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Dynamic network data envelopment analysis model grounded on game theory

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Abstract

Among the most significant management and policy-making elements is improving the efficiency of businesses and companies active in various parts of the industry, as well as management processes and systems. Various methods and models have been proposed to measure the efficiency of firms, processes and systems in a sector. One of the most important models used is network data envelopment analysis, the most widely utilized in measuring the efficiency and productivity of businesses, processes and systems. This is founded on mathematical programming and moreover is among the most powerful techniques for performance evaluation and optimization. In this study, while mathematically modeling and measuring the performance of the study sample, utilizing dynamic network data envelopment analysis, a suitable model for measuring performance, taking into account the stability of the conditions, was designed to be a framework for leading the system to higher objectives. Via utilization, it provides the basis for improving and reducing the adverse effects of the system. Therefore, the data envelopment analysis model in this research is designed as a framework for measuring, analyzing and promoting activities at the network level with the game theory approach as well as in uncertain conditions. Executives/managers will be able to understand the strengths and weaknesses of the system to strengthen and eliminate existing weaknesses plus undertake requisite planning and actions.

Keywords: efficiency, envelopment analysis of network data, stability, game theory, compositional uncertainty
2020 MSC: 91A80

1 Introduction

Estimating the efficiency and ranking of businesses operating in various sectors of the industry as well as prioritizing management processes and systems are among the most crucial tools for executives and policy makers toward enabling them to make decisions. One of the most important models utilized to calculate the efficiency and ranking of enterprises as well as management systems is data envelopment analysis model. Due to the significance of the issue, various methods have been proposed to measure it. In general, there are two categories of parametric and non-parametric methods for evaluating performance. In the parametric method, a specific production function is

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evaluated using statistical methods to evaluate performance. But nonparametric methods do not require estimating the production function. Among the most widely used non-parametric methods is data envelopment analysis, which assesses the relative efficiency of units in comparison with each other [21]. Data envelopment analysis is a mathematical programming model employed to estimate the performance boundary. This method provides a boundary function covering all the data and is therefore called envelopment analysis [5].

Classical data envelopment analysis models view the units under evaluation as a black box that converts inputs into outputs. These models do not focus the structure and internal flow of the companies. Generalized network models are based on these classic models that also take into account the internal structure of the businesses. These models were first introduced by [10]. Conventional data envelopment analysis models are not inclusive of decision-making units and their internal structure. To overcome this problem and negligence as far as calculating efficiency, a network data envelopment analysis model has been proposed to develop this technique. The data envelopment analysis models can be classified in general compliant to Figure 1.

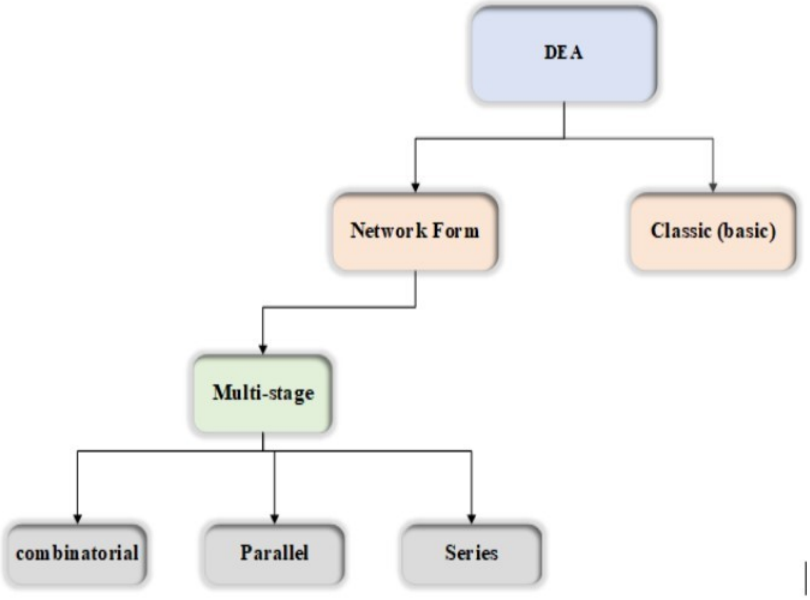

Figure 1: Classification of data envelopment analysis models according to decision-making unit structure (researcher's findings)

In line with Figure 1, multi-stage network data envelopment analysis is divided into three categories: series, parallel and series-parallel. Multi-stage models are mostly utilized in assessing the efficiency of supply chains consisting of several different organizations [26]. Multi-part models are related to the internal structure of an organization, composed of various components.

