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Interacting anisotropic dark energy with time dependent inhomogeneous equation of state

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Abstract

The interaction of dark energy in the LRS Bianchi type-I line element is explored on the background of $f(R,T)$ gravity, where R and T denotes the Ricci scalar together with the trace of energy momentum tensor of matter respectively. Here modified field equations are calculated using $f(R,T) = f_1(R) + f_2(T)$ together with inhomogeneous equation of state (EoS), $p = \omega\rho - \Lambda(t)$, where ω is constant. The solutions of modified Einstein field equations (EFE) obtained are solved by taking a periodic time varying deceleration parameter (DP). Our model shows periodic nature with Big-Bang prevailing at time $t = 0$. An investigation is done on the energy conditions and found that the conditions of null energy and strong energy are found to be violated. We analyse the geometrical and physical behaviours of these models.

Keywords: Anisotropic, Dark energy, Inhomogeneous equation of state, Modified theory of gravity.

1. Introduction

In recent years, many observational evidences have come up for the cosmological acceleration of the Universe's expansion which has brought a revolution in understanding the evolution history of Universe. These important findings are supported by the observational probes like Cosmic Microwave Background Radiation (CMBR), Supernova Ia and Baryon Acoustic Oscillation (BAO) [33, 20, 34, 46, 32]. These observations indicate the existence of two cosmic epoch of accelerating expansion of Universe i.e. the cosmological phase before radiation and the present cosmic phase after the matter dominated era. As we know, it is assert that some unknown energy dubbed as the dark energy [16] with unusual anti-gravitational force is inferred to be accountable for the late cosmological expansion

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of Universe. This dark energy (DE) which have the unusual characteristics like large negative pressure and negative entropy, can be discovered only through its gravitational effects. Several models like the cosmological constant (Λ) model, quintessence, phantom model, chaplygin gas, tachyon model and many more [30, 31, 9, 29, 19] have been proposed to study ambiguous nature of DE. The late time behaviour of Universe is a riddle to the cosmologists up to the present time. General relativity (GTR) still fails to clarify the driving force of the present Universe's accelerating expansion. Several alternative theories of gravitation are introduced by applying the Einstein-Hilbert action as a modified theories to GTR. In particular, some of the popular modified gravities are $f(R)$ gravity [15, 10, 45], $f(\tau)$ gravity [8, 27] and $f(G)$ gravity [6, 35] where τ, R and G denote respectively the torsion scalar, curvature scalar and the Gauss-Bonnet scalar. Harko in 2011, established an alternative construction of $f(R)$ theory known as $f(R, T)$ gravity, where R is the Ricci scalar and T is the trace of the energy momentum tensor (EMT) of matter [21]. The inclusion of stiff fluid with time dependent vacuum energy density in Bianchi type II Universe is studied and discussed the detailed solutions of the model [4]. The behaviours of geometrical and physical solutions in the anisotropic model in the background of $f(R, T)$ theory is investigated [1]. Bianchi type-V model is investigated in $f(R, T)$ gravity by taking the form $f(R, T) = f_1(R) + f_2(T)$ [2]. The constant (Λ) is expressed in respect of trace of the EMT of matter T. Modified $f(R, T)$ theory in Bianchi type-I model is investigated by adopting a variable $\Lambda(T)$ with some fascinating results [39]. The geometrical and physical solutions of anisotropic models in $f(R, T)$ theory was investigated by adopting varying DP [38].

