Int. J. Nonlinear Anal. Appl. 14 (2023) 2, 133–140
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.6404

Some dominating results of the topological graph

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(Communicated by Madjid Eshaghi Gordji)

Abstract

Let $G_\tau = (V, E)$ be a topological graph constructed from the topological space (X, τ). In this paper, several types of dominating parameters are applied on the topological graph G_τ. Such as independent domination, total domination, connected domination, doubly connected domination, restrained domination, strong domination and weak domination. Also, the inverse domination of all these parameters was proved.

Keywords: Topological graph, dominating set, domination number, inverse domination, connected graph.
2010 MSC: 05C69

1 Introduction

The graph is $G = (V, E)$ where $V(G)$ is a set contains all vertices of G, and $E(G)$ is a set contains all edges of G. The degree of vertex u is the number of all edges that incident on it. The maximum degree denoted by $\Delta(G)$ and the minimum degree denoted by $\delta(G)$. For any two vertices $w, v \in V(G)$ are adjacent vertices if there is an edge between them, otherwise they are not adjacent. The graph G is called connected graph if for any two vertices w, v belong to $V(G)$ there is a path between them. The vertex of degree 0, is called isolated vertex, to learn more see [22]. The subset D is called dominating set if for each vertex of $V - D$ is adjacent to one or more vertices of D. The domination number $\gamma(G)$ is the order of a minimum dominating set. Let G be a graph with a minimum dominating set of D. Such that the subset of $V - D$ is an inverse dominating set with respect to D, if it is also dominating set in G denoted by D^{-1}. The inverse domination number denoted by $\gamma^{-1}(G)$ is the order of a minimum inverse dominating set [32]. For more information about domination, see references [23, 24, 25]. The subset D is independent dominating set if all vertices of $G[D]$ are isolated vertex [21]. The subset D is a total dominating set if $G[D]$ has no isolated vertex [19]. Also, it is called a connected dominating set if $G[D]$ is connected [36]. And D is a doubly connected if both $G[D]$ and $G[V - D]$ are connected subgraphs [20]. A dominating set D is called a restrained dominating set if every vertex in $V - D$ is adjacent to another vertex in $V - D$[33]. Let $wv \in E(G)$ and w, v dominate each other, if $\deg(w) \geq \deg(v)$. This mean the vertex w strongly dominates v and v weakly dominates w. The subset D is strong dominating set if every vertex in $V - D$ is strongly dominated by at least one vertex in D. Also, it is called weak dominating set if every vertex in $V - D$ is weakly dominated by at least one vertex in D [37]. To learn more about parameters of domination, see references [1]-[18],[26]-[31],[34, 35]. Many properties of the discrete topological graph are discussed by us in [30]. And studied the general domination of G_τ in [29]. Here, many parameters of domination are applied on the topological graph and study the inverse for each dominating parameters.

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Received: January 2022 *Accepted:* March 2022

2 Some General Properties of Topological Graph

In this section, some properties of the discrete topological graph G_τ are given.

Definition 2.1. [30] Let X be a non-empty set and τ be a discrete topology on X. The discrete topological graph denoted by $G_\tau = (V, E)$ is a graph of the vertex set $V = \{A; A \in \tau \text{ and } A \neq \emptyset, X\}$ and the edge set $E = \{AB; A \nsubseteq B \text{ and } B \nsubseteq A\}$.

Proposition 2.2. [30] Let $|X| = n$ and G_τ be a discrete topological graph. Then, the graph G_τ has $n-1$ complete induced subgraphs.

Theorem 2.3. [30] Let G_τ be a discrete topological graph of a non-empty set X. Then, G_τ is a connected graph.

Theorem 2.4. [30] Let $|X| = n$, then

1) $\delta(G_\tau) = \sum_{i=1}^{n-1} \binom{n-1}{i}$, for $n \geq 3$.

2) $\Delta(G_\tau) = \binom{n}{\lceil \frac{n}{2} \rceil} - 1 + \sum_{i=1}^{\lceil \frac{n}{2} \rceil - 1} \left[\binom{n}{i} - \binom{\lceil \frac{n}{2} \rceil}{i} \right] + \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n-1} \left[\binom{n}{i} - \binom{n - \lceil \frac{n}{2} \rceil}{i - \lceil \frac{n}{2} \rceil} \right]$, for $n \geq 4$

3) $\delta(G_\tau) = \Delta(G_\tau) = 3$, if and only if $n = 3$.

