Suggested threshold to reduce data noise for a factorial experiment

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Abstract

In this research, a factorial experiment (4*4) was studied, applied in a completely random block design, with a size of 2^5 observations, where the design of experiments is used to study the effect of transactions on experimental units and thus obtain data representing experiment observations that The difference in the application of these transactions under different environmental and experimental conditions It causes noise that affects the observation value and thus an increase in the mean square error of the experiment, and to reduce this noise, multiple wavelet reduction was used as a filter for the observations by suggesting an improved threshold that takes into account the different transformation levels based on the logarithm of the base J and obtaining several values for the suggested threshold and applying then Haar wavelet function With the cut-off hard and mid threshold and Comparing the results according to several criteria.

Keywords: Design, Noise, Factorial Experiment, experimental units, Wavelet Transformation

1. Introduction

The subject of data noise is one of the very important topics that affect the results of experiments in various branches of applied statistics. This prompted the researchers to find new methods or link different methods and employ them according to the experiment in order to obtain data free of noise as much as possible, through which accurate decisions can be made in the different experiences.

Noise is an unexplained variance within data or observations [17], and The procedure followed in designing experiments is initialize the appropriate circumstances to reduce the variance occurring

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between experimental units by using the concept of blocking or repeater to group homogeneous experimental units within blocking or replicate to reduce the effect of these differences that cause noise to the observations [14], but this procedure is not controlled in many experiments and it does not eliminate all noise due to the presence of uncontrollable circumstances and variables during the experiment. For this reason, multiple wavelet reduction was used as a filter to reduce noise and for all observations of the experiment and apply Haar wavelet function with a threshold cutoff to isolate the desired signal from the undesirable one.

The selection of the threshold is very important in noise reduction algorithms, and the performance of noise reduction is directly affected by this selection; therefore; in this research, a threshold was suggested that depends on the levels of analysis and application on a factorial experiment (4*4) using the concept of Shrinking Wavelet and for the number of observations $2^j = 2^5$ and comparison between the results using several criteria.

2. Shrinking wavelet

The shrink wavelet with a threshold cutoff is used to smooth the feedback and isolate the noise E_{ij} from the data x_{ij} , as shown in the following equation [15].

$$x_{ij} = z_{ij} + E_{ij} \qquad i = 1, 2 \ldots, t \quad ; j = 1, 2, .., r. \tag{2.1}$$

Where's x_{ij} : Experiment observation, z_{ij} : Noise-free observation, E_{ij} : Represents the noise

The shrinking wavelet is highly efficient for processing noise-containing observations by separating the noise from the experiment observations depending on the wavelet coefficients, which include the Target Signal and Noise Using the appropriate threshold value for the design and applying one of the types of threshold pieces.

3. Haar Wavelet

In 1909 it was suggested by the mathematician Alfried Haar, and she is one of the simplest types of discrete wavelets, as it is built from the father function (father wavelet) and the mother function (mother wavelet), in the following form [18, 4].

Father wavelet Haar

$$\varnothing (x) = \begin{cases} 1 & if \quad 0 \leq x < 1 \\ 0 & ow \end{cases} \tag{3.1}$$

Where; s Haar scaling function coefficients $f_0 = \frac{1}{\sqrt{2}}$, $f_1 = \frac{1}{\sqrt{2}}$

$\varnothing (x) = f_0 \sqrt{2} \varnothing (2x - 0) + f_1 \sqrt{2} \varnothing (2x - 1)$

Mother wavelet Haar

$$\psi (x) = \begin{cases} 1 & if & 0 \leq x < \frac{1}{\sqrt{2}} \\ -1 & if & \frac{1}{\sqrt{2}} \leq x < 1 \\ 0 & 0w \end{cases} \tag{3.2}$$

Where; s Haar wavelet function coefficients $s_0 = f_1 = \frac{1}{\sqrt{2}}, s_1 = f_0 = -\frac{1}{\sqrt{2}}$

$\psi (x) = s_0 \sqrt{2} \varnothing (2x - 0) + s_1 \sqrt{2} \varnothing (2x - 1)$