In the evolution of the Universe, there are two types of singularities i.e. Big Bang singularity and finite time singularity which take place at the final stage of the DE dominated Universe or a future Crunch. It is understandable that models with DE also undergo the finite time singularity problem. Various cosmological models have been developed to escape from these singularities, like the cyclic Universe [47, 48, 26], the ekpyrotic model [24, 18] and the bouncing Cosmological model [12]. The main motivation for the introduction of inhomogeneous equation of state is to develop the alternative theory by modifying general relativity. In the proposed EoS, the inhomogeneous term is used to examine the behaviours of distinct forms of singularities transition to others, future singularity, phantom epoch, and the phantom barrier crossing. The inhomogeneous term's dependency on the Hubble parameter is useful for discussing the emerging oscillating Universe. The inhomogeneous equation of state is introduced through viscosity terms and modified gravity theory [28]. They discovered that the presence of an inhomogeneous term makes it easier for many models to cross the phantom barrier. The Weierstrass and Jacobian elliptic functions were used to examine the EoS and cyclic Friedmann-Robertson-Walker (FRW) Universes [7]. The Universe with time dependent inhomogeneous EoS may occur in the accelerated phase of quintessence or phantom type [14]. They assume the time dependent inhomogeneous EoS as $p = \omega\rho(t) + \Lambda(t)$ where ω and Λ are time-dependent parameters. The same inhomogeneous EoS was used to observe the transition of the Universe from a phantom to a non-phantom era, as well as the occurrence of singularities [13]. The exact solutions of viscous FRW model interacting with variable inhomogeneous EoS by assuming viscous form and quadratic form of Hubble parameter is discussed [23]. The periodic cosmologies had been investigated in many ways by many physicists. Investigation of periodic as well as an inhomogeneous EoS for dark energy fluid leads to the development of an oscillating Universe. Hubble parameter with periodic behaviour can be obtained for both the early Universe's inflation as well as the late phase cosmic acceleration [36]. According to the findings, oscillating DE models can alleviate the coincidence problem by introducing periodic acceleration that varies naturally and is consistent with observations [17]. They can be considered as good candidates for unifying early Universe's inflation together with accelerated expansion at late epoch. The cosmological solutions of $f(R, T)$ theory in respect of periodic varying deceleration parameter was found and physical kinematic properties of the model are discussed [40]. Again, LRS Bianchi-I transit Universe is studied by considering periodic time dependent DP in the $f(R,T)$ theory [11]. The cosmological parameters of anisotropic DE model in the construction of $f(R,T)$ gravity constrained with latest observational data of Plank are investigated and calculated the values of Ω_m and Ω_{de} [43]. The phase transition of an anisotropic cosmological model in $f(R,T)$ gravity from deceleration in early Universe to late time acceleration was examined using a specific type of a time changing deceleration parameter which expressed in terms of Hubble parameter [49]. Many authors [5, 44] study the DP in order to learn more about the Universe's accelerated expansion. FRW Universe is the generalisation of the flat Universe. However, Universe at early epoch is assumed to be not uniform, as evidenced by CMB measurements. Also, according to WMAP data analysis, the Universe is anisotropic and a slightly anisotropic shape. Thus early Universe is assumed to be anisotropic because of this background anisotropy. Although there may be expansion or contraction based on direction, the simplest model of the anisotropic flat Universe is the Bianchi type-I model.

With reference to an inhomogeneous EoS, $p=\omega\rho-\Lambda(t)$ and time dependent DP which is periodic, we are focussed to investigate a LRS Bianchi type-I in $f(R,T)$ theory where $f(R,T)=\lambda R+\lambda T$ with Ricci scalar (R) and trace of EMT (T) as well as constant (λ) The work is presented as follows: The gravitational field equations of $f(R,T)$ theory is formulated in Sec.2. In Section 3, we derived the field equations and looked at the solutions. In Section 4, we look at DE models with a periodic DP. In Section 5, we talk about our model's energy conditions and several observational factors. In Section 6, we give the paper's final remark.

2. $f(R,T)$ gravity and its general formulation

With reference to the Hilbert-Einstein action, modified EFEs of $f(R,T)$ gravity are deducted. For $f(R,T)$ theory, Harko et al. [21] apply the following action

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \tag{2.1}$$

Here R, T, L_m and g denotes the Ricci scalar, the trace of the EMT of matter, the matter Lagrangian density and the metric determinant respectively. The stress-energy tensor of matter is given by

$$T_{ih}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2.2}$$

Taking the variation of action S $w.\ r.\ t.$ metric tensor g_{ij}, the gravitational field equation is obtained as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\Box-\nabla_i\nabla_j)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_R(R,T)\Theta_{ij} \tag{2.3}$$

with the Ricci tensor $(R_{ij}), f_R(R,T)=\frac{\partial f(R,T)}{\partial R}, \Box$ denotes D Alembert's operator and Θ_{ij} is defined as

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{2.4}$$

The energy momentum tensor (T_{ij}) is expressed as

$$T_{ij} = (\rho, -p, -p, -p) \tag{2.5}$$

Here, p together with ρ denoted the pressure and energy density of the fluid respectively. Here we take the matter Lagrangian $L_m = -p$. Thus equation (2.4) may be rewritten as

$$\Theta_{ij} = -2T_{ij} - pg_{ij} \tag{2.6}$$

Several models based on different forms of $f(R,T)$ may be studied with respect to the nature of the matter source. Harko in 2011 [21] established the functional from of $f(R,T)$:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}$$

Here, the following special form is considered and is given by

$$f(R,T) = f_1(R) + f_2(T) \tag{2.7}$$

with $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$, where λ is arbitrary constant [2, 43]. We employ the aforementioned reconstruction method to represent the rapid expansion of the late time Universe by adding the corresponding function based on the trace of the stress-energy tensor.

