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Stability and some results of triple effect domination

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Abstract

Let $G=(V,E)$ be a graph without isolated vertices. A subset $D \subseteq V$ is a triple effect dominating set if every vertex in D dominates exactly three vertices of $V-D$. The triple effect domination number $\gamma_{te}(G)$ is the minimum cardinality of overall triple effect dominating sets in G. In this paper, the triple effect domination number $\gamma_{te}(G)$ is studied to be changing or not after adding or deleting edge or deleting vertex. Some conditions are put on the graph to be affected or not with several results and examples. Then, the triple effect domination and its inverse are applied on several graphs obtained from complement, join and corona operations.

Keywords: Dominating set, triple effect domination, inverse triple effect domination.
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1 Introduction

Let $G=(V,E)$ with order n and size m, such that n the number of all vertices in G and m the number of all edges in G. The degree of any vertex v in G is the number of edges incident on v and denoted by $\deg(v)$. If $\deg(v)=0$, then v is said isolated vertex. $\Delta(G)$ is the maximum degree in G and $\delta(G)$ is the minimum degree in G. Link two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$, such that induced a graph having the vertex set $V_1 \cup V_2$ and the edge set $E_1 \cup E_2 \cup \{v_1v_2 : \forall v_1 \in V_1, v_2 \in V_2\}$. This linking is called join operation and denoted by G_1+G_2, see [17] for theoretic terminology and basic concepts of graph. In G, a set D of V is said a dominating set if every vertex out of it, is adjacent to one vertex or more of D. For more details see [18, 19, 20]. There are many papers deals different types of domination, such as [1]-[12],[15, 16], [21]-[27]. In previous paper [13], authors studied a new model of domination called triple effect domination and put several theorems and properties of this model. Also, they studied the inverse triple effect domination and given several properties in [14]. In this paper, the triple effect domination number $\gamma_{te}(G)$ is studied to be changing or not after adding or deleting edge or deleting vertex. Some conditions are putted on the graph to be affected or not. Then, the triple effect domination and its inverse is applied on several graphs obtained from complement, join and corona operations.

2 Stability of triple effect domination number

We delete a vertex or edge or adding an edge to any graph G to study the changing on the triple effect domination number $\gamma_{te}(G)$. So, if $G-v$ has a triple effect dominating set, then v belongs to $(V^0 \cup V^+ \cup V^-)$, such that: $V^0 =$

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$\{v \in V : \gamma_{te}(G-v) = \gamma_{te}(G)\}$, $V^{+} = \{v \in V : \gamma_{te}(G-v) > \gamma_{te}(G)\}$ and $V^{-} = \{v \in V : \gamma_{te}(G-v) < \gamma_{te}(G)\}$. Similarly, edges set can be partitioned as:

$$E_{*}^{0} = \{e \in E : \gamma_{te}(G * e) = \gamma_{te}(G)\}, E_{*}^{+} = \{e \in E : \gamma_{te}(G * e) > \gamma_{te}(G)\}$$

and

$$E_{*}^{-} = \{e \in E : \gamma_{te}(G * e) < \gamma_{te}(G)\}$$

, where $* = \begin{cases} -, & e \in G \\ +, & e \in \bar{G} \end{cases}$

Theorem 2.1. For some graphs G having γ_{te}-set, there exists a vertex v such that $G-v$ has a triple effect domination, then $V_{-}^{*} \neq \emptyset$, where $* = 0$ or $-$ or $+$.

Proof .

A. Assume that D is $\gamma_{te}-$ set in G, when $v \in D$ we show that V_{-}^{*} is non-empty set as follows:

Case 1: $V_{-}^{0} \neq \emptyset$: Let $v \in D$, a vertex v dominates three vertices say w_1, w_2 and w_3 in $V-D$, such that only one vertex say $w_1 \in pn[v, D]$ and it is adjacent to exactly three vertices in $V-D$, then we can add this vertex w_1 to set $D-\{v\}$. It is obvious that $(D-\{v\}) \cup \{w_1\}$ is a minimal triple effect dominating set and $\gamma_{te}(G-v) = \gamma_{te}(G)$. For example see Fig.1.(a).

Case 2 $\cdot V_{-}^{+} \neq \emptyset$: Let w_1, w_2 and w_3 are adjacent vertices and belong to $pn[v, D]$. Each one of w_2 and w_3 is adjacent to exactly two vertices in $V-D$ without w_1. Then, we add the vertices w_2 and w_3 to the set $D-\{v\}$. It is obvious that $D = (D-\{v\}) \cup \{w_2, w_3\}$ is a minimal triple effect dominating set and $\gamma_{te}(G-v) > \gamma_{te}(G)$. For example see Fig.1.(b).