Theorem 2.5. [29] Let G_τ be a discrete topological graph where $|X| = n$. Then, G_τ has dominating set where $\gamma(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Theorem 2.6. [29] Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has inverse dominating set and $\gamma^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

3 New results of domination in the topological graph

In this section, many types of domination. Such as: independent domination, total domination, connected domination, doubly connected domination, restrained domination, strong domination and weak domination are applied on the topological graph G_τ. Inverse domination for all previous types are studied also.

Theorem 3.1. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has an independent dominating set and $\gamma_i(G_\tau) = n - 1$.

Proof . Let $D \subseteq V(G_\tau)$, so for all $u, v \in D$ thus u not adjacent to v according to the definition of independent dominating set. Let S be a set of all vertices of singleton element such that $|S| = n$. Then, $G[S] = K_n$ from proof of Proposition 2.2. Thus, for any vertex in S say u_1 dominates all vertices of S, so that $u_1 \in D$. Again, let S' be a set of all vertices that have two elements such that $|S'| = \binom{n}{2}$. So, from proof of Proposition 2.2, then $G[S'] = K_{\binom{n}{2}}$. Thus, there is at least one vertex in S' say u_2 not adjacent to u_1. So that, u_2 dominates all vertices of a set $S', u_2 \in D$. Also, if S'' be a set of all vertices that have three elements such that $|S''| = \binom{n}{3}$. Thus, $G[S''] = K_{\binom{n}{3}}$ from proof of Proposition 2.2, so that there is at least one vertex in a set S'' say u_3 not adjacent with u_1 and u_2. Since $u_3 \in K_{\binom{n}{3}}$, then u_3 dominates all vertices of a set S'' so that $u_3 \in D$ and so on. The last step if S^{n-1} be a set of all vertices that have $n-1$ elements such that $\left|S^{n-1}\right| = \binom{n}{n-1} = n$. Hence, $G\left[S^{n-1}\right] = K_n$ from proof of Proposition 2.2, again there is at least one vertex in a set S^{n-1} say u_{n-1} not adjacent with vertices

$u_1, u_2, u_3, \ldots, u_{n-2}$. Since $u_{n-1} \in K_n$, then the vertex u_{n-1} dominates all vertices of a set S^{n-1} so that $u_{n-1} \in D$ and $D = \{u_1, u_2, u_3, \ldots, u_{n-1}\}$. Now, to prove set D is minimum independent dominating set. Assume that D is not a minimum independent dominating set. So, there exist a minimum dominating set D' such that $|D'| < |D|$. Then, there exist one or more vertices in $V - D$ do not dominated by any vertex of D'. So, it is clear that D' is not a minimum independent dominating set. Therefore, D is an independent dominating set and $\gamma_i(G_\tau) = n - 1$, as an example. See Figure 1 . □

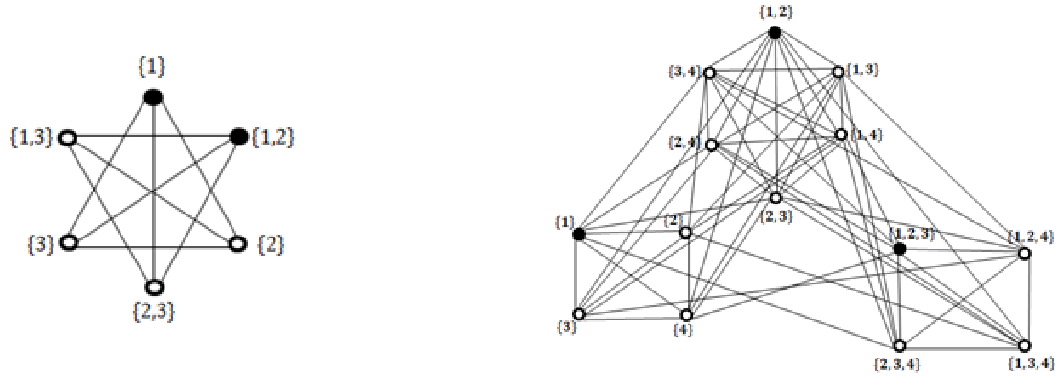

Figure 1: The independent dominating set in G_τ for $|X| = 3, 4$.