Where it is considered the basis for all types of wavelets and it is a peaceful function that can be represented as follows:
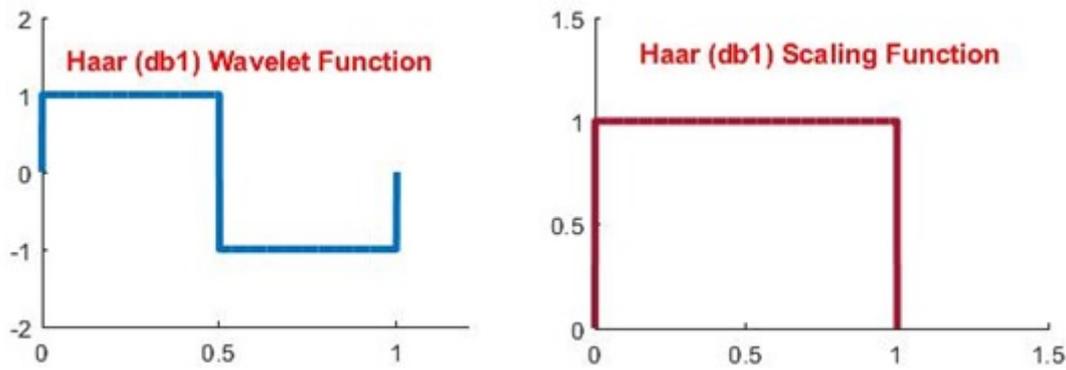
Figure 1: It shows the scale function and the wave function of a Haar wave

Through the expansion and displacement of the function $\psi(t)$, we can get the remainder of the basic functions through the principle of orthogonality by the following formula:

$$\left\langle \psi(x)_{jk}, \psi(x)_{j'k'} \right\rangle = \int_{-\infty}^{+\infty} \psi_{jk}(x)\,\psi_{j'k'}(x)\,\mathrm{dx} = 0 \tag{3.3}$$

The wavelet function must satisfy the following conditions:

$$\int_{-\infty}^{+\infty} \psi(x)\,\mathrm{dx} = 0, \qquad \int_{-\infty}^{+\infty} |\psi(x)|^2\,\mathrm{dx} = 1$$

4. Universal Threshold (UT)

The calculation of the global threshold value level depends on finding the absolute median of the wave coefficients, because the standard deviation of noise is not known in most scientific applications and therefore must be estimated. Assuming there is noise in the observation, The estimator for the standard deviation will be highly sensitive to the affected parameters [5]. This method was 4 introduced by Donoho and Jonstone. The following formula represents the calculation of the global threshold level [8, 13, 20].

$$\delta_{tu} = \widehat{\sigma}_{\mathrm{MAD}} \sqrt{2Log\ N} \tag{4.1}$$

Where's N : number of observation

$$\widehat{\sigma}_{\mathrm{MAD}} = \frac{\mathrm{median}\,|w_i|}{0.6745} \tag{4.2}$$

Where's w_i : Represents the first measurement ; median (w_i) : it is the first measurement median

5. Suggested Threshold (ST)

A threshold has been suggested that takes into account the different levels of transformation by using this concept and for several levels and based on the logarithm of the base J, which can be calculated from the experiment data.

$n = 2^J$

The suggested formula represents an Universal Threshold multiplied by the formula $\text{Log}_J(j+1)$ It can be written as follows

$$\delta_{st(j+1)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_J(j+1), \qquad j = 1, 2, \ldots, J \tag{5.1}$$

Where's Log_J : Logarithm of base J , j : Data decomposition levels

Through formula (5.1), it is possible to calculate more than one value for the suggested threshold and thus overcome the use of one threshold, and assuming that we have an experiment that contains 32 observations.

$$n = 32 = 2^J = 2^5 \ , \qquad j = 1, 2, 3, 4, 5$$

We have a number of threshold values confined during the period

$$\delta_{\text{st}(2)} < \delta_{\text{st}(3)} < \delta_{\text{st}(4)} < \delta_{\text{st}(5)} = \ Universal\ Threshold < \delta_{st} \tag{5.2}$$

Where's

$$\delta_{st(2)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_5(1+1) \tag{5.3}$$

$$\delta_{st(3)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_5(2+1) \tag{5.4}$$

$$\delta_{st(4)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_5(3+1) \tag{5.5}$$

$$\delta_{st(5)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_5(4+1) = Universal\ Threshold \tag{5.6}$$

$$\delta_{st(6)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2Log\ N} * \text{Log}_5(5+1) \tag{5.7}$$

The suggested threshold period depends on the number of trial observations.