The subunits of a decision-making department, the existing constituents of that department, can be utilized in series or parallel or series-parallel. Such steps within decision-making units are often far from reality. In order to evaluate the performance of a unit with its subsets (in relation to its past or to others), various techniques and methods can be used. In order to guide them, the most key issue for executives/managers in relation to decision-making units is to be informed about their performance and efficiency. Without a scientific approach, executives can not be aware of the operation of subordinate units and make appropriate decisions to improve efficiency and productivity. In practice, decision-making units can have more complex relationships. A decision-making unit consists of several sub-units and the output of some sub-units is the input of another sub-unit, hence the set of sub-units forms a network. These types of units can be examined via utilization of network data envelopment analysis methods.

Most unit activities (for instance: the emergency system), in addition to the desired outputs, have undesirable outputs such as patients waiting in long queues and the numerous medication errors made by medical staff. All this will have social, economic and environmental consequences. Therefore, the modeling of these outputs is important.

In classical methods of production theory in general expression and data envelopment analysis in specific expression, at the technology level the aim is to minimize inputs and maximize outputs. While units and organizations such as factories, hospitals, etc. in the process of activity may, in addition to producing intended/desired outputs, also produce undesirable outputs. The existence of undesirable outputs plays an important role in estimating the efficiency of these units. In evaluating such units, the objective is to utilize a method that, in addition to adapting to the concepts of production theory, can help reduce undesirable outputs and increase desirable outputs. Modeling of adverse factors requires focus not only in terms of measuring efficiency, but also in terms of estimating harmful factors from social, economic and environmental angles.

In this article, the application of series-parallel multi-stage envelopment analysis model of the stability network data was performed utilizing the game theory approach in combined uncertainty conditions.

2 Review of previous studies

With due attention to the fact that the research conducted in the field of data envelopment analysis is quite extensive, and since the focus of this research is on performance evaluation of stability as well as performance measurement utilizing the game theory approach, in Tables 1 and 2, studies performed on the application of data envelopment analysis models in assessing the performance stability of systems and game theory have been reviewed and examined.

Table 1: Extracted articles on sustainability performance

Year	Author	Title	Model
2007	Bosetti et al [3]	Efficiency Assessment-Economic Sustainability Of Tourist Destinations	DEA
2011	Bruni et al [4]	Efficiency Assessment-Italy's Sustainable Development	DEA
2011	Martinez et al [25]	Efficiency Assessment-Agricultural Industry's Sustainability	DEA
2012	De Koeijer et al [8]	Efficiency Assessment-Agricultural System's Sustainability	DEA
2012	Hoang&Alauddin [16]	Efficiency Assessment-Agricultural System's Sustainability	DEA
2012	Haron & Chellakumar [14]	Efficiency Assessment-Stability Of A Manufacturing Company In Korea	DEA
2013	Gerdessen & Pascucci [13]	Efficiency Assessment-Agricultural Sustainability In Europe	CRS
2013	Zhu & Zhu [33]	Efficiency Assessment-R&D Performance	DEA - Bargaining Game Theory
2014	Dobos&Vrsmarty [9]	Selection Of Supplier With Sustainability Criteria	DEA
2014	Hou et al [17]	Calculation Of Sustainable Performance	DEA- Regression
2015	Gadanakis et al [11]	Efficiency Assessment-Sustainability Of Agricultural Lands	DEA - Slack
2016	Alfonso & Martinez [1]	Efficiency Assessment-Sustainability Of City/Urban Development	DEA
2016	Dash & Balachandra [7]	Efficiency Assessment-Sustainability Of City/Municipality	DEA
2016	Thore & Tarverdyan [29]	Efficiency Assessment-Sustainability Of Countries	DEA
2016	Kocmanová et al [23]	Efficiency Assessment-Sustainability Of Government Value Added	DEA - Slack
2016	Hinojosa et al [15]	Ranking Of Efficient Decision Makers	DEA - Participatory Game Theory
2017	Yu et al [31]	Efficiency Assessment-Sustainability Of City/Urban Development	DEA - Slack
2018	Choi et al [6]	Efficiency Assessment-Sustainability Of Steel Industry	DEA - Slack
2018	Izadikhah et al [20]	Efficiency Assessment-Sustainability Of Suppliers	DEA
2018	Badiezadeh et al [2]	Efficiency Assessment-Supply Chain Performance Sustainability	DEA - Multi Level
2018	Izadikhah & Saen [18]	Efficiency Assessment-Supply Chain Performance Sustainability	DEA - Dual Level
2019	Shabanpour et al [27]	Ranking Of Contractors In Terms Of Sustainability	DEA
2019	Kalantary et al [22]	Efficiency Assessment-Supply Chain Sustainability	NDEA
2019	Yue et al [32]	Productivity Evaluation Of Several Chinese Provinces	DEA
2019	Lombardi et al [24]	Efficiency Assessment-Sustainability Of Country's Water Sector	DEA
2019	Yousefi et al [30]	Efficiency Assessment-Sustainability Of Power Plants	DEA-NN
2019	Izadikhah & Saen [19]	Ranking Suppliers As Far As Sustainability	DEA
2020	Tian et al [28]	Efficiency Assessment- Regional Transportation Sustainability	DEA