Case 3. $V_{-}^{-} \neq \emptyset$: If the three vertices w_1, w_2 and w_3 are dominated by other vertices in D, then $\gamma_{te}(G-v) < \gamma_{te}(G)$ and $V_{-}^{-} \neq \emptyset$. For example see Fig.1.(c).

B: When $v \in V-D$, we show that V_{-}^{*} is non-empty set as follows:

Case 1: $V_{-}^{0} \neq \emptyset$: The vertex in D that dominates the vertex v say u dominates other two adjacent vertices in $V-D$ say w_1 and w_2. Any vertex from $\{w_1, w_2\}$ is adjacent to exactly one vertex in $(V-D)-\{v\}$, then we can take it instead of the vertex u in D. Hence, $\gamma_{te}(G-v) = \gamma_{te}(G)$. For example see Fig.1.(d).

Case 2: $V_{-}^{+} \neq \emptyset$: The vertex in D that dominates the vertex v say u dominates other two non-adjacent vertices in $V-D$ say w_1 and w_2. Such that $w_1, w_2 \in pn[u, D]$, where at least one of $\{w_1, w_2\}$ are adjacent to exactly two vertices in $(V-D)-\{v\}$, then we can take them instead of the vertex u. For example see Fig.1.(e).

Case 3: $V_{-}^{-} \neq \emptyset$: If the vertex v is dominated by two vertices in D say u_1 and u_2 and its adjacent other two vertices from $V-D$. Such that u_1 adjacent with u_2 and dominates v and other two vertices from $V-D$, and $pn[u_1, D] = \{u_2\}$, and u_2 adjacent with u_1 and dominates v and other two vertices from $V-D$. When we delete v, then u_1 dominates u_2 and other two vertices from $V-D$. Then $\gamma_{te}(G- v) < \gamma_{te}(G)$. For example see Fig.1.(f).

□

Theorem 2.2. For any graph G having γ_{te}-set. If $e \notin E$ and $G+e$ having a triple effect domination, then $E_{+}^{*} \neq \emptyset$, where $* = 0$ or $-$ or $+$.

Proof . If e is added to $G[D]$ or $G[V-D]$, it is obvious that the set D is not influenced by such addition, i.e. $\gamma_{te}(G) = \gamma_{te}(G+e)$. Thus $e \in E^{0}$.

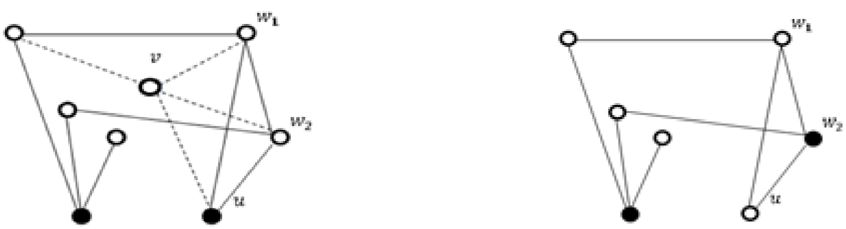
If e is added to G, such that one vertex incident with e say v belongs to D and the other vertex say w belongs to $V-D$. Let u_1, u_2 and u_3 are three vertices in $V-D$ which are dominated by vertex v. Assume that D is a $\gamma_{te}-$ set, then we show that E_{+}^{*} is non-empty set as follows:

Case 1: $E_{+}^{0} \neq \emptyset$.If the vertex v dominates three vertices in $V-D$ say u_1, u_2 and u_3 and we add an edge e between v and w, such that $w \in V-D$ is adjacent with u_2 and u_3. Where u_1 is dominated by other vertex from D and every vertex in D that dominates w is adjacent with v. When we add $e = vw$, then w will replace the vertex v in the set D. Therefore, $\gamma_{te}(G+e) = \gamma_{te}(G)$. For example see Fig.2.(a).