3. Field equations for the functional form $f(R,T) = f_1(R) + f_2(T)$

The line element of LRS Bianchi type-I is considered in this model as

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2) - B^2(t)dz^2 \tag{3.1}$$

Expressions of average scale factor (a), the spatial volume (V) as well as Hubble parameter (H) are obtained as

$$a = (A^2B)^{\frac{1}{3}},\ V = a^3 = A^2B,\ H = \frac{1}{3}(2H_x + H_z) \tag{3.2}$$

Here, $H_x = H_y = \frac{\dot{A}}{A}$ and $H_z = \frac{\dot{B}}{B}$. Now equation (2.4) can be rewritten as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(p + \frac{1}{2}T\right) g_{ij} + \left(\frac{8\pi + \lambda}{\lambda}\right) T_{ij} \tag{3.3}$$

From the equation (3.3) for the metric (3.1) with the use of equations (2.5) and (2.7), the consequent equations are obtained as

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = \left(p + \frac{1}{2}T\right) + \alpha\rho \tag{3.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = \left(p + \frac{1}{2}T\right) + \alpha p \tag{3.5}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = \left(p + \frac{1}{2}T\right) - \alpha p \tag{3.6}$$

Here an overhead dot denotes first derivative together with the overhead dots $(\cdot\cdot)$ denote the second derivative $w.\ r.\ t.\ 't'$ and $\alpha = \frac{8\pi+\lambda}{\lambda}$. Many authors have studied the inhomogeneous equation of state [28, 14, 13, 23] and are proved to be quite useful in describing various cosmological phenomena. Here the inhomogeneous EoS is supposed in the following form

$$p = \omega\rho - \Lambda(t) \tag{3.7}$$

where $-1 \leq \omega \leq 1$, is constant and Λ denotes the time-dependent cosmological parameter. Also, the trace of the EMT(T) in this model is obtained as $T = \rho - 3p$. Then the above field equations in terms of Hubble directional parameters become

$$H_x^2 + 2H_xH_z = \frac{1}{2}\Lambda + \frac{1}{2}\rho(1 - \omega + 2\alpha) \tag{3.8}$$

$$\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_xH_z = \left(\frac{1}{2} + \alpha\right)\Lambda + \frac{1}{2}\rho(1 - \omega - 2\omega\alpha) \tag{3.9}$$

$$2\dot{H}_x + 3H_x^2 = \left(\frac{1}{2} + \alpha\right)\Lambda + \frac{1}{2}\rho(1 - \omega - 2\omega\alpha) \tag{3.10}$$

On solving equations (3.4), (3.5) and (3.6), we get

$$A = c_2^{\frac{1}{3}} a\ exp\left(\frac{c_1}{3}\int\frac{dt}{a^3}\right) \tag{3.11}$$

$$B = c_2^{\frac{-2}{3}} a\ exp\left(\frac{-2c_1}{3}\int\frac{dt}{a^3}\right) \tag{3.12}$$

where c_1 and c_2 are integration constants. The expressions of cosmological term, energy density and pressure are calculated using eqns. (3.8),(3.9),(3.10) as

$$\Lambda = \frac{1}{\alpha(\alpha+1)}[(\omega\alpha - \omega + 3\alpha + 1)H_x^2 + (1 - \omega + 2\alpha)\dot{H}_x + (2\omega\alpha + \omega - 1)H_xH_z] \tag{3.13}$$

$$\rho = \frac{1}{\alpha(\alpha+1)}[(\alpha - 1)H_x^2 - \dot{H}_x + (2\alpha + 1)H_xH_z] \tag{3.14}$$

$$p = -\frac{1}{\alpha(\alpha+1)}[(3\alpha + 1)H_x^2 + (2\omega + 2\alpha - 1)\dot{H}_x - H_xH_z] \tag{3.15}$$