3 Determining the multi-stage structure of the emergency system

In order to unveil the multi-stage structure of the emergency system based on existing and prevailing facts, interviews with hospital experts were conducted. The experts were managers, executives and administrators with more than a decade's experience working in the health system, and were fully familiar with most hospital issues, especially emergency activities. Ultimately, compliant with library studies and the findings of interviews with experts, the general network model for emergency operations was obtained (Figure 2).

By determining the multi-stage structure of the emergency and how these parts relate to each other, Figure 2 can be said that the above model falls into the category of network models of the combined multi-stage type.

4 Methodology

4.1 Research's mathematical model

In real-world network envelopment analysis, decision-making components utilize multiple inputs for multiple outputs, and within the network, the output of one decision-making unit at one input stage and the next stage is considered to be the same decision-making unit. In a network structure, increasing the output of a decision-making unit will not increase the efficiency of the system, and it has the ability to demonstrate well the relationship and interdependence between internal processes, and properly calculate efficiency (in its entirety & in separate stages).
Four categories were presented toward measuring the performance of multi-stage systems: the standard DEA approach, performance evaluation, network DEA, and game theory approaches.

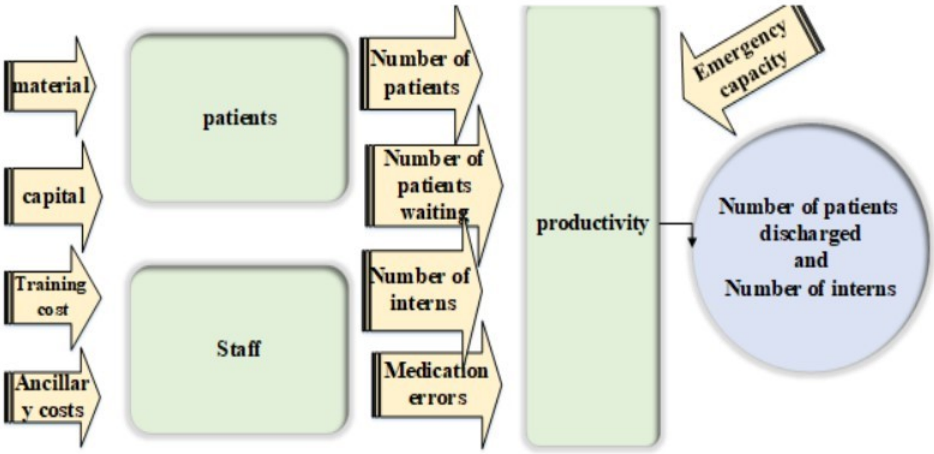

Figure 2: Multi-stage structure of the emergency service system

The three-stage model of network data envelopment analysis is offered with a game theory approach, and taking into account the undesirable output and stabilization and fuzzy approach for the final output and evaluating their efficiency.
The components and activities of the model are as follows.