(a) $\gamma_{te}(G-v)=\gamma_{te}(G)$

(b) $\gamma_{te}(G-v)>\gamma_{te}(G)$

(c) $\gamma_{te}(G-v)<\gamma_{te}(G)$



(d) $\gamma_{te}(G-v)=\gamma_{te}(G)$

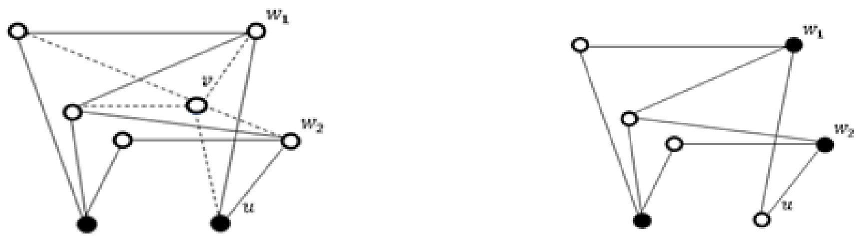
Case 2: $E_+^+ \neq \emptyset$. If the vertex v dominates three vertices in $V-D$ say u_1, u_2 and u_3 and we add an edge e between v and w, such that $w \in V-D$ adjacent with all vertices that dominated by v. And u_1, u_2 and u_3 are adjacent such that $u_3 \in pn[v,D]$, when we add $e=vw$, then w and u_3 will replace the vertex v in the set D. Therefore, $\gamma_{te}(G+e)>\gamma_{te}(G)$. For example see Fig.2.(b).

Case 3: $E_+^- \neq \emptyset$. If the vertex v dominates three adjacent vertices in $V-D$ say u_1, u_2 and u_3 and we will add an edge e between v and w, such that $w \in V-D$ adjacent with u_1 only and dominated by a vertex say x and v don't adjacent with any vertex from D. when we add $e=vw$, then w will replace the vertex v and dominates x, therefore $\gamma_{te}(G+e)<\gamma_{te}(G)$. For example see Fig.2.(c). □

Theorem 2.3. For any graph G having $\gamma_{te}-$ set. If $e \in E$ and $G-e$ having a triple effect domination, then $E_-^* \neq \emptyset$, where $*=0$ or $-$ or $+$.

Proof . If e is deleted from $G[D]$ or $G[V-D]$, it is obvious that the set D is not influenced by such deletion, i.e. $\gamma_{te}(G)=\gamma_{te}(G+e)$. Thus $e \in E_-^0$. If e is deleted from G, such that one vertex incident with e say v belongs to D and the other vertex say w belongs to $V-D$ and let u_1, u_2 and u_3 are three vertices in $V-D$ which are dominated by vertex v. Assume that D is a $\gamma_{te}-$ set, then we show that E_-^* is non-empty set as follows:

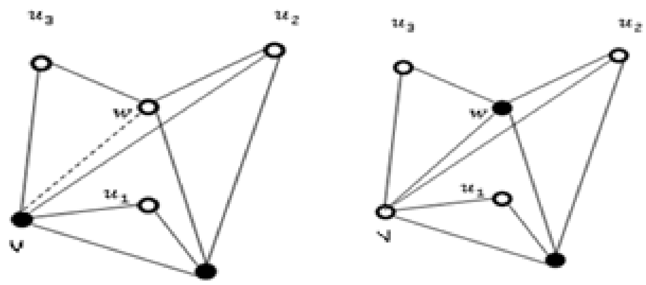
Case 1: $E_-^0 \neq \emptyset$. If the vertex v dominates three vertices in $V-D$ say u_1, u_2 and u_3 and we will delete an edge e between v and u_2, such that $u_2 \in V-D$ and adjacent to u_1 and u_3 and don't dominate by any vertex from D without



(e) $\gamma_{te}(G-v) > \gamma_{te}(G)$

(f) $\gamma_{te}(G-v) < \gamma_{te}(G)$

Figure 2: Minimum triple effect dominating set in G and $G-v$.



(a) $\gamma_{te}(G+e) = \gamma_{te}(G)$

(b) $\gamma_{te}(G+e) > \gamma_{te}(G)$

(c) $\gamma_{te}(G+e) < \gamma_{te}(G)$

Figure 3: Minimum triple effect dominating set in G and $G+e$.

v. When we delete $e = vu_2$, then u_1 will replace the vertex v in the set $V - D$, therefore $\gamma_{te}(G - e) = \gamma_{te}(G)$. For example see Fig.3.(a).

Case 2. $E^+_- \neq \emptyset$. If the vertex w dominates the three adjacent vertices in $V - D$ say u_1, x and v and a vertex x adjacent with two vertices in $V - D$ differently v. And v adjacent with three vertices in $V - D$ say u_1, u_2 and u_3, we will delete an edge e between v and w, when we delete $e = vw$, then v and x will replace the vertex w in the set D, therefore, $\gamma_{te}(G - e) > \gamma_{te}(G)$. For example see Fig.3.(b).