6. Threshold rules used in the experiment

6.1. Hard Threshold

A type of threshold cutoff used to remove noise from experiment observations and It takes the following form [3, 19].

$$\text{HtW}_n = \begin{cases} 0 & if \quad W_n \leq \delta \\ W_n & if \qquad O.w \end{cases} \tag{6.1}$$

Through the formula (6.1), we note that the wavelet coefficients greater than the estimated threshold value remain unchanged. As for the coefficients that are equal to the estimated threshold value and smaller than it, it is compensated for by the value zero, and the non-zero coefficients are represented as a vector with fewer values and that's why it's called the Shrinking wavelet. In general, the Shrinking wavelet that uses the solid threshold cutoff tends to vary greatly in the estimation of the wavelet function.

6.2. Mid Threshold

It is called the intermediate cut, that is, it is between the soft and hard pieces, and it can be written in the following form [6].

$$\text{mt}W_n = sign\,[W_n]\,[|W_n| - \delta]_{++}$$
$$[|W_n| - \delta]_{++} = \begin{cases} 2\,[|W_n| - \delta] & if \quad |W_n| \ < 2\delta \\ |W_n| & if \quad O.w \end{cases} \tag{6.2}$$

It is considered a compromise between hard and soft pieces.

7. The Factorial Experiments

Are experiments in which nearly all factors are of interest to the researcher [9]. These experiments are applied to study a group of factors simultaneously according to the nature of the experimental units so that interactions between factors or effects can be identified with high efficiency, meaning that the same accuracy of effects can be achieved with fewer experiments than is required if each factor is studied in separate experiments [12]. When applying the classical methods in studying each factor separately, while keeping other factors constant [11].

There is a difference between the factor experiments and the experiments of the classes. In the Factorial experiments, the researcher is interested in studying the comparisons of the main effects and interactions. In the experiments of the classes, the comparisons are between different levels of one factor only, It is observed in agricultural experiments that the factors may interact with each other and are generally characterized by being economizing on experimental resources, These experiments allow us to estimate the main effects with the same efficiency and accuracy as if we did a complete experiment for each factor separately, On the other hand, as the size and number of treatments increase, the heterogeneity of factors increases, which leads to a decrease in the accuracy of the estimates. That is, the greater the number of treatments, the more difficult it is to measure the effect of the main feature of interest [9].

8. Tow-Factor Interaction Model

Assuming that each treatment is a combinations of factor A levels with factor B levels [1], the mathematical model of a factorial experiment contains two factors and each factor in it contains several levels.

$$x_{ij} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \rho_k + \alpha\rho_{ik} + \beta\rho_{jk} + \alpha\beta\rho_{ijk} + \varepsilon_{ijk},$$
$$i = 1, \ldots, a, \quad j = 1, \ldots, b, \quad k = 1, \ldots, r \tag{8.1}$$

Where y_{ij} : Viewing value that contains noise, μ The general arithmetic mean of the experiment, α_i The effect of factor levels A , β_j The effect of factor levels B , $\alpha\beta_{ij}$ The effect of interaction levels between factor A and levels of factor B, $\alpha\rho_{ik}$, $\beta\rho_{jk}$ and $\alpha\beta\rho_{ijk}$ are the interactions between A, B, AB and blocks , ε_{ijk} Indicates the error limit for observation.