Theorem 3.2. Let G_τ be a discrete topological graph, where $|X| = n$. Then, G_τ has inverse independent dominating set and $\gamma_i^{-1}(G_\tau) = n - 1$.

Proof . By the same technique of proof of Theorem 3.1. Let $D^{-1} = \{v_1, v_2, v_3, \ldots, v_{n-1}\}$ such that all vertices of $G\left[D^{-1}\right]$ are isolated. Thus, D^{-1} is an inverse independent dominating set and $\gamma_i^{-1}(G_\tau) = n - 1$. See Figure 2. □

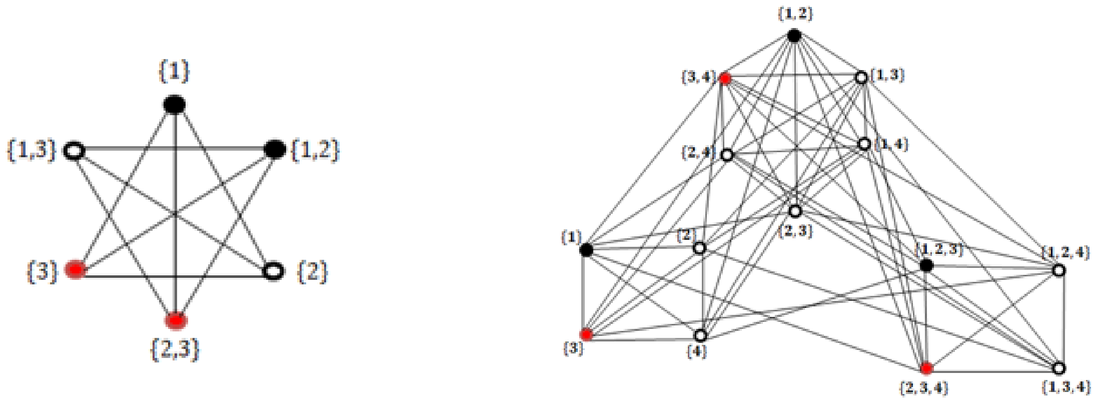

Figure 2: The inverse independent dominating set, for $|X| = 3, 4$.

Proposition 3.3. If $|X| = 2$, then the discrete topological graph G_τ has no total dominating set.

Proof . Since $D = \{u\}$ for $n = 2$, from proof of Theorem 2.5, $G[D]$ has isolated vertex and G_τ has no total dominating set. □

Proposition 3.4. Let $|X| = n (n \geq 3)$ and G_τ be a discrete topological graph. Then, G_τ has total dominating set and $\gamma_t(G_\tau) = 2$.

Proof . By the same technique of proof of Theorem 2.5, let $D = \{u, u^c\}$. Since $u \nsubseteq u^c$ and $u^c \nsubseteq u$, there is an edge between them. Thus, $G[D]$ form a path and D has no isolated vertex. Hence, D is a total dominating set and $\gamma_t(G_\tau) = 2$, as an example. See Figure 3(a). □

Proposition 3.5. Let $|X| = n(n \geq 3)$ and G_τ be a discrete topological graph. Then, G_τ has inverse total dominating set and $\gamma_t^{-1}(G_\tau) = 2$.

Proof . By the same technique of proof of Proposition 3.4. Let $D^{-1} = \{w, w^c\}$ such that $G\left[D^{-1}\right]$ form a path. So, $G\left[D^{-1}\right]$ has no isolated vertex. Hence, D^{-1} is an inverse total dominating set and $\gamma_t^{-1}(G_\tau) = 2$. See Figure 3$(b)$. □

