, for short) and closure ($iClH$, for short) of a set H of X with respect to τ_i are defined by:*

$$iInt(A) = \cup\{G : G \subseteq H, G \in \tau_i\},$$
$$iCl(A) = \cap\{F : H \subseteq F, \overline{F} \in \tau_i\}.$$

Definition 3.3. *A subset H of IBTS (X, τ_1, τ_2) is*

(1) *intuitionistic (i, j)-ROS if $H = iInt(jCl(H))$,*
(2) *intuitionistic (i; j)-SOS if $H \subset jCl(iInt(H))$,*
(3) *intuitionistic (i; j)-POS if $H \subset iInt(jCl(H))$,*
(4) *intuitionistic (i; j)-αOS if $H \subset iInt(jCl(iInt(H)))$,*
(5) *intuitionistic (i; j)-βOS if $H \subset iCl(jInt(iCl(H)))$.*

Definition 3.4. *A subset H of IBTS (X, τ_1, τ_2) are:*

(1) *intuitionistic (i, j)-regular-closed if $H = iCl(jInt(H))$,*
(2) *intuitionistic (i, j)-semi-closed if $jInt(iCl(H)) \subset H$,*
(3) *intuitionistic (i, j)-pre-closed if $iCl(jInt(H)) \subset H$,*
(4) *intuitionistic (i, j)-α-closed if $iCl(jInt(iCl(H))) \subset H$,*
(5) *intuitionistic (i, j)-β-closed if $iInt(jCl(iInt(H))) \subset H$.*

Definition 3.5. *Let (X, τ_1, τ_2) be IBTS and $H \subset X$, $\kappa \in X$ is intuitionistic ($\hbar$, $\Im$)-Θ-Closure of H, denoted by ($\hbar$, $\Im$)- Cl_Θ (H), if $H \bigcap jCl(U) \neq \varnothing, \forall \tau_i$-IOS U containing $\dot{\kappa}$. $H \subset X$ is intuitionistic ($\hbar$, $\Im$)-Θ-Closure if $H =$ ($\hbar$, $\Im$)- Cl_Θ (H). $H \subset X$ is intuitionistic ($\hbar$, $\Im$)-Θ-OS if $H^c$is an intuitionistic ($\hbar$, $\Im$)$-\Theta$-CS .The intuitionistic ($\hbar$, $\Im$)-Θ-Interior of H, denoted by ($\hbar$, $\Im$)$-\Theta - Int_\Theta(H)$ equals the union of all intuitionistic ($\hbar$; $\Im$)-Θ-OS contained in H.*

Lemma 3.6. *For $H \subset$ IBTS (X, τ_1, τ_2), then*

(1) $(i, j)^c$- $Int_\Theta(H) = (i, j)$-$Cl_\Theta(H^c)$,
(2) $(i, j)^c$-$Cl_\Theta(H) = (i, j)$- $Int_\Theta(H^c)$.

Proof . (1) suppose that (i,j)-Cl$_\Theta$ (H^c) and to prove $(i, j)^c$- Int $_\Theta$(H). Since (i,j)-Cl$_\Theta$ (H^c) is an intuitionistic (i,j)-Θ-CS. Then $(i,j)^c$−-Cl$_\Theta$ is an intuitionistic (i,j)-Θ-OS , also $(i, j)^c$-Cl$_\Theta$ $(H^c) \subset H$. Hence $(i, j)^c$-Cl$_\Theta(H^c) \subset (i,j)$- Int$_\Theta$(H). Conversely, let $\dot{\kappa} \in (i,j)$- Int$_\Theta(H)$. Then there is an intuitionistic (i,j)-Θ-open set G such that $\dot{\kappa} \in G \subset H$. Then G^c is an intuitionistic (i,j)-θ-closed and $H^c \subset G^c$. Since $\dot{\kappa} \notin G^c$, $\dot{\kappa} \notin (i,j)$-Cl$_\Theta$ (H^c) and hence $(i,j)^c$- Int$_\Theta$(H) $\subset (i,j)$-Cl$_\Theta$ (H^c). Therefore, $(i,j)^c -$ Int$_\Theta(A) = (i,j)$-Cl$_\Theta$ (H^c).

(2) This follows from (1) immediately. □

Definition 3.7. *A function $f : (X, \tau_1, \tau_2) \longrightarrow \theta; \sigma_1; \sigma_2$ are*

1. *intuitionistic (i, j)- SO for all τ_i-IOS $\mathcal{B}$ of X, f($\mathcal{B}$) is intuitionistic (i, j)-SO in θ,*
2. *intuitionistic (i, j)-PO if for all τ_i- IOS $\mathcal{B}$ of X, f($\mathcal{B}$) is intuitionistic (i, j)-PO in θ,*
3. *intuitionistic (i, j)-α O if for all τ_i-IOS $\mathcal{B}$ of X, f($\mathcal{B}$) is intuitionistic (i, j)-α O in θ,*
4. *intuitionistic (i, j)-β O if for all τ_i-IOS $\mathcal{B}$ of X, f($\mathcal{B}$) is intuitionistic (i, j)-Iβ O in θ,*
5. *weakly intuitionistic (i, j)-OS if for all τ_i-IOS $\mathcal{B}$ of X, f($\mathcal{B}$) $\subset$ i-Int(f(jCl($\mathcal{B}$))) in θ.*
6. *pair wise weakly intuitionistic OS if f is weakly intuitionistic (1, 2)-OS and weakly intuitionistic (2, 1)-OS .*