Case 3. $E^-_- \neq \emptyset$. If the vertex w dominates three vertices in $V - D$ say u_1, u_2 and v and every vertex in D that is adjacent with w dominates v. Such that $v \in V - D$ and adjacent with three vertices in V different to w, such that u_1, u_2 and u_3 are adjacent and $u_3 \in D$ dominates u_1, u_2 and v and adjacent with w, when we delete $e = vw$, then v will dominate u_1, u_2 and u_3, so it replaces the vertex w in the set D and u_3 will belong to $V - D$. Therefore, $\gamma_{te}(G - e) < \gamma_{te}(G)$. For example see Fig.3.(c). □

(a) $\gamma_{te}(G - e) = \gamma_{te}(G)$

(b) $\gamma_{te}(G - e) > \gamma_{te}(G)$

(c) $\gamma_{te}(G - e) < \gamma_{te}(G)$

Figure 4: Minimum triple effect dominating set in G and $G - e$.

3 Triple effect domination and its inverse for complement graphs

In this section, we apply the triple effect domination and its inverse on several graphs obtained from complement, join and corona operations.

Theorem 3.1. Let P_n a path graph, then $\bar{P}_n$ has triple effect domination if and only if $6 \leq n \leq 9$ such that:

$$\gamma_{te}\left(\bar{P}_n\right) = \begin{cases} 2 & \text{if } n = 6, 7 \\ n - 5 & \text{if } n = 8, 9 \end{cases}$$

Proof . Since $\deg(v_i) \leq 2 \forall \bar{P}_i, i = 2, 3, 4$, $\bar{P}_i$ has no triple effect domination. When $n = 5$ if $D = \{v_1\}$, there is one vertex is not dominated by D. If $D = \{v_5\}$, there is one vertex that is not dominated by D. If $D = \{v_1, v_5\}$.

Then every one of them dominates two vertices, all above cases are contraction our definition, so $\bar{P}_5$ has no triple effect domination. If $n = 6$, let $D = \{v_1, v_6\}$ or $D = \{v_2, v_3\}$ or $D = \{v_4, v_5\}$. If $n = 7$, then $D = \{v_i, v_j\}$ such that $d(v_i, v_j) = 2$ and $i, j \neq 1, 7$. If $n = 8$, let $D = \{v_2, v_4, v_6\}$ or $D = \{v_3, v_5, v_7\}$, then all vertices of D are adjacent together and dominate exactly three vertices. If $n = 9$, let $D = \{v_2, v_4, v_6, v_8\}$, such that these vertices not pendant in P_n then all vertices of D is adjacent together and dominate exactly three vertices. In all the above cases, D is a minimum triple effect dominating set. Thus, D is a $\gamma_{te}-$ set of $\bar{P}_n$, for example, see Fig.4.

If $n \geq 10$, then every dominating set D has at least one vertex that dominates less than three vertices or dominates more than three vertices. □

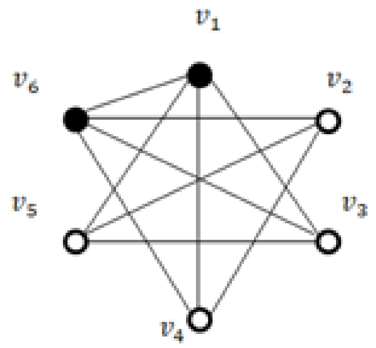


Figure 5: Triple effect dominating set in $\bar{P}_n$.

Theorem 3.2. Let P_n be a path graph, then $\bar{P}_n$ has inverse triple effect domination if and only if $n = 6, 7, 8$, such that:

$$\gamma_{te}^{-1}(\bar{P}_n) = \begin{cases} 2 & \text{if } n = 6, 7 \\ 3 & \text{if } n = 8 \end{cases}$$

Proof . From proof of Theorem 3.1., if $n = 6, 7, 8$, then $\bar{P}_n$ has more than one triple effect dominating set, then it has inverse triple effect dominating set (for example, see Fig.5). □

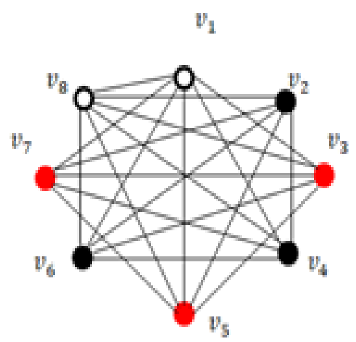


Figure 6: Inverse triple effect domination in $\bar{P}_n$.

