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The regressions of least absolute shrinkage and selection operator with applications

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Abstract

Lasso regression model is a causal model based on providing a more accurate estimator through the model's dependence on shrinkage, in which the data values were reduced towards the data center. This model solves the problems of multicollinearity presence of high relationships between the explanatory variables of the model. In this research, a number of factors (sample size, number of explanatory variables and pollution rate) were adopted in order to observe the ability of these factors to effect Lasso regression.

Keywords: LASSO regressions, Mean Square Error, multicollinearity
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1 Introduction

In the case of a descriptive dependent variable, the Lasso regression is needed to represent the effect of a number of independent variables on the response variable. Many researches have been carried out The research submitted by (robert)in the year (1996) in which it is located the research a new method was hypothesized based on reducing the effect of non-influential variables by adopting their own zero parameters. The research included a number of simulation experiments, the results of which proved the ability of the proposed method to represent the desired effect [9].

The research submitted by (Rahim Alhamzawi,) and others in the year (2012) in which it is located In this research, the Bayesian Lasso regression was studied and for the case of the descriptive response variable, the method of greatest possibility was adopted by taking the gamma distribution as a primary distribution [1].

The research submitted by (Chao Sun) and others in the year (2022) in which it is located The research aims to show the effect of a group of factors related to the density of minerals and its effect on the maintenance of water. Bone health The research included data for a period of time (2015-2018). In this research, a regression model was presented for comparison according to the data collected. The data included a number of explanatory variables equal to (18) and finding Lasso estimators to determine their effect on the response variable The models showed the ability of the proposed models to provide estimations for the assumed models [8].

In this research, the Lasso regression method has been applied to the explanatory variables (17) and their effect on the response variable, which is the number of children who are born dead or are born alive and then died.

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2 Problem of Research

The research problem is that some factors, such as the presence of strong relationships between the explanatory variables of the model, the insufficient sample size and the presence of pollution within the sample.

3 Aim of Research

The research aims to know the extent of the ability of the Lasso regression to deal with the variables (sample size, number of explanatory variables and pollution rate).

4 LASSO Regressions

Regression analysis is based on showing and explaining the effect of one or more explanatory variables on changing the response variable and includes the following linear formula assumption [2]

$$E(Y/X) = X\beta \tag{4.1}$$

with
Y represent the response variable with ($n * 1$)
X represent the explanatory variables with $(n * p)$
β represent the unknown parameters with $(p * 1)$
p represent no of explanatory variables.

The linear assumption of the effect can lead to misleading results if there is a non-linear effect. Therefore, a non-parametric regression is taken, which takes into account the non-linear effect, but it has problems, including when the number of explanatory variables increases. Therefore, the single-indicator model is relied upon to solve this problem, and this model is one of the semi-parametric models by relying on the following formula [3]

$$E(Y/X = x) = g\left(X^T\beta\right). \tag{4.2}$$

The previous model has more flexibility than the parametric models because it allows for non-linear relationships between the change index $\left(X^T\beta\right)$ and the response variable while preserving the new properties in addition to the ability to reduce the dimensions and reduce the risk of determining the link function.

Least Absolute Shrinkage and Selection Operator (Lasso) method was first presented in the geophysical literature in the year (1982), then it was independently rediscovered in the year (1996) by the researcher Robert Tibshirani, who formulated this method and provided many ideas about its performance. which is a penalty function for the linear regression model, which is a method for estimating the parameters of the regression model as well as for selecting and organizing the variables included in the model to increase the explanatory accuracy of the regression models used in analyzing the phenomenon under study through model fit operations. To choose a subset of the covariates in the final model instead of using them all, in the Lasso method, the sum of squares of random errors is minimized with the sum of the absolute values of the regression model coefficients.

