Int. J. Nonlinear Anal. Appl.
Volume 12, Special Issue, Winter and Spring 2021, 2223-2230
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2021.6114

Rayleigh-uniform odd distribution and estimation of its parameters

Ban M. Tuij[a], Ahmed AL-Adilee[b,]

[a]*Department of Mathematics, Faculty of Education for Girls, University of Kufa, Iraq*
[b]*Department of Mathematics, Faculty of Education, University of Kufa, Iraq*

(Communicated by Madjid Eshaghi Gordji)

Abstract

This study is concerned with generating odd distribution by combining Rayleigh distribution to uniform distribution, denoted by (RUOD). We drive the distribution function (df), its probability density function (pdf), and we discuss some other properties like the survival function, moments and graphs of such functions. Also, we estimate the parameters model of the generated distribution RUOD by using the maximum likelihood method to find the approximate values that fit the desired distribution. Eventually, we analyze some data set by the generated RUOD and compare the results upon the goodness-of-fit measures with respect to Rayleigh distribution to decide the best distribution that fits the data set.

Keywords: Continues distributions, Odd techniques, Moments, Estimation method, Data analysis

1. Introduction

The RUOD is an odd distribution that is generated from combining a uniform distribution with Rayleigh distribution to obtain the requested odd distribution. Moreover, we estimate the new parameter model resulting from this combination by substituting the location parameter of the Rayleigh distribution with uniform distribution. A comparison of RUOD with Rayleigh distribution has been discussed via the measures of goodness-of-fit. This method appears to improve the analysis of data sets that have been used to compare the Rayleigh distribution using statistics genuine data sets. The usage of maximum likelihood method to estimate the parameter for a one parameter Rayleigh distribution with estimation of three parameters model of RUOD. We have also utilized the goodness-of-fit measures to find the optimal distribution that matches the data set among others.

Email addresses: ahmeda.aladilee@uokufa.edu.iq (Ban M. Tuij), banm.tuij@student.uokufa.edu.iq (Ahmed AL-Adilee)

Received: October 2021 *Accepted:* December 2021

Rayleigh distribution has been applied in a variety of fields, including life testing, dependability, and survival analysis. Another field of research that relies heavily on the Rayleigh distribution is modeling the lifespan of random occurrences. Rayleigh was the one who originally introduced us to each other (1880), [2].

Maximum probability is a very straightforward method of constructing the unknown parameter estimator o. In 1912, R.A Fisher, an eminent English mathematical statistician, introduced it. The Maximum Likelihood Estimation (MLE) may be used to solve a wide range of issues, has a natural gravitational value, and frequently yields a same estimate of 0. Furthermore, if the sample size is big, the approach will yield a good estimate of 0. Because of these factors, the most extensively used estimating approach in statistics is the maximum likelihood method. In the current age, there has been a growing interest in using at least one extra shape parameter event for the baseline distribution to provide novel generators of univariate distributions.

This additional parameter value was demonstrated in the tail properties in addition to increasing the proposed family suitability [1]. Some common estimation in weighted Rayleigh [2], the odd Fréchet-G [12], beta- gamma [13], Estimation of the Rayleigh [4], Exponential-Weibull Exponential [7], Kumaraswamy-Weibull [3], gamma-uniform [10], Weibull-exponential [8] and gamma- uniform [11].

Eventually, we refer to parts of this paper: in the next part, we review some basic notions related to the statistical concepts like pdf, df, and others. Part three consists of the main ideas concerned with this study. In part four, we present several sentences that summarize the whole work.