The procedure followed in designing experiments to reduce noise, and as we mentioned at the beginning of the research, is to use the block concept to reduce the impact of differences between experimental units that cause noise to the observations [14]. That is, the work of the block is to reduce the differences between the experimental units, and what concerns the researcher, especially in the field of agricultural experiments, is to study the main effects and interactions between their levels, and therefore we assume that there are no interactions with blocks

$$(A * Block , \ B * Block, \ AB * Block) = 0.$$

This procedure increases the degrees of freedom in the denominator of the F test and thus increases the strength of the test for the main effects and interactions A, B and A B, the mathematical model would be as follows [16].

$$x_{ij} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \rho_k + +\varepsilon_{ijk}, \qquad i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, r \tag{8.2}$$

Below are the analysis of variance table for a two-factor experiment

Table 1: Sources of variance for a two-factor experiment

S.O.V	D.f	S.s	M.s	F
Blocks	$r - 1$	SS_r	MS_r	
A	$a - 1$	SS_A	MS_A	F_A
B	$b - 1$	SS_B	MS_B	F_B
AB	$(a - 1)(b - 1)$	SS_{AB}	MS_{AB}	F_{AB}
Error	$(r - 1)(t - 1)$	SS_e	MS_e	
Total	$abr - 1$	SS_T		

9. The wavelet transform of the tow-factor interaction model

The choice of blocks design is due to the difference between the experimental units which is one of the reasons for the noise of the views. Thus, obtaining inaccurate results Therefore, the experimental units are grouped into blocks that are at least homogeneous, But use blocks ,does not cancel this noise, but rather reduces it. To treat this noise, which may be between blocks or other effects, we will use the Discrete wavelet transformation to process design observations and apply a Haar wavelet with a Hard and Mid threshold cut.

9.1. The Blocks In One Area

It is intended to address the noise within the sectors due to the presence of causes, including the change in environmental conditions during the experiment and its impact on all observations in the region and the difference of experimental units between replicates or sectors, and each cause varies according to the conditions of the experiment.

To illustrate this method, we have factorial experiment 4 * 4, and the Figure 2 shows the scheme of the experiment
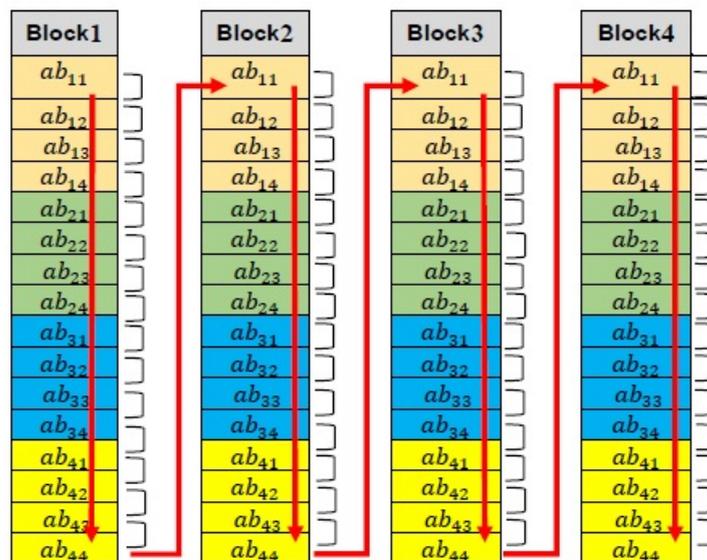

Figure 2: Experiment Diagram

We note from Figure 2 that the conversion will be done on the observations, assuming that the noise is focused on the difference in the impact of the environmental conditions in one region, in addition to the effect of the difference in the experimental units. The conversion will be done using the vector $\underline{x}$.

$$\begin{aligned}\underline{x} =[&ab_{11}\ , ab_{12},\ ab_{13}, ab_{14}, ab_{21}, ab_{22}, ab_{23}, ab_{24}, ab_{31}, ab_{32}, ab_{33}, ab_{34},\quad ab_{41}, ab_{42}, ab_{43}, ab_{44}, ab_{11}, ab_{12}, \\ &ab\ ab_{13}, ab_{14,21}\ , ab_{22}, ab_{23}, ab_{24},\ ab_{31},\ ab_{32}, ab_{33}, ab_{34}, ab_{41}, ab_{42}, ab_{43}, ab_{44}, ab_{11}\ , ab_{12},\ ab_{13}, ab_{14}, \\ &ab_{21}, ab_{22}, ab_{23}, ab_{24}, ab_{31}, ab_{32}, ab_{33}, ab_{34}, ab_{41}, ab_{42}, ab_{43}, ab_{44}, ab_{11}\ , ab_{12},\ ab_{13}, ab_{14}, ab_{21}, ab_{22}, ab_{23}, \\ &ab_{24}, ab_{31}, ab_{32}, ab_{33}, ab_{34}, ab_{41}, ab_{42}, ab_{43}, ab_{44}]\end{aligned}$$