In (2008) Chen Lei Leng proposed an estimation method based on minimizing the following function

$$\sum_{i=1}^{n}\left[y_i - \sum_j x_{ij}\beta_j\right]^2 + \gamma\sum_{j=1}^{p}|\beta_j| \tag{4.3}$$

which provides a solution to quadratic programming problems and that (γ) represents a tuning parameter in which the strength of the penalty is controlled, which is basically the size of the shrinkage. For the previous formula when $(\gamma = 0)$ the estimators are equal to the estimators of least squares. When the value of (γ) is increased, more parameters are excluded by making them equal to zero and when $(\gamma = \infty)$ all the parameters are excluded increase in the value of (γ) corresponds to an increase in the bias values of the parameter estimators decrease in the value of (γ) corresponds to an increase in the variance of the estimators

Multicollinearity problem

Originally designed for Least squares models, LASSO reveals a large amount of estimator behavior via the Lasso parameter or the so-called Soft threse holding, including the relationship of the Lasso estimator with the Ridge Regression estimator and the Best subset selection estimator Which is analogous to the stepwise selection method, and also reveals (as in linear regression) that estimates of the Lasso coefficient should not be unique if the explanatory variables suffer from a poly linear problem.

he Lasso method has the ability to choose a subset that depends on the constraint formula, and although Lasso has been defined for least squares, the Lasso method can easily be used in a wide range of statistical models, including generalized linear models, generalized estimation coefficients, relative risk models and M estimators. Lasso can be used in many fields such as geometry, Bayesian statistics, and convex analysis.

Before the Lasso regression method, the most used method for choosing the explanatory variables that are included within the model was the Stepwise Selection method, which improves the accuracy of the model in certain cases, especially when some explanatory variables have a strong relationship with the response variable, which makes the prediction inaccurate, as well as The most popular Ridge Regression method that is used to improve the prediction accuracy of the regression model.

It improves prediction error by reducing large regression coefficients in order to reduce redundancy, but it does not perform co-selection and thus does not help make the model more interpretable. While Lasso can achieve both goals by making the set of the absolute values of the regression coefficients have less than a fixed value, forcing some coefficients to be equal to zero, while choosing a simpler model that does not include those coefficients.

5 Principle of LASSO Regression

The principle of the Lasso regression method is to reduce the sum of squares of the residuals according to a constraint that represents the absolute sum of the coefficients that are less than a certain constant. In order to do this, Lasso applies the process of shrinkage (regulation), as it makes a penalty for the regression coefficients and reduces some of them to zero, and during the process of selecting the variables, the variables whose coefficients are non-zero will be determined after the shrinkage process and they will be part of the model and the goal of this process is to reduce the prediction error.

In the Lasso method, there is an adjustment parameter (settlement) that controls the power of penalization (penalty) of the regression coefficients and occupies great importance in that. That is, the larger the value of the setting parameter, the greater the number of coefficients equal to zero. And if the adjustment parameter is equal to zero, we get an OLS Regression.

6 Advantages of LASSO Regression

There are many advantages to using the Lasso method, including the following [5]:

a. Lasso can provide very good predictive accuracy because reducing and removing variables can reduce variance without significantly increasing bias, and this is especially useful when we have a small number of observations and a large number of variables.

b. Lasso helps to increase the possibility of interpreting the model by eliminating irrelevant variables that are not related to the response variable.

Thus, the Lasso method is a method for selecting and organizing the variables included in the regression model.

7 Formula of LASSO Regression

The parameters of Lasso regression were estimated according to the principle of least squares from the basic formula as follows [4]:

Let us have a sample of (N) states, each state consisting of (P)of explanatory variables and one intentional variable (yi), and let (x) represent the vector of explanatory variables for the $\left(j^{\text{th}}\right)$ state, then the goal of Lasso gradient is to solve the following equation:

$$\min\left\{\frac{1}{N}\sum_{i=1}^{N}\left(yi-\beta_0-{x_i}^{\mathrm{T}}\beta\right)^2\right\} \tag{7.1}$$

subject to

$$\sum_{\mathrm{j}=1}^{\mathrm{p}}|\beta_{\mathrm{j}}|\leq \mathrm{t}$$

(t) : is a preset free parameter that specifies the amount of flattening (shrink).
(X) : matrix of explanatory variables.

$$X_{ij}=(x_i)_j$$

and $\left(\mathrm{x_i}_{\mathrm{T}}^{\mathrm{T}}\right)$ is the $\left(\mathrm{j^{th}}\right)$ row of the matrix (X).