2. Basic notions

We review some basic notions related to uniform distribution and Rayleigh distribution. The pdf, the df, survival function S(x), the hazard function h(x), [5].

$$f(x) = \frac{x}{\sigma^2} e^{(\frac{-1}{2\sigma^2} x^2)}, 0 \leq x \leq \infty \tag{2.1}$$

$$F(x) = 1 - e^{(\frac{-1}{2\sigma^2} x^2)}, 0 < x < \infty \tag{2.2}$$

$$s(x) = e^{(\frac{-1}{2\sigma^2} x^2)}, 0 < x < \infty \tag{2.3}$$

$$h(x) = \frac{f(x)}{s(x)} \tag{2.4}$$

Also, we need only to review the pdf, the df, and the survival function distribution, respectively.

$$g(x) = \frac{1}{b-a}, a \leq x \leq b, a, b > 0 \tag{2.5}$$

where a and b are scale parameter.

$$G(x) = \frac{x-a}{b-a}, a \leq x \leq b, a, b > 0 \tag{2.6}$$

$$\overline{G}(x) = \frac{b-x}{b-a}, a \leq x \leq b, a, b > 0 \tag{2.7}$$

Also, we can be defined as anew random variable that is used to generate the odd distribution by the odd ratio and remember as follows:

$$X = \frac{G(x)}{\overline{G}(x)}, a < x < b, a, b > 0 \tag{2.8}$$

A well-known approach for estimating parameters is the method of parameter estimation. The final concept is to demonstrate the quality measures that we need to compare the individual distribution that was created by combining a uniform distribution with a Rayleigh distribution. These measures of goodness-of-fit are known as Akanke information cirterion (AIC), and consistent Akanke information the criterion (CAIC), cirterion (CAIC), Haman-Quinn information cirterion (HQIC), and Bayesian information cirterion (BIC). They're commonly used to figure out which distribution fits data the best, among other things, see [1, 9].

3. Derivations, discussion, and analysis

3.1. The generating of RUOD

In this part, we present the combination of Rayleigh distribution with uniform distribution to generate the RUOD. By using the formula in equation (2.8), we obtain a random variable that has the following form.

$$X = \frac{G(x)}{\overline{G}(x)} = \frac{x-a}{b-x'} \tag{3.1}$$

By substituting such random variable into the distribution function in (2.2), so this yields that

$$F(x) = p(X \leq x) = p(X \leq \frac{x-a}{b-x}) = \int_{-\infty}^{\frac{x-a}{b-x}} f(x)dx = F(\frac{x-a}{b-x}) \tag{3.2}$$

Which means that we have a df of RUOD, that is:

$$F(x) = 1 - e^{[\frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2]}, a < x < b, a, b, \sigma^2 > 0 \tag{3.3}$$

By differentiate $F(x)$ with respect to x, we obtain pdf of RUOD, that is

$$f(x) = \frac{dF(\frac{G(x)}{\overline{G}(x)})}{dx} = \frac{dF(\frac{x-a}{b-x})}{dx} = \frac{f(x)g(x)}{[G(x)]^2} \tag{3.4}$$

Then, the pdf has the following form

$$f(x) = \frac{(x-a)(b-a)}{a^2(b-x)^3} e^{[\frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2]}, a < x < b, a, b, \sigma^2 > 0 \tag{3.5}$$

From the df and pdf in equations (3.3), and (3.5) we can obtain other properties like the survival function $S(x)$, the hazard rate function $h(x)$, reversed hazard rate function $r(x)$, and the cumulative hazard rate function $H(x)$ of the random variable X are, respectively given by:

$$s(x:a,b,\sigma^2) = 1 - F(X) = e^{[\frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2]} \tag{3.6}$$

$$h(x:a,b,\sigma^2) = \frac{f(x:a,b,\sigma^2)}{\overline{F}(x:a,b,\sigma^2)} = \frac{(x-a)(b-a)}{a^2(b-x)^3} \tag{3.7}$$

$$r(x:a,b,\sigma^2) = \frac{f(x:a,b,\sigma^2)}{F(x:a,b,\sigma^2)} = \frac{(x-a)(b-a)e^{[\frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2]}}{a^2(b-x)^3(e^{[\frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2]} - 1)} \tag{3.8}$$

$$H(x:a,b,\sigma^2) = -Ln[1 - F(x:a,b,\sigma^2) = \frac{-1}{2\sigma^2}(\frac{x-a}{b-x})^2 \tag{3.9}$$