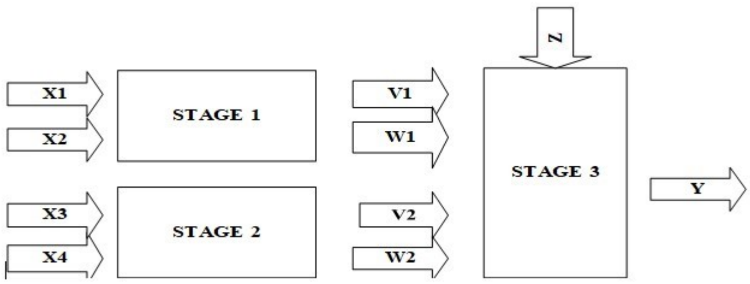

Figure 3: Emergency system's inputs & outputs

Consistent with Figure 3, the model has three stages (with desirable & undesirable inputs & outputs). The symbols used are as follows:

X_1: Amount of consumables

X_2: Level of capital

X_3: Training fee

X_4: Auxiliary costs

V_1: Number of patients

W_1: Number of patients in the waiting line

V_1: Number of interns

W_2: Medication errors

Z_1: Emergency capacity

Y_1: Number of patients discharged annually

Y_2: Number of interns per year

4.2 Proposed hybrid uncertainty model

Due to the ambiguity and uncertainty in their definition and understanding, real world problems usually have a complex structure. For systems with high complexity and uncertainty for which sufficient and accurate information is unavailable, solid and fuzzy optimization approaches are proposed. The main premise of mathematical programming is to develop a model based on explicitly defined data equal to a nominal value. However, in such models, the impact of data uncertainty has no effect on the quality and feasibility of the responses. Consequently, in real-world problems, changing one of the data may violate a large number of constraints and make the answer non-optimal or even impossible. The answer to this primary question is "solid optimization", which is resistant to this data uncertainty. In the fuzzy approximate reasoning approach, Professor Lotfi-Zadeh introduced the theory of fuzzy sets and provided the preconditions for inaccurate information modeling and approximate reasoning with mathematical equations. Inputs to fuzzy systems can be inaccurate (fuzzy) information, and system processes can be performed fuzzily utilizing approximate reasoning.
In the proposed model, assuming up to &, input data of desirable & undesirable outputs are

$$x_k = (x_{1K}, \dots, x_{NK}) \geq 0, \quad v_k = (v_{1K}, \dots, v_{MK}) \geq 0, \quad w_k = (w_{K1}, \dots, w_{JK}) \geq 0 \tag{4.1}$$

And assuming that & & . Production technology shall be as follows

$$p(x) = \left\{ (v, w)\, x \dots (v, w)\,, x \in R_+^N \right\} \tag{4.2}$$

Definition 4.1. Outputs (favorable & unfavorable) are poorly usable once if & only if and imply $(v, w) \in p\,(x)$, and;

$$0 \leq \theta \leq 1 \text{imply}\,(\theta_v, \theta_w) \in p\,(x)\,, x \in R_+^N \tag{4.3}$$

and Faragosefe proposed the following technique via the BBC assuumption:

$$T_{PG} = \left\{ \begin{array}{l} (v, w, x) | \sum_{k=1}^{k} \theta z^k v_m^k \geq v_m, m = 1, \dots, M, \\ \sum_{k=1}^{k} \theta z^k w_j^k = w_j, j = 1, \dots, J, \sum_{k=1}^{k} z^k x_n^k \leq x_n, n = 1, \dots, N, \sum_{k=1}^{k} z^k = 1, z^k \geq 0, 0 \leq \theta \leq 1, k = 1, \dots, K \end{array} \right\} \tag{4.4}$$

The conflict parameter θ is traced back to Scheferd's definition of poor accessibility. This parameter enables the integration of good and bad outputs at the same time. As Cosmanon points out, this model utilizes a uniform reduction factor for all companies. He proposed the following production technology:

$$T_k = \left\{ \begin{array}{l} (v, w, x) | \sum_{k=1}^{k} \theta^k z^k v_m^k \geq v_m, m = 1, \dots, M, \\ \sum_{k=1}^{k} \theta^k z^k w_j^k = w_j, j = 1, \dots, J, \sum_{k=1}^{k} z^k x_n^k \leq x_n, n = 1, \dots N, \sum_{k=1}^{k} z^k = 1, z^k \geq 0, 0 \leq \theta^k \leq 1, k = 1, \dots, K \end{array} \right\} \tag{4.5}$$