4. Relationships among (i, j)-intuitionistic open mappings

Definition 4.1. *A map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is pair wise intuitionistic open if the induced functions $f_1 : (x, \tau_1) \longrightarrow (\theta; \sigma_1)$ and $f_2 : (X; \tau_2) \longrightarrow (\theta; \sigma_2)$ are IOS .*

Definition 4.2. *A map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is almost intuitionistic (i, j)-OS if f(U) is an intuitionistic σ_i-IOS in θ for all intuitionistic (i, j)-ROS U of X.*
A function $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is almost pair wise intuitionistic open if it is almost intuitionistic (1, 2)-open and almost intuitionistic (2, 1)-open.

Remark 4.3. *It is clear that pair wise intuitionistic openness $\Longrightarrow$ almost pair wise intuitionistic openness $\Longrightarrow$ weakly pair wise intuitionistic openness, but reverse not true.*

Example 4.4. *Let $X = \{\rho, \mu\}$, $\tau_1 = \left\{\widetilde{\Phi},\ \widetilde{X}, \Theta\right\}$, where $\Theta = \langle\ x, \{\rho\}, \{\mu\}\rangle$, $\tau_2 = \left\{\widetilde{\Phi},\ \widetilde{X}, \psi\right\}$, where $\langle\ x, \{\mu\}, \{\rho\}\rangle$. Also, $\theta = \{\rho, \mu\}$, $\sigma_1 = \left\{\widetilde{\Phi},\ \widetilde{X}, \Theta\right\}$ and $\sigma_2 = \left\{\widetilde{\Phi},\ \widetilde{X}, \psi\right\}$. Define a map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ by $f(\dot{\rho}, \dot{\mu}) = (\dot{\mu}, \dot{\rho})$. Then f is almost pair wise intuitionistic open , because f is almost intuitionistic (1, 2)- open and almost intuitionistic (2, 1)-open. But f does not satisfy pair wise intuitionistic openness, because $f_1 : (X, \tau_1) \longrightarrow (\theta; \sigma_1)$ and $f_1 : (X, \tau_1) \longrightarrow (\theta; \sigma_2)$ are not IOS.*

Example 4.5. *Let $X = \{\rho, \mu\}$, $\tau_1 = \left\{\widetilde{\Phi},\ \widetilde{X}, \Theta\right\}$, where $\Theta = \langle\ x, \phi, \{\rho\}\rangle$, $\tau_2 = \left\{\widetilde{\Phi},\ \widetilde{X}, \psi, \Xi\right\}$, where $\psi = \langle\ x, \phi, \phi\rangle$, $\Xi = \langle\ x, \phi, \{\rho\}\rangle$. Also, $\theta = \{\rho, \mu\}$, $\sigma_1 = \left\{\widetilde{\Phi},\ \widetilde{X}, \Theta\right\}$ and $\sigma_2 = \left\{\widetilde{\Phi},\ \widetilde{X}, \psi, \Xi\right\}$. Then a map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is weakly pair wise intuitionistic open, because f is weakly intuitionistic (i,j)-OS and weakly intuitionistic (j, i)-OS . But f does not satisfy almost pair wise intuitionistic open , because f is not almost intuitionistic (i, j)-OS (resp. almost intuitionistic (j, i)-open).*

Definition 4.6. *An IBTS (X, τ_1, τ_2) is intuitionistic (i, j)-SR if $\forall$ $\dot{\kappa} \in X$ and for all intuitionistic τ_i-IOS U & $\dot{\kappa} \in X$, there is an intuitionistic (i, j)-ROS V of X such that $\dot{\kappa} \in V \subset U$.$(X, \tau_1, \tau_2)$ is pair wise intuitionistic SR if it is intuitionistic (1, 2)-SR and intuitionistic (2, 1)-SR.*

Theorem 4.7. *Let IBTS (X, τ_1 , τ_2) be a pair wise intuitionistic SR. Then a function $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is a pair wise intuitionistic open if and only if it is almost pair wise intuitionistic open.*

Proof . Suppose that f is almost intuitionistic (i, j)-OS. Let U be τ_i- IOS and X is intuitionistic (i, j)-SR, for all $\dot{\kappa} \in U$ there is an intuitionistic (i, j)-ROS $U_{\dot{\kappa}}$ such that $\dot{\kappa} \in U_{\dot{\kappa}} \subset U$. So f is an almost intuitionistic (i, j)-OS, f $(U_{\dot{\kappa}})$ is an intuitionistic σ_i-open in Y. So $f(U)$ is an intuitionistic σ_i-OS. Hence, $f : (X, \tau_i) \longrightarrow (\theta; \sigma_i)$ is IOS. Thus $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is pair wise intuitionistic open.