Moreover, we have derived the formulas of moments about the origin, and the mean of the RUOD, which have the forms below:

$$\mu_r^{'} = \frac{(b-a)\sum_{p=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{3p-i-j-m}a^{2p-j}}{2^p(\sigma^2)^{p+1}p!\sum_{i=0}^{\infty}\sum_{m=0}^{\infty}\binom{3}{i}\binom{2p}{m}b^{3-i+2p-m}}$$
$$\left[\frac{1}{r-i+j-m+2}(b^{r-i+j-m+2} - a^{r-i+j-m+2}) - \frac{a}{r-i+j-m+1}(b^{r-i+j-m+1} - a^{r-i+j-m+1})\right] \tag{3.10}$$

$$\mu_r = E(x-\mu)^m = \frac{(b-a)\sum_{p=0}^{\infty}\sum_{j=0}^{\infty}\sum_{s=0}^{m}(-1)^{3p-i-j-n+m-s}\binom{m}{s}a^{2p-j}\mu^{m-s}}{2^p(\sigma^2)^{p+1}p!\sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\binom{3}{i}\binom{2p}{n}b^{3-i+2p-n}}$$
$$\left[\frac{1}{s-i+j-n+2}(b^{s-i+j-n+2} - a^{s-i+j-n+2}) - \frac{a}{s-i+j-n+1}(b^{s-i+j-n+1} - a^{s-i+j-n+1})\right] \tag{3.11}$$

The plots of (df), (pdf) and survival function of the RUOD for different parameters are given by the following figures.

Also,The plots of the survival function $s(x)$ are given by the following figures.