It should be noted that the first formula is a special case of the second formula with $\theta_1 = ... = \theta_k$. with free accessibility of good inputs and outputs via $\boldsymbol{V}$ nad $\boldsymbol{X}$ use of inequality conditions and is modeled in light of $\mathbf{T}_k$, the nonlinear technology can be expressed as an equal linear shape, utilizing a simple and efficient method. For linearization of Formula (4.5), the resonant weight $\mathbf{Z}_k$ can be divided into two parts, as Cosmanon used the concept to transform production technology (4.5) into the following linear form:

$$T_K^{(L)} = \left\{ \begin{array}{l} (v, w, x) | \sum_{k=1}^{k} \lambda^k v_m^k \geq v_m, m = 1, \dots, M, \\ \sum_{k=1}^{k} \lambda^k w_j^k = w_j, j = 1, \dots, J, \sum_{k=1}^{k} (\lambda^k + \mu^k) x_n^k \leq x_n, n = 1, \dots, N, \sum_{k=1}^{k} (\lambda^k + \mu^k) = 1, \lambda^k, \mu^k, k = 1, \dots, K \end{array} \right\} \tag{4.6}$$

The above formula (4.6) is now a linear form. This technology and the aforementioned linearization method are utilized to model undesirable intermediate criteria in a two-step production process. Poor accessibility in the three-step decision-making process wherein mediation actions include favorable and unfavorable outcomes.

We assume that to DMU_k exists,and information vector on inputs, desirable & undesirable outputs shall be as: $x_k = (x_{1K}, ..., x_{NK}) \geq 0$, $v_k = (v_{1K}, ..., v_{MK}) \geq 0$, $w_k = (w_{K1}, ..., w_{JK}) \geq 0$
(V_k, W_k) outputs are created as inputs for the second stage by $(V_k, W_k) Z_k = (Z_{1k}, ..., Z_{TK})$ an external input vector . The final product DMU_k is represented and shown by $y_k = (y_{1K}, \dots, y_{SK})$. In the following, the non-participatory game theory approach is deployed to assess and solve the model in solid and fuzzy conditions.

4.3 Leader-follower game theory

At this stage, the leader-follower (non-participatory) approach is implemented to test this developed three-step structure. In a non- participatory game, the leader is preferred to the follower. Therefore, the leader determines the most efficient conditions and in pursuance the follower proceeds with their optimal conditions in line with the leader's information. Within the methods, the first stage is the leader and the second and third stages are followers.
Math demonstration of the first stage production technique:

$$T_1 = \left\{ \begin{array}{l} (v,w)|\sum_{k=1}^{k}(\rho^k+\mu^k)x_n^k \geq x_n, n=1,...,N, \\ \sum_{k=1}^{k}\rho^k v_m^k = v_m, m=1,...,M, \sum_{k=1}^{k}\rho^k w_j^k = w_j, j=1,...J, \sum_{k=1}^{k}(\rho^k+\mu^k)=1, \rho^k,\mu^k \geq 0 \end{array} \right\} \tag{4.7}$$

The above linear technology are unknown variables types &. In the proposed model, we intend to examine the efficiency of in terms of the probability of reduction in undesirable outputs. This is obtained as the optimal value of the following model. Furthermore, $\propto$ is the level of confidence for performance satisfaction and objective constraints. Henceforth, the above model becomes a fuzzy model via the size requirement method:

$$\begin{array}{ll} \min & \theta_0 = e_0^{(1)^\star} \\ s.t. & \sum_{k=1}^{k}((\alpha_0)(\chi_n^k)+(1-\alpha_0)(\chi_n^k))(\rho^k+\mu^k) \leq \chi_n^0, \quad n=1,\ldots,N \\ & \sum_{k=1}^{k}((\alpha_0)(v_m^k)+(1-\alpha_0)(v_m^k))\rho^k \geq v_m^0, \quad m=1,\ldots,M \\ & \sum_{k=1}^{k}((\alpha_0)(w_j^k)+(1-\alpha_0)(w_j^k))\rho^k = ((\alpha_0)(w_j^k)+(1-\alpha_0)(w_j^k))\theta_0, \quad j=1,\ldots,J \\ & \sum_{k=1}^{k}(\rho^k+\mu^k)=1, \\ & \rho^k,\mu^k \geq 0, \quad k=1,\ldots,K \end{array} \tag{4.8}$$