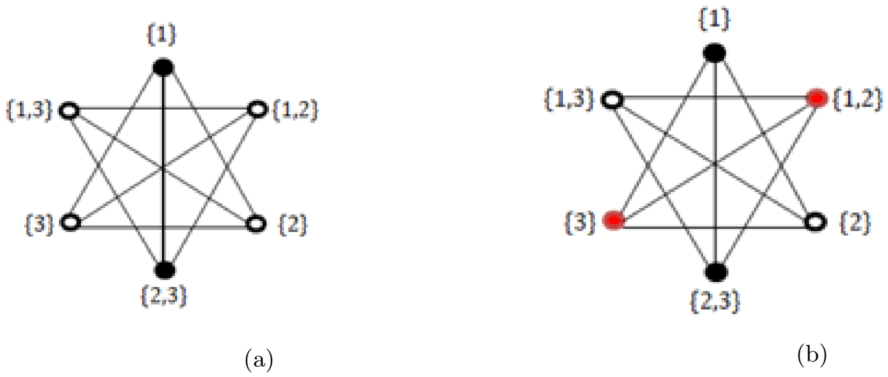

Figure 3: The total domination and inverse total domination of $\overline{C_6}$.

Proposition 3.6. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has a connected dominating set and $\gamma_c(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . From proof of Theorem 2.5, for $n = 2$ the induced subgraph $G[D]$ has only one isolated vertex. And for $n > 2$, since $G[D]$ form a path from proof of Proposition 3.4, $G[D]$ is connected for all value of n. See Figure 3(a). □

Proposition 3.7. Let $|X| = n$, then G_τ has inverse connected dominating set and $\gamma_c^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . From proof of Theorem 2.6, for $n = 2$ since $G\left[D^{-1}\right]$ has only one isolated vertex and its form a path for $n > 2$ from proof of Proposition 3.5. Then, $G\left[D^{-1}\right]$ is connected for all value of n. See Figure 3(b). □

Proposition 3.8. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has a doubly connected dominating set where $\gamma_{cc}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . By the same technique of proof of Theorem 2.5. Let $D = \{v\}$ for $n = 2$ and Proposition 3.4, let $D = \{u, u^c\}$ for $n > 2$. Thus, $G[D]$ is connected for all value of n from proof of Proposition 3.6. Now in the induced subgraph $G[V-\text{ D}]$ and by the same technique of proof of Theorem 2.3, $G[V - D]$ is connected. Hence, D is doubly connected dominating set. See Figure 3(a). □

Proposition 3.9. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has inverse doubly connected dominating set where $\gamma_{cc}^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . By the same technique of proof of Theorem 2.6. Let $D^{-1} = \{u\}$ for $n = 2$ and Proposition 3.5, let $D^{-1} = \{w, w^c\}$ for $n > 2$. So, from proof of Proposition 3.7, $G\left[D^{-1}\right]$ is connected for all value of n. Also, by the same technique of proof of Theorem 2.3, in $G\left[V - D^{-1}\right]$. Then, we get the induced subgraph $G\left[V - D^{-1}\right]$ is connected. Hence, D^{-1} is inverse doubly connected dominating set. See Figure 3(b). □

Proposition 3.10. Let $|X| = n(n > 2)$ and G_τ be a discrete topological graph. Then, G_τ has restrained dominating set such that $\gamma_r(G_\tau) = 2$.

Proof . If $n = 2$, then it is clear $G[V - D]$ has one isolated vertex and has no restrained dominating set. If $n > 2$ by the same technique of proof of Theorem 2.5, let $D = \{u, u^c\}$. Since $G[V - D]$ is connected from proof of Proposition 3.8, $G[V - D]$ has no isolated vertex. Thus, D is restrained dominating set and $\gamma_r(G_\tau) = 2$. See Figure 4 . □

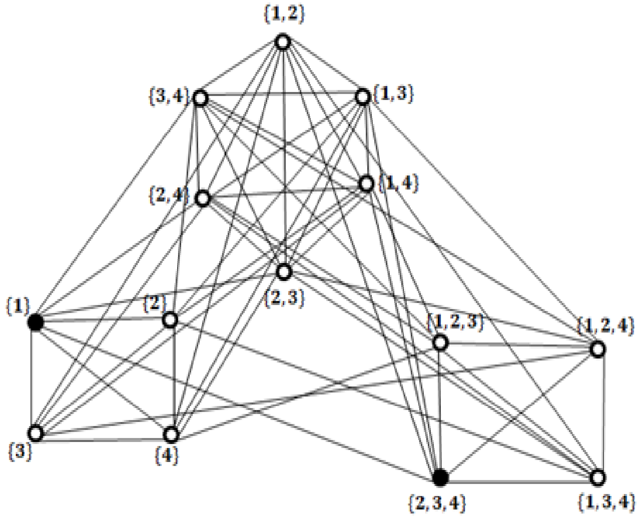

Figure 4: The restrained dominating set for $|X| = 4$.