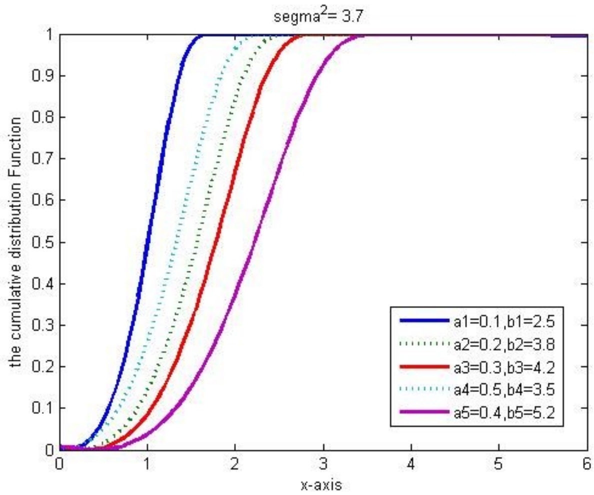

Figure 1: The cdf of RUOD

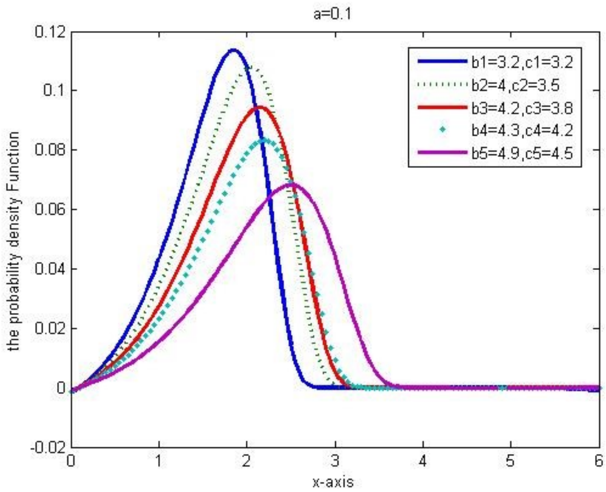

Figure 2: The pdf of RUOD

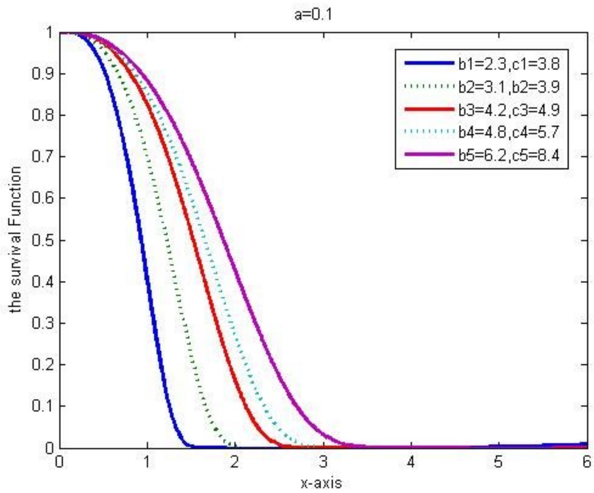

Figure 3: The survival function of RUOD

3.2. Estimation of RUOD parameters

We investigate the estimation of the parameters model of the generated distribution RUOD by the MLE. This method is preferred comparing to other methods because it almost yields unbiased estimators. If$X_1, X_2, \ldots, X_n$ be a random sample from RUOD, then the likelihood function of those odd distribution is given by:

$$L = \prod_{i=1}^{n} f(x : a, b, \sigma^2) \tag{3.12}$$

By substituting the pdf in (3.G5) into equation (3.12), we obtain the likelihood function, that is

$$L = \prod_{i=1}^{n} \frac{(x_i - a)(b-a)}{a^2(b-x_i)^3} e^{[\frac{-1}{2\sigma^2}(\frac{x_i - a}{b - x_i})^2]} = \frac{(b-a)^n \prod_{i=1}^{n}(x_i - a)}{(\sigma^2)^n \prod_{i=1}^{n}(b - x_i)^3} \tag{3.13}$$

Then, the log-likelihood function of L respectively is

$$\ell = ln(l_1) = nln(b-a) - nln\sigma^2 + \sum_{i=0}^{n} ln(x_i - a) - 3\sum_{i=0}^{n} ln(b - x_i) - \frac{-1}{2\sigma^2}\sum_{i=0}^{n}(\frac{x_i - a}{b - x_i})^2 \tag{3.14}$$

parameter a, b, σ^2 and setting the result to zero, we get the following equation:

$$\frac{\partial \ell}{\partial a} = \frac{-n}{(b-a)} - \sum_{i=0}^{n} \frac{1}{(x_i - a)} + \frac{1}{\sigma^2}\sum_{i=0}^{n} \frac{(x_i - a)}{(b - x_i)^2} \tag{3.15}$$

$$\frac{\partial \ell}{\partial b} = \frac{n}{(b-a)} - 3\sum_{i=0}^{n} \frac{1}{(b - x_i)} + \frac{1}{\sigma^2}\sum_{i=0}^{n} \frac{(x_i - a)^2}{(b - x_i)^3} \tag{3.16}$$

$$\frac{\partial \ell}{\partial \sigma^2} = \frac{-n}{\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=0}^{n}(\frac{x_i - a}{b - x_i})^2 \tag{3.17}$$

3.3. Application and discussion

This section discusses the parameter estimation using the data set, as well as the analysis of lifespan data using the Rayleigh distribution with estimated value of σ^2, and the generated RUOD, in order to compare and explain the obtained conclusions of the investigated data. Using the goodness of fit metrics like AIC, BIC, and others, the best eyesight may be obtained by comparing the Rayleigh distribution and the RUOD. The data taken from SMITH and NAYLOR (1987) studied the strength of 1.5cm glass fibers measured at the national physical laboratory, England, the data comprises of 63 observations [6].