The lasso formula can be written as:

$$\min_{\beta_0,\beta}\left\{\frac{1}{N}\|y-X\beta\|_2^2\right\} \tag{7.2}$$

subject to $\|\beta\|\leq \mathrm{t}$ and

$$||\beta||_{\mathrm{p}}=\left(\sum_{\mathrm{i}=1}^{\mathrm{N}}|\beta_{\mathrm{i}}|^{\mathrm{p}}\right)^{\frac{1}{\mathrm{p}}} \tag{7.3}$$

and when $(\mathrm{P}=1)$, then $(\|\beta\|_1)$ becomes the standard length (ℓ^{p}) and $(\mathrm{I_N})$ the units vector (Nx1). ($\overline{\mathrm{x}}$) stands for the standard mean of the data points $(\mathrm{x_i})$ and $(\overline{\mathrm{y}})$ the mean of the dependent variable (the response variable $(\mathrm{y_i})$) and the estimate

$$\hat{\beta}_0=\overline{\mathrm{y}}-\overline{\mathrm{x}}_{\mathrm{i}}^{\mathrm{T}}\beta \tag{7.4}$$

since:

$$\mathrm{yi}-\beta_0-\mathrm{x_i^T}\beta=\mathrm{yi}-\left(\overline{\mathrm{y}}-\overline{\mathrm{x}}_{\mathrm{i}}^{\mathrm{T}}\beta\right)-\mathrm{x_i^T}\beta \tag{7.5}$$

$$\mathrm{yi}-\beta_0-\mathrm{x_i^T}\beta=(\mathrm{yi}-\overline{\mathrm{y}})-(\mathrm{x_i}-\overline{\mathrm{x}})^{\mathrm{T}}\beta \tag{7.6}$$

Thus, it is natural to work with the variables that have been centralized (making their mean equal to zero) in addition to the explanatory variables being Typically standardizes

$$\left(\frac{1}{N}\sum_{\mathrm{i}=1}^{\mathrm{N}}{\mathrm{x_i}}^2=1\right)$$

and

$$\left(\frac{1}{N}\sum_{\mathrm{i}=1}^{\mathrm{N}}\mathrm{x_i}=0\right)$$

The above formula can be rewritten as follows: -

$$\min_{\beta_0,\beta}\left\{\frac{1}{N}\|y-X\beta\|_2^2\right\} \tag{7.7}$$

subject to $\|\beta\|_1\leq \mathrm{t}$. It has the form of a Lagrange multiplier as follows:

$$\min_{\beta\in\mathrm{R}^P}\left\{\frac{1}{\mathrm{N}}\|\mathrm{y}-\mathrm{X}\beta\|_2^2+\lambda\|\beta\|_1\right\} \tag{7.8}$$

(λ) is the parameter that controls the penalty force (contraction) on the gradient parameters.

8 Properties of Lasso Estimator

There are some characteristics of a lasso estimator that we can list as follows [7]:

(1-8)-Orthonormal Covariates

Suppose the covariates are naturally orthogonal, such that

$$(\mathrm{x_i} \mid \mathrm{x_j}) = \delta_{\mathrm{ij}} \tag{8.1}$$

such that the inner product ((.—.)), the Kroncher delta (δ_{ij}) and $\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

By using the iterative sub gradient method, which is one of the multiple analysis methods to solve the least convex problems, we get

$$\hat{\beta}_{\mathrm{j}} = \mathrm{S}_{\mathrm{N}\lambda}\left(\widehat{\beta}_{\mathrm{j}}^{\mathrm{oLS}}\right) = \hat{\beta}_{\mathrm{j}}^{\mathrm{oLS}} \ \mathrm{Max}\left(1 - \frac{\mathrm{N}\lambda}{\left|\hat{\beta}_{\mathrm{j}}^{\mathrm{OLS}}\right|}\right) \tag{8.2}$$

and

$$\hat{\beta}_{\mathrm{j}}^{\mathrm{OLS}} = \left(\mathrm{X^T X}\right)^{-1} \mathrm{X^T Y} \tag{8.3}$$

($\mathrm{S}_{\mathrm{N}\lambda}$) represent Smooth Threshold.