The vector $\underline{x}$ represents the observations of the binary interaction model, this experiment has size 16 and is written according to the Discrete wavelet transform condition in the following figure

$$t * r =\ 64 = 2^6$$

By applying the discrete Haar wavelet transform to obtain the discrete wavelet coefficients as follows:

$$\underline{W} = \{\text{Haar Wavelet}\}\ \rightarrow \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ V_{j0} \end{bmatrix} \begin{array}{c} \text{Detail coefficients} \\ \text{and} \\ \hline \text{Approximation coefficient} \end{array} \tag{9.1}$$

Formula (9.1) represents the detail coefficients computed from the rate of variation of the design observations at each measurement , After that, the threshold value is calculated from the of equations (4.1) and (5.1) based on the detail coefficients of the first measurement W_1 in the following form

$$W_1 \rightarrow \left[\ \widehat{\sigma}_{\text{MAD}} = \frac{\text{median } |w_i|}{0.6745}\ \right]\ \rightarrow \left\{ \begin{array}{c} \delta_{tu} = \widehat{\sigma}_{\text{MAD}}\sqrt{2 Log\ N} \\ \delta_{sm(j+1)} = \widehat{\sigma}_{\text{MAD}}\sqrt{2 Log\ N} * \text{Log}_J (j+1) \end{array} \right\} \tag{9.2}$$

Through the formula (9.2) we have a value for the Universal Threshold with a number of values for the suggested threshold where are used in the hard and mid threshold pieces, and they are applied to the detail coefficients to separate noise from it to make a reverse transformation to get the observations of the treated experiment from noise as shown.

$$\begin{bmatrix} \text{Hard Universal Threshold} (\text{HtW}_n) \\ \text{Mid Universal Threshold} (\text{MtW}_n) \\ \text{Hard Suggested Threshold} (\text{HtW}_n) \\ \text{Mid Suggested Threshold} (\text{MtW}_n) \end{bmatrix} \rightarrow \begin{pmatrix} \underline{X}_{\text{Hhut}}\ ;\ \text{Hmtu} \\ \underline{X}_{\text{Hh2st}}\ ;\ \text{Hm2st} \\ \underline{X}_{\text{Hh3st}}\ ;\ \text{Hm3st} \\ \underline{X}_{\text{Hh4st}}\ ; \text{Hm4st} \\ \underline{X}_{\text{Hh6st}}\ ; \text{Hm6st} \end{pmatrix} . \tag{9.3}$$

Through the formula (9.3) containing 10 vectors, each vector represents the observations of the experiment, on which the Haar wavelet transformation was applied using the Universal threshold and the suggested threshold, where the vector $\underline{X}_{\text{Hhut}}$ represents the observations of the experiment, on which the Haar wavelet transform was applied with a Hard threshold cutoff and a Universal threshold value Also, the vector $\underline{X}_{\text{Hh2st}}$ represents the experiment notes A Haar wave is applied to it by cutting a hard threshold and a suggested threshold.

10. Evaluation Criteria

For the purpose of evaluating the results and comparison, criteria related to the design of experiments, in addition to criteria related to wavelet transform, were applied to the observations, and the best ones were selected according to the nature of the experiment and the design used.

First: Design evaluation criteria

Based on two evaluation criteria for the design and analysis of the experiments, the first mean square error of the experiment applied through comparison before and after the transformation

$$Mse = \frac{\text{SSe}}{\text{df}}.$$ (10.1)

Where's SSe : The sum of the squares of the error of a design df : Degrees of freedom for design error. The second criterion is the coefficient of variation(cv), which measures the residual variance in the data as a percentage of the general average of the experiment [14]. That is, the amount of variance in the variable relative to its mean, and it can be calculated using the following formula [10, 2].

$$C.V = \frac{\sqrt[2]{\text{Mse}}}{\mu} * 100$$ (10.2)

Where's Mse : The mean of the error squares for the experiment, μ : The general arithmetic mean of the experiment.