4. Dark energy model having periodic varying deceleration parameter

The value of deceleration parameter becomes positive at decelerated epoch and negative at accelerated epoch. At redshift $z \sim 1$, cosmic movement of the phase from decelerated universe to accelerated phase of Universe is observed. This phenomenon of Universe clearly suggests the possibility for variable periodic deceleration parameter. This character of periodic deceleration parameter determines the scenario of signature flip. Because of this signature flipping behaviour of the Universe's evolution, DP which is a function of time as well as periodic is considered as

$$q = m\ \cos\ kt - 1 \tag{4.1}$$

Here both $m > 0$ and $k > 0$. k stands for periodic property of the periodic deceleration parameter. It can be treated as frequency parameter. Here, m increases the magnitude of the periodic deceleration parameter. At initial epoch, DP $(q = m - 1)$ is positive (for $m > 1$) and enter into a maximum negative value of $q = -m - 1$. Following this maximum negative value, this raises again, then returns to the early conditions. We can observe that a periodic evolutionary behaviour of the deceleration parameter is repeated periodically. Thus we can develop the model that begins from a decelerating epoch that evolves into an accelerating epoch of Universe. Integration of equation (4.1) gives the relation as

$$H = \frac{k}{m\ \sin\ kt + k_1} \tag{4.2}$$

Here k_1 denotes integration constant. We assume $k_1 = 0$ as special case and Hubble parameter becomes

$$H = \frac{k}{m\ \sin\ kt} \tag{4.3}$$

Here H shows the periodic character of expansion. The scale factor a is found by integrating (4.3) as

$$a = a_0 \tan^{\frac{1}{m}} \left(\frac{kt}{2}\right) \tag{4.4}$$

with the scale factor (a_0) at present time and may be taken as unity. The scale factor appears to rise over time, with no evident variation for different k values. Since it is very difficult to solve for A and B from the field equations, we fixed $m = 3$ for simplicity. This result to

$$A = c_2^{\frac{1}{3}} \tan^{\frac{1}{3}} \left(\frac{kt}{2}\right) \sin^{\frac{2c_1}{3k}} \left(\frac{kt}{2}\right) \tag{4.5}$$

$$B = c_2^{\frac{-2}{3}} \tan^{\frac{1}{3}} \left(\frac{kt}{2}\right) \sin^{\frac{-4c_1}{3k}} \left(\frac{kt}{2}\right) \tag{4.6}$$

These show that the scale factor expands along $x,\ y$ and z axes with different rates in the periodic form. And the Hubble directional parameters are obtained as

$$H_x = \frac{1}{3} \left[k\ \csc(kt) + c_1 \cot\left(\frac{kt}{2}\right)\right] \tag{4.7}$$

$$H_z = \frac{1}{3} \left[k\ \csc(kt) - 2c_1 \cot\left(\frac{kt}{2}\right)\right] \tag{4.8}$$

Also the anisotropy parameter of our model is calculated as

$$\Delta = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H}\right)^2 = \frac{8c_1^2}{k^2} \cos^2 \left(\frac{kt}{2}\right) \tag{4.9}$$

We consider here the dark energy cosmological model $(\omega = -1)$. Taking $\omega = -1$, above values of H_x and H_z in equation (3.13), (3.14) and (3.15), the following parameters are calculated as follows:

$$\Lambda = \frac{2}{3\alpha} \left[c_1^2 \cot^2 \left(\frac{kt}{2}\right) - k^2 \csc(kt) \cot(kt)\right] \tag{4.10}$$

$$\rho = \frac{1}{3\alpha(\alpha+1)}\left[\alpha k^2 \csc^2(kt) - (\alpha+1)c_1^2 \cot^2\left(\frac{kt}{2}\right) + k^2 \csc\left(\frac{kt}{2}\right)\cot(kt)\right] \tag{4.11}$$

$$p = -\frac{1}{3\alpha(\alpha+1)}\left[\alpha k^2 \csc^2(kt) + (\alpha+1)c_1^2 \cot^2\left(\frac{kt}{2}\right) - (1+2\alpha)k^2 \csc\left(\frac{kt}{2}\right)\cot(kt)\right] \tag{4.12}$$

In this work, we plotted all the graphs by taking $\alpha = 1, c_1 = 0.5$ and $k = 1.05, 1.10, 1.15$. In Figure 1, the evolutionary aspect of the DP (q) is plotted. It is clearly depicted the model has periodic nature and oscillates in between 2 and -4. The geometrical behaviours of scale factor and Hubble parameter are shown in Figure 2 and 3. Here the scale factor increases with the evolution of time for varied value of k. On the contrary, the Hubble parameter demonstrates periodic nature with time. As the scale factor as well as the Hubble parameter are determined by the tangent and sine functions, both can be positive or negative at various epochs. It is observed that derived model becomes anisotropic at early phase then enters into the isotropic phase at $t = \frac{(2n+1)\pi}{k}$ for all $n \in I$ in Figure 4. Thus we see the periodic nature that begins from anisotropy to isotropy. This nature is repeated periodically.