The converse is clear. □

Definition 4.8. *An IBTS (X, τ_1 , τ_2) is intuitionistic (i; j)-almost regular if for all $\dot{\kappa} \in X$ and for all intuitionistic (i, j)-ROSH containing $\dot{\kappa}$, there is an intuitionistic (i, j)-ROS Θ of X such that $\dot{\kappa}$ $\in \Theta \subset$ jCl(Θ) $\subset$ H. (X,τ_1, τ_2) is pair wise almost intuitionistic regular if it is intuitionistic (i, j)-almost regular and intuitionistic (j, i)-almost regular.*

Definition 4.9. *An IBTS (X, τ_1 , τ_2) is intuitionistic (i, j)-R if for all $\dot{\kappa} \in X$ and for all intuitionistic τ_i-OS H containing $\dot{\kappa}$, there is an intuitionistic τ_i-OS Θ such that $\dot{\kappa}$ $\in\Theta \subset$ jCl(Θ) $\subset$ H. (X,τ_1, τ_2) is pair wise intuitionistic regular if it intuitionistic (i, j)-regular and intuitionistic (j, i)-regular.*

Definition 4.10. *A map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is intuitionistic strongly continuous if for all $H \subseteq X$, then f(ICl(H)) $\subset$ f(H).*

Proposition 4.11. *If $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is weakly intuitionistic (i, j)-O-map and strongly intuitionistic j-continuous, then f is intuitionistic i-OS.*

Proof . Let Γ is a τ_i-OS of X, so f is weakly intuitionistic (i, j)-OS and strongly intuitionistic j-continuous, so $f(\Gamma) \subset iInt(f(jCl(\Gamma))) \subset iInt(f(\Gamma))$.

Thus, $f(\Gamma) = iInt(f(\Gamma))$ and $f(\Gamma)$ is σ_i-IOS. So that f is intuitionistic i-OS. □

Definition 4.12. *A map f : (X, τ_1, τ_2) $\longrightarrow$(θ; σ_1; σ_2) is intuitionistic (i, j)- contra-CS if f(F) is intuitionistic σ_i-OS in θ for all intuitionistic τ_j-CS F of X.*

Corollary 4.13. *If a map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is intuitionistic (i, j)- SO and intuitionistic (i, j)- contra-OS. Then f is intuitionistic i-IOS.*

Proof . Straightforward. □

Definition 4.14. *A map $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is intuitionistic (i, j)- contra-continuous if $f^{-1}(\Lambda)$ is intuitionistic(i, j)-CS in X $\forall$ Intuitionistic σ_i- OS Λ of θ.*

Corollary 4.15. *If $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is an intuitionistic PO and intuitionistic SO bijection, then f is weakly IOS.*

Proof . Straightforward. □

Definition 4.16. *An IBTS (X, τ_1, τ_2) is pair wise connected if it cannot be expressed as the union of two sets Ω, Ψ such that Ω is τ_i-IOS, Ψ is τ_j-IOS and $\Omega \cap \Psi = \varnothing$.*

Theorem 4.17. *If IBTS (Y; σ_1; σ_2) is pair wise connected and $f : (X, \tau_1, \tau_2) \longrightarrow (\theta; \sigma_1; \sigma_2)$ is a pair wise weakly intuitionistic open bijection, then the IBTS (X, τ_1 , τ_2) is pair wise connected.*

Proof . Suppose that the wise connected. Then (X, τ_1, τ_2) is not pair wise connected. Then there exist τ_i-IOS $\mathfrak{R}_1$ and intuitionistic τ_J-OS U_2 such that $\mathfrak{R}_1 \neq \varnothing, \mathfrak{R}_2 \neq \varnothing$ and $\mathfrak{R}_1 \bigcap \mathfrak{R}_2 = X$, we have $f(\Omega) \neq \varnothing, f(\mathfrak{R}_2) \neq \varnothing$, $f(\mathfrak{R}_1) \bigcap f(\mathfrak{R}_2) = \varnothing$ and $f(\mathfrak{R}_1) \cup f(\mathfrak{R}_2) = \varnothing$ (because f is bijective). Since f is pair wise weakly IOS $f(\mathfrak{R}_1) \subset iInt(f(jCl(\mathfrak{R}_1)))$ and $f(\mathfrak{R}_2) \subset jInt(f(iCl(\mathfrak{R}_2)))$. Since R_1 is τ_j-ICS and $\mathfrak{R}_2$ is τ_i-ICS, so $f(\mathfrak{R}_1) \subset iInt(f(\mathfrak{R}_1))$ and $f(\mathfrak{R}_2) \subset jInt(f(\mathfrak{R}_2))$, so $f(\mathfrak{R}_1) = iInt(f(\mathfrak{R}_1))$ and $f(\mathfrak{R}_2) = jInt(f(\mathfrak{R}_2))$. Thus, $f(\mathfrak{R}_1)$ is σ_i-IOS and $f(\mathfrak{R}_2)$ is σ_i-IOS.

This is contrary to the hypothesis that an IBTS (θ, σ_1, σ_2) is pair wise connected. Therefore, the IBTS (X, τ_1, τ_2) is pair wise connected. □

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