Since it converts the values to zero (making it completely zero if it is small enough) instead of setting smaller values to zero and leaving the largest values untouched as a solid threshold factor symbolized by ($\mathrm{H}_{\mathrm{N}\lambda}$), and this can be compared with the regression of the letter, so the goal is to reduce the following amount

$$\min_{\beta \in \mathrm{R^P}} \left\{ \frac{1}{\mathrm{N}} \|\mathrm{y} - \mathrm{X}\beta\|_2^2 + \lambda \|\beta\|_2^2 \right\} \tag{8.4}$$

and

$$\hat{\beta}_j = (1 + N\lambda)^{-1} \widehat{\beta}_j^{\mathrm{oLS}} . \tag{8.5}$$

Since the letter regression reduces all coefficients by the variable factor of $\left((1 + N\lambda)^{-1}\right)$. It does not set any of the coefficients with a zero

$$\min_{\beta \in R^P} \left\{ \frac{1}{N} \|y - X\beta\| \Big|_2^2 + \lambda \|\beta\|_0 \right\} \tag{8.6}$$

and ($\|_0$ |) ($\|$) represent the height $\left(\ell^0\right)$ who is known as ($||\mathrm{z}|| = \mathrm{m}$) If (m) are exactly components of (z), then they are nonzero, and you can write as follows:

$$\widehat{\beta}_{\mathrm{j}} = \mathrm{H}_{\sqrt{\mathrm{N}\lambda}}\left(\widehat{\beta}_{\mathrm{j}}^{\mathrm{OLS}}\right) = \left(\left|\widehat{\beta}_{\mathrm{j}}^{\mathrm{OLS}}\right| > \sqrt{\mathrm{N}\lambda}\right) \tag{8.7}$$

such that $\left(\mathrm{H}_{\sqrt{\mathrm{N}\lambda}}\right)$ represent steel threshold limit and (I) represent indicator function. We note from this that Lasso estimates combine the advantages of character regression and partial best-choice regression. It converts all coefficients to zero with a fixed value and sets it to zero if it reaches it.

(2-8)-Correlated Covariates

Returning to the general form of the lasso in which the different covariates may not be explanatory, in the case where two explanatory variables ($i; j$) are identical for each case so that [6]:

$$x_i = x_j .$$

The values of parameters $\left(\hat{B}_i\right)$ and (B_j) that reduce the lasso objective function are not uniquely defined, but rather there is $\left(\widehat{B}_i; \hat{B}_j \geq 0\right)$, then if ($\mathrm{s} \in [0, 1]$). Then replace ($B_i$) with $\left(s\left(\hat{B}_i + \hat{B}_j\right)\right)$ and replacing $\left(\hat{B}_j\right)$ with $\left(s\left(\hat{B}_i + \hat{B}_j - 1\right)\right.$ keeping the rest of $\left(\hat{B}_i\right)$ and we get a new solution and the lasso function continues to reduce coefficients.

9 Data of Research

research data included the information available in the records of women's visits to the health centers of the Department of Health, where data were collected on (100) women and the research variables were as follows:

(y) represents the dependent variable, which is the number of children born (live + born and then died) (x_1) age of the woman, (x_2) Age at marriage, (x_3) women's academic achievement (1-8), (x_4) husband's academic achievement $(1-8)$, (x_5) Women's weight, (x_6) Women's use of contraceptives (1 - use - 2 - do not use), (x_7) Women smoking (1- smoke 2- do not smoke), (x_8) husband's age, (x_9) Husband's profession (1- 4), (x_{10}) Marriage period, (x_{11}) Number of dead children, (x_{12}) Number of hours of exercise per week, (x_{13}) Thyroid disease (1infected 2- unaffected), (x_{14}) number of hours a woman sleeps per day, (x_{15}) Women taking medicines (1 - took - 2 - did not take), (x_{16}) The duration of breastfeeding, (x_{17}) Mother's profession (1- housewife 2- employee).

10 Experimental Results

After applying Lasso regression to the data of research, the following results appeared

Table (1) It represents the estimations of the Lasso regression for the explanatory variables