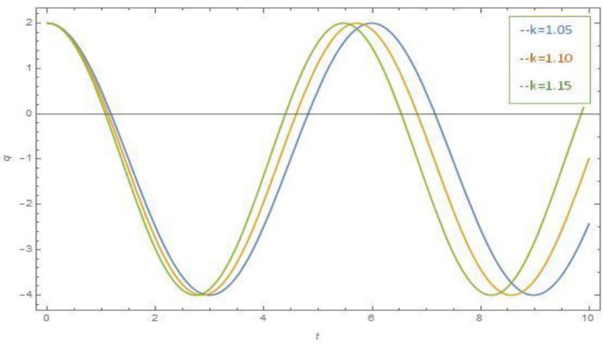

Figure 1: Deceleration parameter (q) vs time (t) for $k = 1.05, 1.10$ and 1.15.

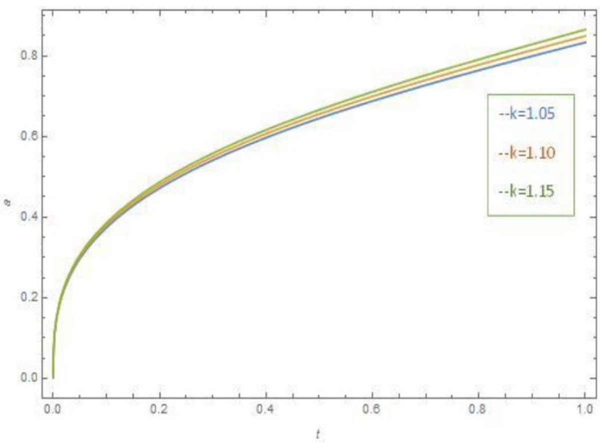

Figure 2: Scale factor a vs time (t) for $k = 1.05, 1.10$ and 1.15

The behaviour of Λ is shown in Figure 5. It is found that Λ gives positive value always and periodic in nature which shows that Universe is expanding. It is clearly seen that it predicts singularities which is periodic at time $t = \frac{2n\pi}{k}$, n is non-negative integer. It begins with a low value $\Lambda_{\min}$ at the beginning

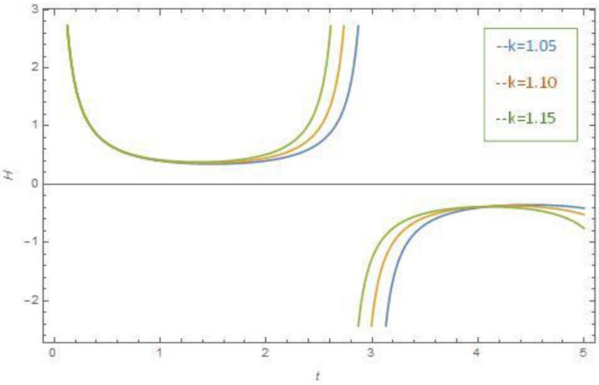

Figure 3: Hubble parameter (H) vs time (t) for $k = 1.05, 1.10$ and 1.15.

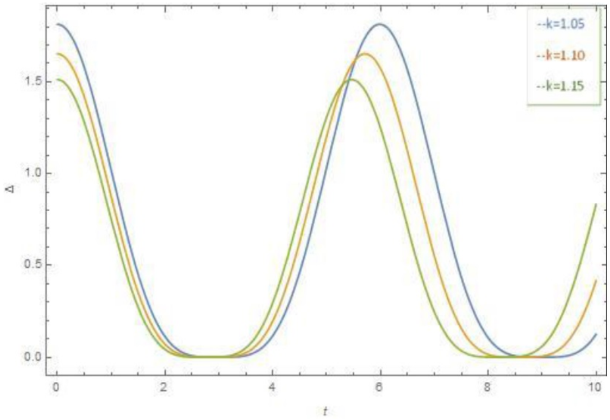

Figure 4: Anisotropic parameter (Δ) vs time (t) for $k = 1.05, 1.10$ and 1.15 and $c_1 = 0.5$