The thematic function minimizes the equal-proportional reduction factor for all undesirable outputs from protecting the current level of the desired inputs and outputs. Clearly, Model 5 is a linear programming problem and is always possible and limited. For an inefficient leader in DMU_0 (Step 1) we have:

$$\begin{array}{l} \sum_{k=1}^{k}(\rho^k+\mu^k)x_n^k = x_n^0 - s_n^{(x)} \\ \sum_{k=1}^{k}\rho^k v_m^k = v_m^0 + s_m^{(v)} \\ \sum_{k=1}^{k}\rho^k w_j^k = \theta_0 w_j^0 \\ S_m^v \end{array} \tag{4.9}$$

In this formula S_n^x, S_m^v are respectively the first and second condition negligible variables in (4.9). Step 1 can be improved by eliminating outputs and removing large amounts of undesirable output and eliminating output shortcomings. It is simple to show that the leader is now efficient. By obtaining the first step's performance, step 2 is evaluated and the performance status of the first step is maintained. By complying with Cosmanon, minimal accessibility technology can be formulated in all DMU. Under these assumptions, the experimental production set $p_2(z,y)$ can be written as follows:

$$T_2 = \left\{ \begin{array}{l} (v,w,y)|\sum_{k=1}^{k}\theta^k\lambda^k v_m^k \leq v_m, m=1,\ldots,M, \\ \sum_{k=1}^{k}\theta^k\lambda^k w_j^k = w, j=1,\ldots,J, \\ \sum_{k=1}^{k}\lambda^k y_r^k \geq y_r, r=1,\ldots,S, \sum_{k=1}^{k}\lambda^k z_t^k \leq z_t, t=1,\ldots,T, \sum_{k=1}^{k}\lambda^k=1, \lambda^k \geq 0, \theta^k \geq 1, k=1,\ldots,K \end{array} \right\} \tag{4.10}$$

Similar to poor accessibility assessment, Formula (4.10) utilizes θ_k stop factors that reduce good and bad outputs by a proportion, as in the case of poor accessibility.We are now transforming nonlinear technology (4.7) into a linear one using the same Cosmanon method. By rearranging the expressions in $\theta^k\lambda^k = \beta^k$ and $\alpha^k = \left(1-\theta^k\right)\lambda^k$ and $\beta^k+\alpha^k=\lambda^k$ we acquire an equivalent representation for production technology (4.7) as follows:

$$T_2 = \left\{ \begin{array}{l} (v,w)|\sum_{k=1}^{k}\beta^k v_m^k \leq v_m, m=1,\ldots,M, \\ \sum_{k=1}^{k}\beta^k w_j^k = w_j, j=1,\ldots,J, \sum_{k=1}^{k}(\beta^k+\alpha^k)y_r^k \geq y_r, r=1,\ldots,S, \sum_{k=1}^{k}(\beta^k+\alpha^k)z_t^k \leq z_t, t=1,\ldots,T, \\ \sum_{k=1}^{k}(\beta^k+\alpha^k)=1; \beta^k,\alpha^k \geq 0, k=1,\ldots,K \end{array} \right\} \tag{4.11}$$