Proposition 3.11. Let $|X| = n(n > 2)$ and G_τ be a discrete topological graph. Then, G_τ has inverse restrained dominating set such that $\gamma_r^{-1}(G_\tau) = 2$.

Proof . If $n = 2$, so it is clear $G\left[V - D^{-1}\right]$ has one isolated vertex and D^{-1} has no inverse restrained dominating set. Also, by similar to proof of Theorem 2.6, let $D^{-1} = \{w, w^c\}$ for $n > 2$. And since $G\left[V - D^{-1}\right]$ connected from proof of Proposition 3.9, we get $G\left[V - D^{-1}\right]$ has no isolated vertex. Hence, D^{-1} is inverse restrained dominating set and $\gamma_r^{-1}(G_\tau) = 2$. See Figure 5 . □

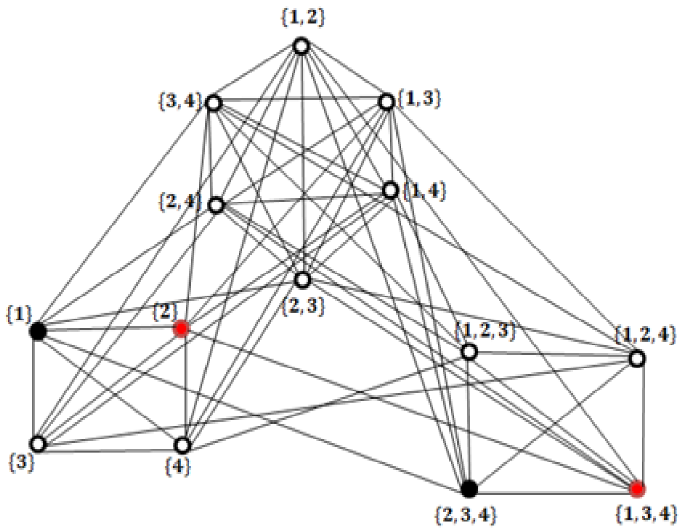

Figure 5: The inverse restrained dominating set for $|X| = 4$.

Proposition 3.12. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has strong dominating set and $\gamma_s(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . If $n = 2$, then by similar to proof of Theorem 2.5, let $D = \{u\}$. So, it is clear a strong domination number for K_2 is one. Hence, $\gamma_s(G_\tau) = 1$. If $n > 2$ also by the same technique of proof of Theorem 2.5, let $D = \{u, u^c\}$. Since the maximum degree is founded in each vertex have $\frac{n}{2}$ elements by Theorem 2.4, (2). So, there is two casas as following:

Case 1: If order of X is odd number, let u is a vertex have $\lfloor \frac{n}{2} \rfloor$ elements. Thus, u^c is a vertex of $\lceil \frac{n}{2} \rceil$ elements.

Case 2: If order of X is even number. Let u is a vertex have $\frac{n}{2}$ elements, then u^c also vertex have $\frac{n}{2}$ elements.

From two cases above these two vertices of D have a maximum degree and dominate all vertices have degree less than or equal to them. So, for any $v \in V - D$ is strongly dominated by these one or two vertices. Such that $\deg(u) = \deg(u^c) \geq \deg(v)$. Therefore, D is strong dominating set and $\gamma_s(G_\tau) = 2$. See Figure 3(a) and Figure 6. $\square$

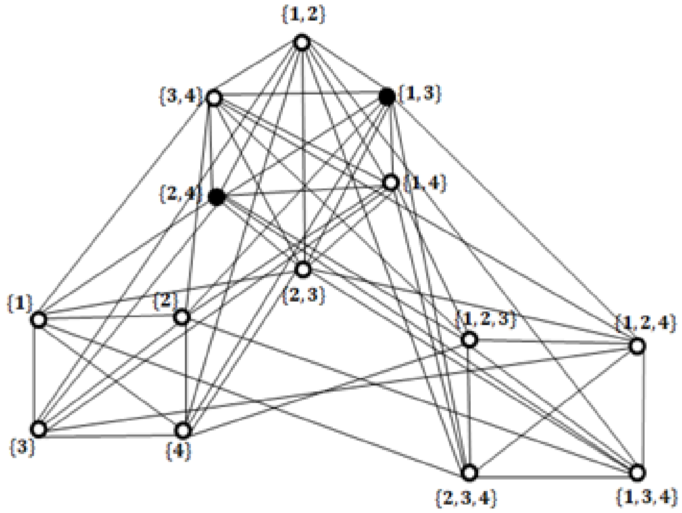

Figure 6: The strong dominating set, where $|X| = 4$.