11. Wavelet Transformation Evaluation Criteria

Several criteria are used to measure the effect of noise reduction on the data, the first being the mean of squares between the original data of the experiment and the data to which the wavelet transform was applied as a filter [7].

$$\text{Mse}_{(w)} = \frac{\sum_{i=1}^{n} \left(y_i - \widetilde{y}_i\right)^2}{n}$$ (11.1)

Where's y_i: original experiment data, $\widetilde{y}_i$: Experiment data after transformation, n The number of observation of the experiment.
When criterion $\text{Mse}_{(w)}$ is small in this case, the noise reduction effect using wavelet transformation is good. If the criterion $\text{Mse}_{(w)}$ is large, the effect of noise reduction using wavelet transformation is weak [8].
The second criterion for evaluation is the signal-to-noise ratio (SNR), which is energy the data and noise , and it represents the ratio between the desired and unwanted information in the views, and it is calculated through the following formula [7].

$$SNR = 10 * \log\left(\frac{\sigma^2}{D}\right)$$ (11.2)

Where's $\sigma^2 = \frac{\sum_{i=1}^{n}(y_i - \overline{y}_i)^2}{n}$, y_i : observation value of experiment, $\overline{y}_i$: The general arithmetic mean of the experiment. $D = \frac{\sum_{i=1}^{n}(y_i - \widetilde{y}_i)^2}{n}$ $= \text{Mse}_{(w)}$ $\widetilde{y}_i$: Experiment data after transformation.
When the SNR criterion is small in this case, the noise reduction effect is low, but if the SNR criterion large, in this case, the noise reduction effect is large, that is, the work of each standard is different from the second.

12. Application

A 4 * 4 factorial experiment was conducted for cultivating broccoli and the experiment factors were

1. growth stimulator IBA contains four levels (0%,15%,30%,45%).

2. A compound fertilizer DAP that contains four levels(0%,10%,20%,30%).

The experiment contains two blocks, the size of each block is 16 experimental units, and the results are shown in the following table

Table 2: Represents the amount of production of broccoli for a factorial experiment 4*4

A	Levels(A)		B	Replicate	
			Levels(B)	Block(1)	Block(2)
		0%	0%	65.84	77.58
			15%	72.28	75.36
			30%	68.87	70.69
			45%	76.09	93.83
		10%	0%	88.4	92.62
			15%	85.89	85.23
			30%	85.77	94.38
			45%	93.66	90.92
		20%	0%	74.62	79.6
			15%	70.98	80.66
			30%	90.82	72.65
			45%	81.06	81.11
		30%	0%	60.69	77.42
			15%	78.54	78.7
			30%	91.62	73.64
			45%	94.61	91.87

By applying equations (4.1), (5.1)-(6.1) and (10.1)-(11.2) to the experiment observations, the results were as in the following table.

Table 3: Comparison results for the Hard cut-off threshold with the Universal threshold and suggested

TH	Observation	Mse	$Mse_{(w)}$	Cv	SNR	**
	X	51.374		0.0883		A,B
UT	X_{Hmth}	29.008	25.1853	0.0663	5.2497	A,B
ST	X_{Hm2st}	48.781	6.647	0.0860	11.0345	A,B
ST	X_{Hm3st}	43.471	11.626	0.0812	8.6068	A,B
ST	X_{Hm4st}	37.932	16.818	0.0759	7.0032	A,B
ST	X_{Hm6st}	29.008	25.185	0.0663	5.249	A,B