This technology is in terms of anonymous variables α, β linear. The second stage, based on the leader game hypothesis for the three-stage process, only takes into account the optimal solutions that consider the first stages of performance situations. For this purpose, the triple stage (v, w, z) is designated as the subject of this constraint in order for the efficiency score of the first stage to remain optimal. To analyze the second step, we solve the following linear programming problem in solid and fuzzy state:

$$\begin{array}{ll} \min & \varphi = e_0^{(2)^\star} \\ s.t. & \sum_{k=1}^{k}((\alpha_0)(\chi_n^{k1}) + (1-\alpha_0)(\chi_n^{k2}))\tau^k \leq \chi_n^0, \quad n = 1, \ldots, N \\ & \sum_{k=1}^{k}((\alpha_0)(v_m^{k1}) + (1-\alpha_0)(v_m^{k2}))\tau^k \geq v_m^0, \quad m = 1, \ldots, M \\ & \sum_{k=1}^{k}((\alpha_0)(w_j^{k1}) + (1-\alpha_0)(w_j^{k2}))\tau^k = \varphi_0 w_j^0, \quad j = 1, \ldots, J \\ & \sum_{k=1}^{k}(\tau^k + \sigma^k) \leq 1, \\ & \tau^k, \sigma^k \geq 0, \quad k = 1, \ldots, K \end{array} \tag{4.12}$$

In this model, the second stage assumes the mean of $\boldsymbol{m}$ as the desired output and the mean of $\boldsymbol{j}$ as the undesirable output of constant $\varphi_0 w_j^0$ and $\sum_{k=1}^{k} \tau^k$. These conditions are the optimal output values of the first stage of, hence to the right is the first condition in (4.12) the maintenance of stage 2 performance. It must be determined that a system is efficient if and only if the three processes that constitute it are efficient. Additionally, the final output of the third stage is assumed as indefinite, modeled using both solid and fuzzy optimization approaches:

$$\begin{array}{ll} \min & \phi_0 = e_0^{(3)^\star} \\ s.t. & \sum_{k=1}^{k}((\alpha_0)(y_r^k) + (1-\alpha_0)(y^k))(\beta^k + \alpha^k) \\ & \quad +((\alpha_0)f(r) + (1-\alpha_0)f(r))\sum_k((\alpha_0)(p_r^k) + (1-\alpha_0)(p_r^k)) \leq y_r^0, \quad r = 1, \ldots, R \\ & g_r + ((\alpha_0)(p_r^k) + (1-\alpha_0)(p_r^k)) \geq ((\alpha_0)(\alpha_r^k) + (1-\alpha_0)(\alpha_r^k))(\beta^k + \alpha^k), \quad r = 1, \ldots, R \\ & y_r^k = y_r^k \pm \alpha_r^k \\ & p_r^k, g_r, f(r) \geq 0, \quad r = 1, \ldots, R \end{array} \tag{4.13}$$

5 Model implementation findings

To measure the efficiency of the three stages, the undesirable outputs are defined as the final outputs as exhibited in Figure 3, with the desired $\boldsymbol{V}$ criteria of the first and second stages being used for the third stage, and the undesirable criteria $\boldsymbol{W}$ being removed from the system. As a consequence, in this method, our aim was to improve the efficiency of the first and second stages by increasing the desired outputs, impacting the efficiency of the third stage and ultimately overall efficiency.

In model (4.13), the enveloping analysis of network data in the case of combined uncertainty has the 3 stages of patients, staff and productivity. The described stability performance was measured and obtained, the findings of which can be observed in the below table:

As demonstrated in Table (4.2), the DMU performance of all three steps is calculated with alpha 0, 0.25, 0.5, 0.75 & 1 for all 7 DMUs. The third stage (productivity stage) has a better performance, and therefore, it should be stated that the higher the level of confidence, the smaller the numbers become, proving that the performance has been properly and accurately calculated. The findings for the performance of all three steps (plotted bar graphs for each of the 7 DMUs) is observable in Figure 4.

In the final model (4.13), the envelopment analysis of network data in conditions of combined uncertainty is assessed and examined toward obtaining the efficiency of the entire hospital emergency service system in 5 alpha. The findings are shown in the table below:

As delineated in Tables 2 & 3, the findings obtained from the envelopment analysis of network data are in such a manner where the larger the alpha, the smaller the numbers, hence the lower the performance.