Proposition 3.13. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has inverse strong dominating set and $\gamma_s^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.

Proof . By the same technique of proof of Proposition 3.12. Let $D^{-1} = \{u\}$ for $n = 2$ and let $D^{-1} = \{w, w^c\}$ for $n > 2$. So, for any $t \in V - D^{-1}$ is strongly dominated by these one or two vertices. And $\deg(w) = \deg(w^c) \geq \deg(t)$. Then, D^{-1} is inverse strong dominating set and $\gamma_s^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$. See Figure 3$(b)$ and Figure 7 . $\square$

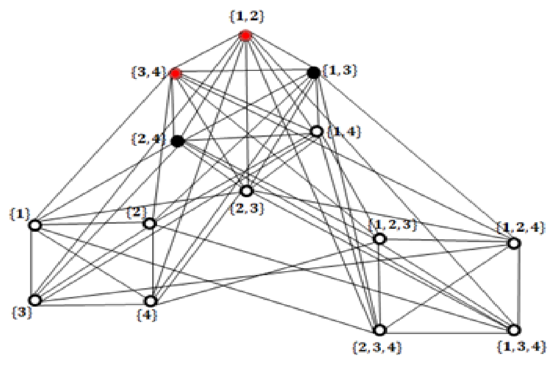

Figure 7: The inverse strong dominating set, for $|X| = 4$.

Proposition 3.14. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has weak dominating set and $\gamma_w(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n = 2 \\ 2, \text{ if } n > 2 \end{array} \right\}$.—

Proof . If $n = 2$, then by similar to proof of Theorem 2.5, let $D = \{v\}$. So, it is clear a weak domination number of K_2 is one. Hence, $\gamma_w(G_\tau) = 1$. If $n > 2$, then the same technique of proof of Theorem 2.5, let $D = \{u, u^c\}$. Since the minimum degree of G_τ exist in each vertex of singleton element and each vertex have $n-1$ elements from proof of Theorem 2.4 (1). So, let u is a vertex of singleton element, then u^c is a vertex of $n-1$ elements. Hence, D has two vertices of minimum degree and it is dominates all remaining vertices that have degree bigger than or equal to them. Thus, for any $t \in V-D$ is weakly dominated by the vertices of D such that $\deg(t) \geq \deg(u) = \deg(u^c)$. Therefore, D is weak dominating set and $\gamma_w(G_\tau) = 2$. See Figure 3(a) and Figure 4 . □

Proposition 3.15. Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has inverse weak dominating set where $\gamma_w^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n=2 \\ 2, \text{ if } n>2 \end{array} \right\}$.

Proof . By the same technique of proof of Proposition 3.14. Let $D^{-1} = \{u\}$ for $n=2$ and let $D^{-1} = \{w, w^c\}$ for $n>2$. So, for any $v \in V - D^{-1}$ is weakly dominated by the vertices of D^{-1} which is $\deg(v) \geq \deg(w) = \deg(w^c)$. Hence, D^{-1} is inverse weak dominating set and $\gamma_w^{-1}(G_\tau) = \left\{ \begin{array}{l} 1, \text{ if } n=2 \\ 2, \text{ if } n>2 \end{array} \right\}$. See Figure 3($b$) and Figure 5 . □

4 Conclusions

Many parameters of domination are applied on the discrete topological graph G_τ. Such as: the independent dominating set, total dominating set, restrained dominating set, strong dominating set, weak dominating set, connected dominating set and doubly connected dominating set. The inverse domination of all previous types are proved.

5 Open problems

Study other types of domination on the topological graph G_τ, like co-independent domination, complementary tree domination, bi-domination, triple effect domination and pitchfork domination.

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