13. Results

1- The following comments aim to link the outputs in Table 3 with theoretical developments. We note that the use of the discrete wavelet transform led to a decrease in the value of the mean error squares MSe of the design used by applying the Haar wavelet transform with the hard-threshold

cutoff, in addition to obtaining low values for the $\text{Mse}_{(w)}$ criterion. In the case of using the suggested threshold with an increase in the value of the SNR criterion, and this indicates a significant improvement in reducing data noise through the criteria used and the charts from (1-4) explain each criterion.
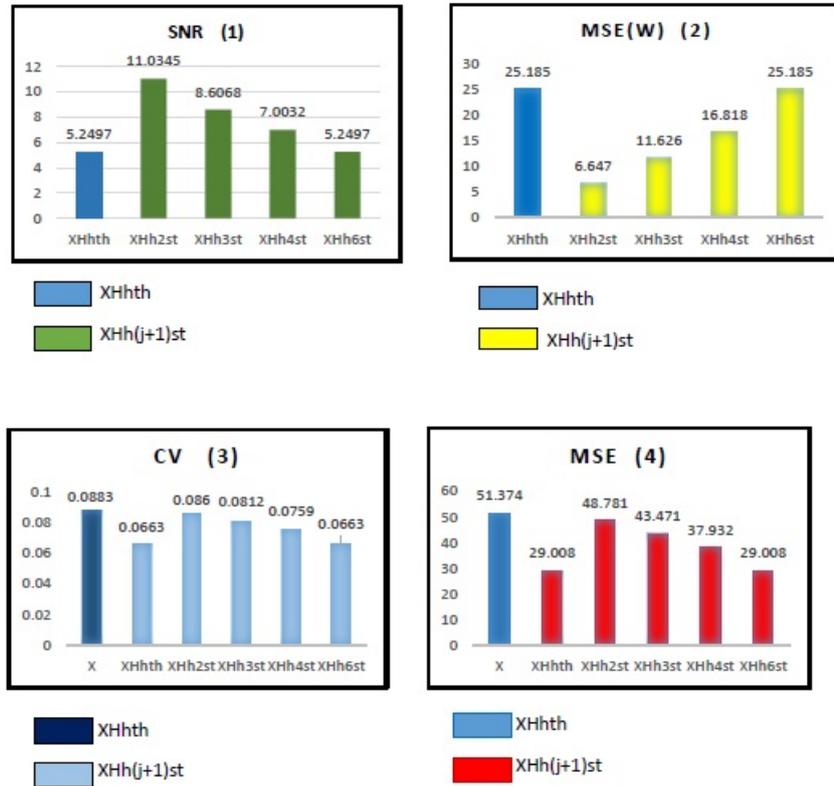

Figure 3: the criteria using the cut-off Hard threshold with Universal threshold and suggested

We notice from Figure 3, which contains diagrams (1-4), a significant improvement in the criteria values in the case of using the discrete wavelet transform as a filter for the data, especially in the case of applying the suggested threshold.
By applying equations (4.1), (5.1)-(5.7), (6.2) and (10.1) -(11.2) to the experiment observations, the results were as in the following table.

Table 4: Comparison results for the mid cut-off threshold with the Universal threshold and suggested

TH	Observation	Mse	$Mse_{(w)}$	Cv	SNR	**
	X	51.374		0.0883		A,B
UT	X_{Hmth}	29.008	25.1853	0.0663	5.2497	A,B
ST	X_{Hm2st}	37.932	16.818	0.0759	7.003	A,B
ST	X_{Hm3st}	29.008	25.185	0.0663	5.249	A,B
ST	X_{Hm4st}	29.008	25.185	0.06637	5.249	A,B
ST	X_{Hm6st}	29.008	25.185	0.0663	5.249	A,B

The following comments aim to link the outputs in Table 4 with theoretical developments. We note that the use of the discrete wavelet transform led to a decrease in the value of the mean error squares MSe of the design used by applying the Haar wavelet transform with the mid cut-off threshold, in addition to obtaining low values for the $\text{Mse}_{(w)}$ criterion. In the case of using the