Theorem 3.3. Let C_n be a cycle graph of order $n \geq 3$, then $\bar{C}_n$ has triple effect domination if and only if $n = 6, 7, 8, 9, 10, 12$, such that

$$\gamma_{te}(\bar{C}_n) = \begin{cases} 2 & \text{if } n = 6 \\ n - 5 & \text{if } n = 7, 8, 9, 10 \\ 8 & \text{if } n = 12 \end{cases}$$

Proof . Since $\deg(v) \leq 2$, for all $v \in \bar{C}_i, i = 2, 3, 4, 5$, $\bar{C}_n$ has no triple effect domination. If $n = 6$, then D have any two consecutive vertices for $\bar{C}_6$. If $n = 7$, then $D = \{v_i, v_j\}$ such that $d(v_i, v_j) = 3$. If $n = 8$, let $D = \{v_2, v_4, v_6\}$ or $D = \{v_3, v_6, v_8\}$ or $D = \{v_2, v_5, v_7\}$, then all vertices of D are adjacent together and dominate exactly three vertices.

If $n = 9$, then $D = \{v_2, v_4, v_6, v_8\}$ or $D = \{v_3, v_5, v_7, v_9\}$, then all vertices of D are adjacent together and dominate exactly three vertices. If $n = 10$, then $D = \{v_2, v_4, v_6, v_8, v_{10}\}$ or $D = \{v_1, v_3, v_5, v_7, v_9\}$, then all vertices of D is adjacent together and dominate exactly three vertices. If $n = 12$, then $D = \{v_2, v_3, v_5, v_6, v_8, v_9, v_{11}, v_{12}\}$. In all the above cases, D is a minimum triple effect dominating set. Thus, D is a $\gamma_{te}-$ set of $\bar{C}_n$. For example, see Fig.6. If $n = 11$ or $n > 12$, then every dominating set D has at least one vertex that dominates less than three vertices or dominates more than three vertices. □

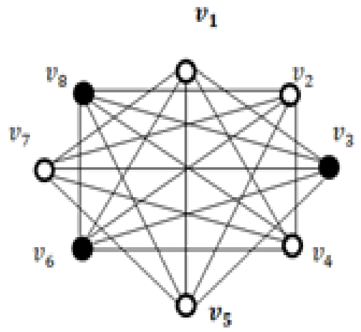


Figure 7: Minimum triple effect dominating set in $\bar{C}_n$.

Theorem 3.4. Let C_n be a cycle graph of order $n \geq 3$, then $\bar{C}_n$ has inverse triple effect domination if and only if $6 \leq n \leq 10$, such that:

$$\gamma_{te}^{-1}\left(\bar{C}_n\right) = \begin{cases} 2 & \text{if } n = 6 \\ n-5 & \text{if } n = 7, 8, 9, 10 \end{cases}$$

Proof . Similar to proof of Theorem 2.3 (For example, see Fig.7). □

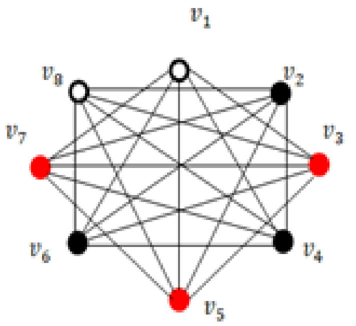


Figure 8: Inverse triple effect domination in $\bar{C}_n$.

Theorem 3.5. Let $K_{n.m}$ be a bipartite graph, then $\bar{K}_{n,m}$ has triple effect domination if and only if $n \geq 4$ and $m \geq 4$, such that: $\gamma_{te}\left(\bar{K}_{n,m}\right) = n + m - 6$.

Proof . If $n < 4$ or $m < 4$, there is no triple effect domination. If $n \geq 4$ and $m \geq 4$, there are two graphs K_n and K_m, such that $K_{n,m} = K_n \cup K_m$ and every graph of them has triple effect domination. Then, we have $\gamma_{te}\left(\bar{K}_{n,m}\right) = n + m - 6$. □

Theorem 3.6. Let $K_{n.m}$ be a bipartite graph has inverse triple effect domination if and only if $m \geq 4$, and $n = 4, 5, 6$ such that: $\gamma_{te}^{-1}\left(\bar{K}_{n,m}\right) = n + m - 6$.