6 Conclusion

Heretofore, multiple studies have been conducted in the field of data envelopment analysis as well as the development of network data envelopment analysis models with the game theory approach and its application. In the development of envelopment analysis models (multi-stage data with input and output nature based on solid and fuzzy optimization techniques with inputs), intermediates and uncertain outputs have not been performed in any of the studies. However, the fact is that in the real world, the existence of uncertain data in the process of data envelopment analysis is

Table 2: Efficiency of the three stages

	STAGE	∝ =0	∝ =0.25	∝ =0.5	∝=0.75	∝=1
DMU 1	1	0.0621	0.0584	0.0538	0.0498	0.0462
2	0.3443	0.3115	0.2819	0.2549	0.2305	
3	1	0.9033	0.7838	0.7403	0.6568	
DMU 2	1	0.0273	0.0247	0.0223	0.0202	0.0182
2	0.3885	0.3515	0.318	0.2876	0.2601	
3	1	0.9048	0.8185	0.7404	0.6694	
DMU 3	1	0.0265	0.024	0.0217	0.0196	0.0177
2	0.2018	0.1826	0.1652	0.1494	0.1351	
3	0.9461	0.8561	0.7745	0.7005	0.6333	
DMU 4	1	0.0262	0.0237	0.02146	0.0194	0.01754
2	0.20121	0.18206	0.1647	0.1489	0.1346	
3	0.1365	0.1235	0.1117	0.101	0.0913	
DMU 5	1	0.02	0.0181	0.01637	0.0148	0.01339
2	0.2827	0.2558	0.2314	0.2093	0.1893	
3	1	0.9048	0.8185	0.7404	0.6694	
DMU 6	1	0.02285	0.02068	0.01871	0.01692	0.0153
2	0.49976	0.4522	0.4091	0.37002	0.3345	
3	0.1388	0.1256	0.1136	0.10279	0.0929	
DMU 7	1	1	0.9048	0.8185	0.7404	0.6694
2	1	0.9048	0.8185	0.7404	0.6694	
3	0.081	0.0736	0.0666	0.0603	0.0545	

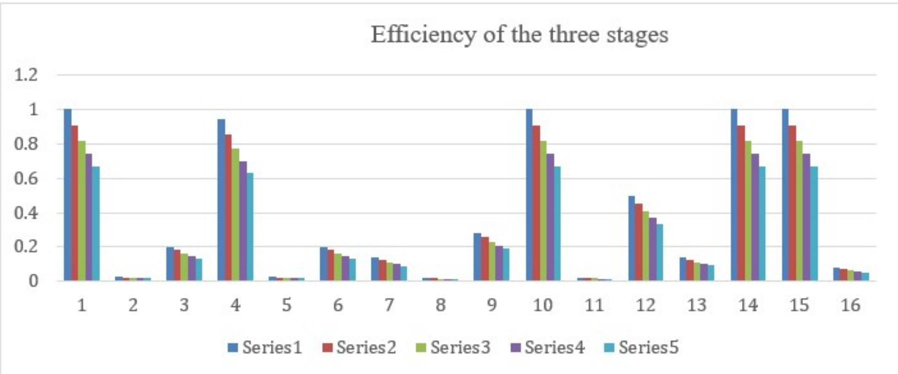

Figure 4: Efficiency of the three stages within network

Table 3: Total efficiency

	Level Of Confidence				
DMU	a=0	a=0.25	a=0.5	a=0.75	a=1
DMU 1	1	0.9048	0.8185	0.7403	0.6693
DMU 2	1	0.9048	0.8186	0.7404	0.6694
DMU 3	1	0.9048	0.8185	0.7403	0.6693
DMU 4	1	0.9048	0.8186	0.7404	0.6694
DMU 5	1	0.9048	0.8185	0.7404	0.6694
DMU 6	1	0.9048	0.8186	0.7403	0.6693
DMU 7	1	0.9048	0.8185	0.7403	0.6694

inevitable. Utilizing ***DEA*** in health system and modeling emergency operations in combined uncertainty conditions (solid-fuzzy optimization) with game theory (non-participatory) approach, the current research was able to not only reduce inputs and increase optimal outputs, it furthermore reduced the undesirable outputs pertinent to sustainability issues.

With the aim of validating the obtained findings, the results were reviewed and analyzed with the cooperation of three hospital administrators. In this study, the findings of the presented model as well as the models in the literature were submitted as a questionnaire to experts and they were requested to compare the findings with their expert/socialist views emanating from their long experiences. Concerning the findings of the envelopment analysis model of network data in the case of combined uncertainty in the health system, it is believed and concluded that the proposed model in this study is more in line with the real world.

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