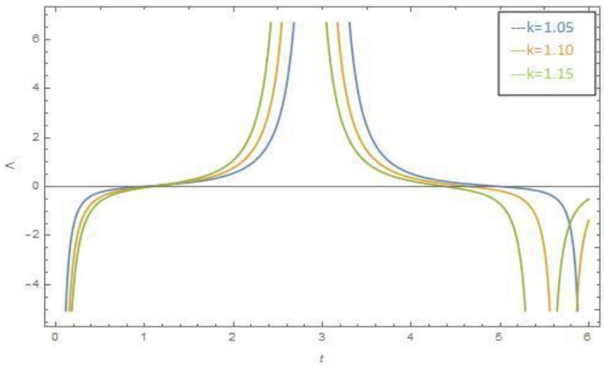

Figure 5: Cosmological constant (Λ) vs time (t) for $k = 1.05, 1.10$ and 1.15 and $\alpha = 1$.

of the cycle and gradually grows with time. With the decrease of value of k, there occurs a decrement in $\Lambda_{\min}$. Figure 6 shows the behaviour of density. The energy density's positivity condition also holds, which is consistent with observations. It is also seen to have periodic type of singularities at time $t = \frac{2n\pi}{k}$, n is non negative integer. It starts with a higher value at the beginning of a cycle, declines to a minimum $\rho_{\min}$, and then grows with time. When the value of k decreases, the value of $\rho_{\min}$ decreases. Similarly Figure 7 shows the behaviour of pressure. It is also found to have a periodic nature of singularities. During a specific period, the negative value of pressure declines at first, then moves to almost positive before becoming more negative, confirming the presence of DE as well as the Universe's accelerating expansion.

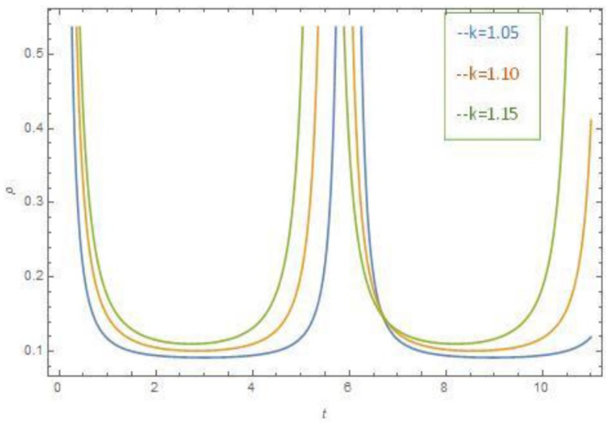

Figure 6: Energy density (ρ) vs time (t) for $k = 1.05, 1.10$ and 1.15, $\alpha = 1$ and $c_1 = 0.5$

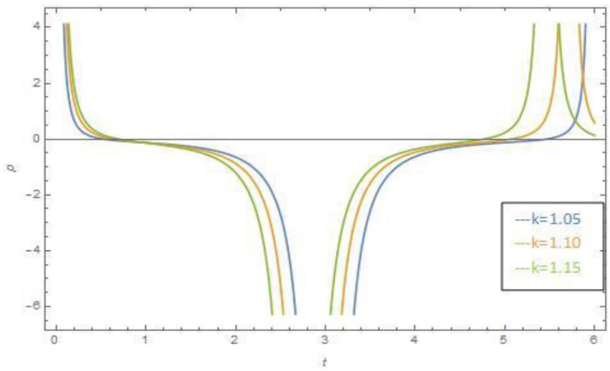

Figure 7: Pressure (p) vs time (t) for $k = 1.05, 1.10$ and 1.15, $c_1 = 0.5$ and $\alpha = 1$.