Proof . If $n \geq 4$ and $m \geq 4$, there are two graphs $K_n and K_m$ and every graph of them has inverse triple effect domination, then $\gamma_{te}^{-1}\left(\bar{K}_{n,m}\right) = n + m - 6$. The complete graph $K_n(n \geq 4)$ has an inverse triple effect domination if and only if $n = 4, 5, 6$. Furthermore, $\gamma_{te}^{-1}\left(K_n\right) = \gamma_{te}\left(K_n\right) = n - 3$) If $n \geq 7$, then has no inverse triple effect domination by (For any graph G having order n and triple effect dominating set, if $\gamma_{te}(G) > \frac{n}{2}$, then G has no inverse triple effect domination), since $\gamma_{te}\left(\bar{K}_{n,m}\right) > \frac{n}{2}$. Then, D^{-1} is minimum inverse triple effect dominating set. Hence, D^{-1} is a $\gamma_{te}^{-1}-$ set of $\bar{K}_{n,m}$. □

4 Triple effect domination for corona and join operations generated graphs

The triple effect domination and the inverse triple effect domination are studied in this section for some graphs constructed by corona or join operations.

Proposition 4.1. For any graph G of order n, then

$$\gamma_{te}\left(G \odot G'\right) = \gamma_{te}(G \odot \bar{G}) = \gamma_{te}(\bar{G} \odot G) = \gamma_{te}(\bar{G} \odot \bar{G}) = n,$$

where G is a graph with order three.

Proof . Since every $v \in G$ is adjacent to three vertices of G or $\bar{G}$, $v \in D$. Therefore, every $v \in D$ dominates exactly three vertices. Thus, D is $\gamma_{te}-$ set and $D = V(G)$ with order n. □

Proposition 4.2. If G is a graph of order n, then

$$\gamma_{te}^{-1}\left(G \odot G'\right) = \gamma_{te}^{-1}\left(\bar{G} \odot G'\right) = n$$

where G is a graph with order three.

Proof . Since there are n complete graphs of order four and $\gamma_{te}^{-1}\left(K_4\right) = 1$. □

Proposition 4.3. For any two graphs G_1 of order $n \geq 1$ and G_2 of order $m \geq 3$, such that:

1. If $m = 3$, then $\gamma_{te}\left(G_1 + G_2\right) = n$.
2. If $m > 3$, then $n + m - 6 \leq \gamma_{te}\left(G_1 + G_2\right) \leq n + m - 3$.

Proof . To prove the lower bound, suppose that G_1 and G_2 are two null graphs having as few edges as possible. Then, $\gamma_{te}\left(G_1 + G_2\right) = \gamma_{te}\left(K_{n,m}\right) = n$ or $n + m - 6$. Also, to prove the upper bound, suppose that G_1 and G_2 are two complete graphs. Then, the join between them gives a complete graph with order $n + m$. Thus, $\gamma_{te}\left(G_1 + G_2\right) = \gamma_{te}\left(K_{n+m}\right) = n + m - 3$. □

Proposition 4.4. For any two graphs G_1 of order $n \geq 1$ and G_2 of order $m > 3$, then $n + m - 6 \leq \gamma_{te}^{-1}\left(G_1 + G_2\right) \leq n + m - 3$.

Proof . To prove the lower bound, suppose that G_1 and G_2 are two null graphs having as few edges as possible. Then, $\gamma_{te}^{-1}\left(G_1 + G_2\right) = \gamma_{te}^{-1}\left(K_{n+m}\right) = n$ or $n + m - 6$. Also, to prove the upper bound, suppose that G_1 and G_2 are two complete graphs. Then, the join between them gives a complete graph with order $n + m$. Thus, $\gamma_{te}^{-1}\left(G_1 + G_2\right) = \gamma_{te}^{-1}\left(K_{n,m}\right) = n + m - 3$. □

Observation 4.5. Let G be a disconnected graph with $T_1, T_1, \ldots, T_n$ components, then:

1. $\gamma_{te}(G) = \sum_{i=1}^{n} \gamma_{te}\left(T_i\right)$
2. $\gamma_{te}^{-1}(G) = \sum_{i=1}^{n} \gamma_{te}^{-1}\left(T_i\right)$.

5 Conclusion

The triple effect domination and the inverse triple effect domination are determined for some graphs. Stability of triple effect domination is studied here.

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