Table 1: The data from Aarset

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2	0.74
1.04	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01	0.77	1.11
1.28	1.42	1.5	1.54	1.6	1.62	1.66	1.69	1.76	1.84	2.24	0.81	0.13	1.29
1.48	1.5	1.55	1.61	1.62	1.66	1.7	1.77	1.84	0.84	1.24	1.3	1.48	1.51
1.55	1.61	1.63	1.67	1.7	1.78	1.89							

The calculated parameters model of the Rayleigh distribution, and RUOD are compared to the conventional mixed distribution known. Matlab was used to complete this estimate, and Table 3.2 provides the estimated parameter values, which are shown as follows:

Table 2: MLEs of parameters, Log-likelihood

Model	MLE'S of parameters
Rayleigh	$\widehat{\sigma}^2$
RUOD	$\widehat{a} = 1.591, \widehat{b} = 6.139, \widehat{\sigma}^2 = 0.545$

The predicted values of the parameter σ^2 of the Rayleigh distribution and the RUOD are vary and almost not identical. The influence of exceeding the number of parameters in the RUOD increase the efficiency of the analysis of data set. This can be shown by using the required data set with the maximum log-likelihood functions (l), and then compute the AIC, BIC, CACI, and HQIC values. This allows for the necessary comparison of the two distributions. In fact, Table 3.3 displays the values of each quality, that are

Table 3: Log-likelihood, AIC, AICC, BIC and HQIC values of models fitted

Model	$\widehat{\ell}$	AIC	BIC	CAIC	HQIC
Rayleigh	-321.28	644.56	646.70	644.62	645.40
RUOD	-287.17	580.35	586.77	580.75	582.87

It is clear that the values of goodness-of-fit measures (AIC, BIC, CAIC, HQIC) within the RUOD are better than their values within Rayleigh distribution.

4. Conclusion

Modern properties such as a new pdf, df, moments, and so on are produced by combining the two selected distributions (Rayleigh distribution and Uniform distribution), which is defined as RUOD. The RUOD graphs show how the pdf graphs are skewed to the right with varying kurtosis coefficients. The goodness-of-fit to the Rayleigh distribution and the RUOD for the given data set shows that the RUOD is somewhat better than its standard distribution, since its maximum likelihood value is greater than the maximum likelihood of the R-distribution, and the values of the goodness-of-fit measures are better.

References

[1] A.A. Ahmed, *Estimation of parameters model based on a modification of newton-raphson method*, Turk. J. Comput. Math. Educ. 12(13) (2021) 1484–1491.

[2] M. Ajami and S.M.A. Jahanshahi, *Parameter estimation in weighted Rayleigh distribution*, J. Modern Appl. Statist. Methods 16(2) (2017).

[3] A.S. Hassan and M. algraphy, *Kumaraswamy weibull-generated family of distributions with applications*, Adv. Appl. Statist. 48 (2016) 205–239.

[4] C. Kim and K. Han, *Estimation of the scale parameter of the Rayleigh distribution under general progressive censoring*, J. Korean Statist. Soc. 38(3) (2009) 239–246.

[5] F. Mitrovic, *Transmuted rayleigh distribution*, Aust. J. Statist. 42(1) (2013) 21–31.

[6] A. Moesia and M.Y. shatalov, *Exact solutions for a tow-parameter rayleigh distribution*, J. Pure Appl. Math. 13(11) (2017) 8039–8051.

[7] S. Muhammad and K. Abed AL-Kadin, *Odd generalized exponential weibull exponential distribution*, J. Engin. Appl. Sci. 16(11) (2019) 10360–10386.

[8] A. Mustafa, B.S. El-Desouky and S. AL-Girahs, *Weibull generalized exponential distribution*, axis preprint arXiv:1606.07378vl [math ST]. 2 (2016).

[9] Sh. Sharma and A.J. Obaid, *Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox*, J. Interdis. Math. 23(4) (2020) 843–849.

[10] H. Torabi and N.M. Heeds, *The gamma-uniform distribution and its applications*, Cybernetic 48(1) (2012) 16–30.
[11] H. Torabi and N. Montazeri Hedesh, *The gamma-uniform distribution and its applications*, Cybernetika 48(1) (2012) 16–30.
[12] A.M. Ulhas and M. Elgar, *The odd Fréchet-G family of probability distributions*, J. Statist. Appl. Prob. 7(1) (2018) 185–201.
[13] K. Zoografts and N. Balakrishnan, *On families of beta-and generalized gamma-generated distributions and associated inference*, Statist. Method. 6(4) (2009) 344–362.