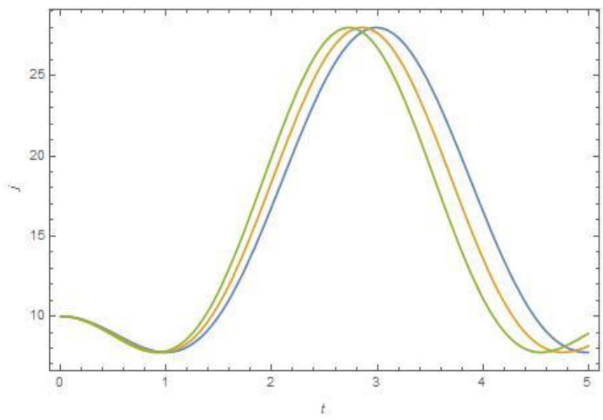

Figure 8: Jerk parameter (j) vs time (t) for $k = 1.05, 1.10$ and 1.15

5. Some observational parameters and energy conditions

The Hubble parameter and the deceleration parameter are the fundamental cosmographic factors that define how the cosmological model and the cosmological principle interact. Another important parameter is the jerk parameter. The jerk parameter [37] is defined as

$$j = \frac{\dddot{a}}{aH^3} 10 - 18\sin^2\left(\frac{kt}{2}\right) + 36\sin^4\left(\frac{kt}{2}\right) \tag{5.1}$$

Figure 8 shows the periodic nature of jerk parameter having same maximum value for varied k. The continuity equation of Fridemann models in general relativity for the energy conservation is given by

$$\dot{\rho} = 3H(\rho + p) = 0 \tag{5.2}$$

which implicates $d(\rho V) = -\rho dV$. Here $V = a^3$ is the volume and ρV denotes the total energy of the Universe. As Universe expands, increase of dark energy is proportional to the expanding volume. When spacetime continues to exist, the total energy will remain constant. Now taking covariant derivative of equation (2.3), we can obtain

$$\nabla^i T_{ij} = \frac{f_T(R,T)}{8\pi - f_T(R,T)}\left[(T_{ij} + \Theta_{ij})\nabla^i \ln f_T(R,T) + \nabla^i \Theta_{ij} - \frac{1}{2} g_{ij} \nabla^i T\right] \tag{5.3}$$

It will be observed that if one considers $f_T(R,T) = 0$ i.e. $\lambda = 0$, then $\nabla^i T_{ij} = 0$. However, for $\lambda \neq 0$, the equation of energy conservation is violated. In recent years, many scholars have looked into the consequences of modified gravity theories violating energy conservation equation (i.e. $\dot{\rho} = 3H(\rho + p) \neq 0$). The non-unitary alterations of quantum physics with spacetime in phenomenological models are expected to lead to the non-conservation of EMT at the Planck scale. This violation of conservation of EMT results in an efficient cosmological term which alters in terms of the creation energy or annihilation energy of cosmological expansion and may become a constant during matter density drops [22]. A non-conservation of energy momentum tensor is found in $f(R,T)$ gravity in the presence of the pressure less cosmic fluid, which may cause Universe's accelerated expansion [41]. Violation of energy conservation equation caused by the factor S is given by

$$S = \dot{\rho} + 3H(\rho + p) \tag{5.4}$$

In case, if $S = 0$ the model satisfies the energy-momentum conservation else the situation leads to non-conservation. The deviation factor S might be negative or positive according to the energy flows into or away from the matter field. The violation of EMT conservation equation with periodic variable deceleration parameters is shown in Figure 9. The energy conditions are expressed as linear combinations of the stress-energy tensor components which will be $+ve$ or at the very least $-ve$. In GTR, null (NEC), weak (WEC), strong (SEC) and dominant (DEC) are the leading energy conditions which are expressed as

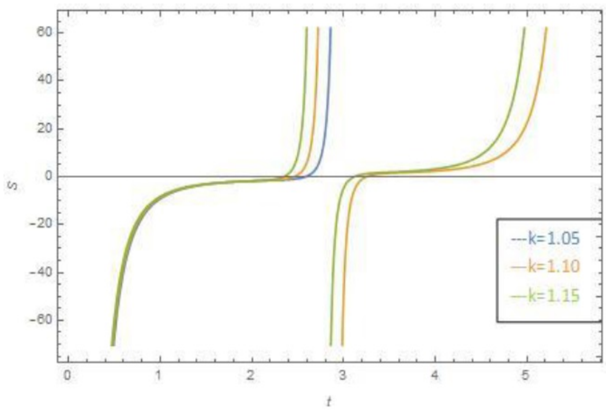

Figure 9: Non-conservation of Energy-momentum

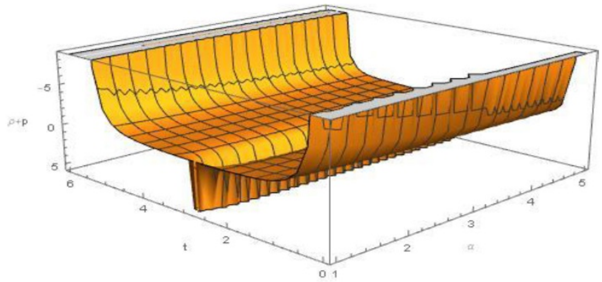

Figure 10: Violation of NEC ($\rho + p \geq 0$) vs. α and t.