suggested threshold with an increase in the value of the SNR criterion, and this indicates a significant improvement in reducing data noise through the criteria used and the charts from (4.1)-(5.2) explain each criterion.
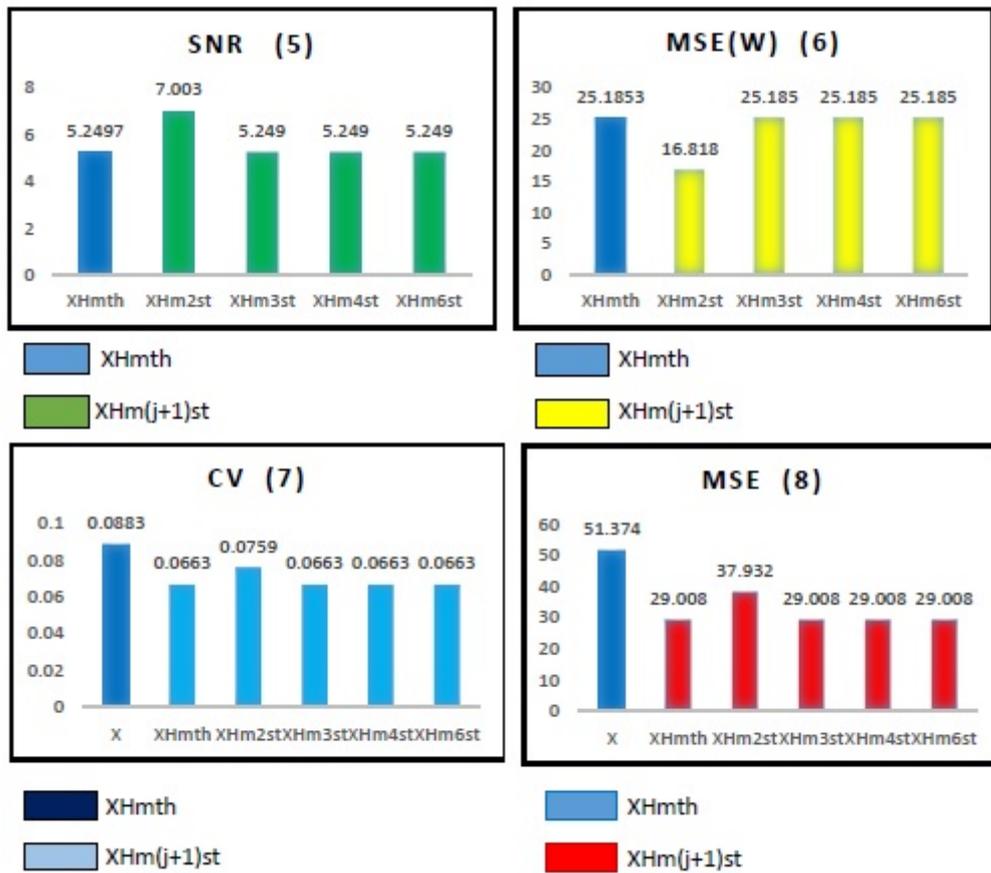

Figure 4: The criteria using the cut-off mid threshold with Universal threshold and suggested

We notice from Figure 4, which contains diagrams (5-8), a significant improvement in the criteria values in the case of using the discrete wavelet transform as a filter for the data, especially in the case of applying the suggested threshold.

14. Conclusion

1. When using hard threshold cut and a mid threshold cut with an Universal threshold to reduce data noise based on the criteria (10.1), (10.2), (11.1) and (11.2). There is an improvement in all the criteria.

2. When using hard threshold cut with Suggest threshold. to reduce data noise based on the criteria (10.1), (10.2), (11.1) and (11.2) better results were obtained from the Universal threshold represented by the following.

   $\underline{X}_{\text{Hh2st}}$: Observations vector processed with a hard threshold cut with suggested threshold and j=2.

   $\underline{X}_{\text{Hh3st}}$: Observations vector processed with a hard threshold cut with suggested threshold and j=3.

$\underline{X}_{\text{Hh4st}}$: Observations vector processed with a hard threshold cut with suggested threshold and j=4.

3. When using mid threshold cut with Suggest threshold. to reduce data noise based on the criteria (10.1), (10.2), (11.1) and (11.2) Better results were obtained from the Universal threshold represented by the following.

$\underline{X}_{\text{Hm2st}}$: Observations vector processed with a mid threshold cut with suggested threshold and j=2.

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