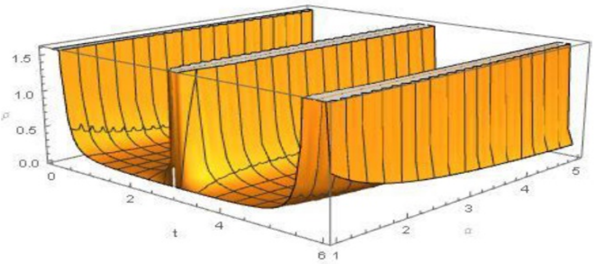

Figure 11: Plot of $\rho \geq 0$ vs. α and t.

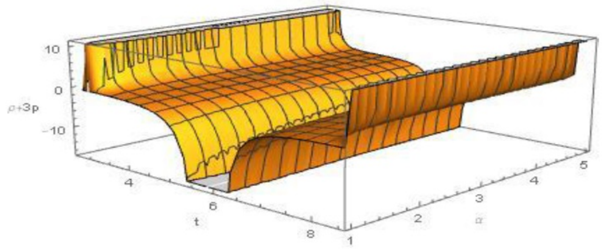

Figure 12: Violation of SEC ($\rho + 3p \geq 0$) vs. α and t.

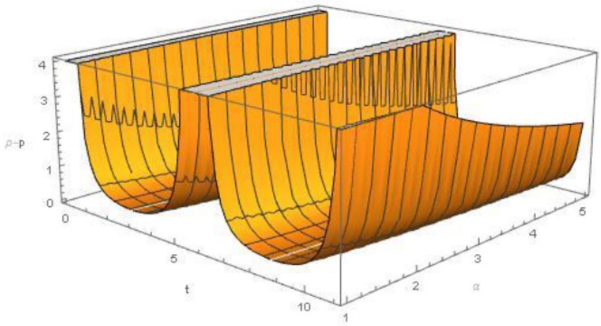

Figure 13: Plot of $\rho - p \geq 0$ vs. α and t.

$$NEC \Leftrightarrow \rho + p \geq 0 \tag{5.5}$$

$$WEC \Leftrightarrow NEC \ and \ \rho \geq 0 \tag{5.6}$$

$$SEC \Leftrightarrow \rho + 3p \geq 0 \tag{5.7}$$

$$DEC \Leftrightarrow \rho - p \geq 0 \tag{5.8}$$

Many authors [3, 42] have analysed the main conditions of energy in $f(R,T)$ theory. Figures 10,11,12,13 show the previously described energy conditions at certain values of k with an acceptable value for $\alpha = \frac{8\pi+\lambda}{\lambda}$. From the plot, it is clear that it is periodic in nature. The conditions of WEC as well as DEC criteria are in agreement, however NEC and SEC are not satisfied. From the plots for NEC and SEC, we can see singularities.

6. Conclusion

The interaction of dark energy is investigated in models of LRS Bianchi type-I on the background of $f(R,T)$ theory. With the help of a periodic varying deceleration parameter, we calculated the solutions of the modified EFEs that involves the inhomogeneous EoS $p = \omega\rho - \Lambda(t)$. The model begins with an epoch of deceleration then enters into a stage of super-exponential expansion with periodic behaviour in this derived model. The model shows initial singularity at $t = 0$. Within a particular cosmic epoch, the cosmological constant, energy density as well as pressure are observed to fluctuate cyclically. Significance of these physical parameters turned out to be infinitely large at some finite time which leads to a future singularity. There also appears to have a Big Rip which takes place cyclically after an interval of $t = \frac{2n\pi}{k}$ where n is positive integer. Also the anisotropic parameter and the jerk parameter are periodic in nature with the evolution of Universe. We studied the energy conditions and found that WEC and DEC are found in agreement, but NEC and SEC are not satisfied. We've also found the violation of conservation energy-momentum equation, which results into the Universe's accelerated expansion. Despite its simplicity, this model may be applicable for a deeper perception about development of Universe.

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