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}
0.04139	0.00009	0	0	0	2.62578	1.12587	0	0	0.28934	1.24871	0	0	0	1.18964	0.17019	1.70064
0.04134	0.00005	0	0	0	2.62552	1.12567	0	0	0.28933	1.24869	0	0	0	1.18961	0.17019	1.70005
0.04129	0.00003	0	0	0	2.62526	1.12548	0	0	0.28932	1.24866	0	0	0	1.18956	0.17018	1.69948
0.04123	0	0	0	0	2.62492	1.12526	0	0	0.28932	1.24863	0	0	0	1.18953	0.17017	1.69879
0.04116	0	0	0	0	2.6246	1.12508	0	0	0.28933	1.24857	0	0	0	1.18948	0.17017	1.69817
0.04103	0	0	0	0	2.62398	1.12474	0	0	0.28935	1.24853	0	0	0	1.1895	0.17017	1.69715
0.04093	0	0	0	0	2.62347	1.12447	0	0	0.28936	1.24846	0	0	0	1.18947	0.17017	1.69627
0.04079	0	0	0	0	2.62281	1.12412	0	0	0.28938	1.2484	0	0	0	1.18947	0.17017	1.69518
0.04066	0	0	0	0	2.62214	1.12377	0	0	0.2894	1.24833	0	0	0	1.18945	0.17016	1.69404
0.04051	0	0	0	0	2.62139	1.12337	0	0	0.28942	1.24825	0	0	0	1.18944	0.17016	1.69278
0.04034	0	0	0	0	2.62056	1.12293	0	0	0.28944	1.24816	0	0	0	1.18942	0.17016	1.69138
0.04015	0	0	0	0	2.61965	1.12244	0	0	0.28946	1.24807	0	0	0	1.1894	0.17016	1.68984
0.03995	0	0	0	0	2.61865	1.1219	0	0	0.28949	1.24796	0	0	0	1.18939	0.17015	1.68815
0.03972	0	0	0	0	2.61754	1.12131	0	0	0.28952	1.24785	0	0	0	1.18937	0.17015	1.68629
0.03948	0	0	0	0	2.61636	1.12066	0	0	0.28955	1.24773	0	0	0	1.18934	0.17015	1.68427
0.03919	0	0	0	0	2.615	1.11992	0	0	0.2896	1.2476	0	0	0	1.18933	0.17014	1.68197
0.0389	0	0	0	0	2.61355	1.11915	0	0	0.28963	1.24745	0	0	0	1.1893	0.17014	1.67953
0.03857	0	0	0	0	2.61195	1.11827	0	0	0.28968	1.24729	0	0	0	1.18927	0.17013	1.6768
0.0382	0	0	0	0	2.61017	1.11732	0	0	0.28973	1.24712	0	0	0	1.18925	0.17013	1.67381
0.03781	0	0	0	0	2.60823	1.11629	0	0	0.28978	1.24692	0	0	0	1.18922	0.17013	1.67056
0.03737	0	0	0	0	2.6061	1.11513	0	0	0.28984	1.24671	0	0	0	1.1892	0.17012	1.66695

From table (2) and figure (2) We note that the advanced variables had variable numbers of zeros, which represent the relative importance of them within the Lasso regression. The lambda-1SE was (φ), the lambda-min deviance (ω), the summation of fit information SE(ϵ), the summation of fit-information Deviance (θ),the summation of fit-information lambda $(\emptyset)$ and the summation of fit-information intercept (α) It can be obtained from the following table From the estimated values we can show that the fit information SE was very small and the lasso regression gives good estimates

11 Conclusions and suggestions

After applying the Lasso regression for research data, many conclusions and recommendations such that

0.03689	0	0	0	0	2.60375	1.11388	0	0	0.2899	1.24647	0	0	0	1.18917	0.17012	1.66302
0.03636	0	0	0	0	2.60118	1.11249	0	0	0.28998	1.24622	0	0	0	1.18913	0.17011	1.65866
0.03578	0	0	0	0	2.59835	1.11097	0	0	0.29005	1.24593	0	0	0	1.1891	0.17011	1.65392
0.03514	0	0	0	0	2.59524	1.10929	0	0	0.29014	1.24563	0	0	0	1.18906	0.1701	1.64867
0.03445	0	0	0	0	2.59182	1.10747	0	0	0.29023	1.24529	0	0	0	1.18903	0.1701	1.64295
0.03368	0	0	0	0	2.58806	1.10547	0	0	0.29033	1.24491	0	0	0	1.18899	0.17009	1.63667
0.03283	0	0	0	0	2.58395	1.10325	0	0	0.29045	1.24451	0	0	0	1.18893	0.17008	1.62973
0.03191	0	0	0	0	2.57941	1.10083	0	0	0.29057	1.24406	0	0	0	1.18889	0.17008	1.62215
0.0309	0	0	0	0	2.57444	1.09818	0	0	0.29071	1.24356	0	0	0	1.18884	0.17007	1.61383
0.02977	0	0	0	0	2.569	1.09524	0	0	0.29086	1.24303	0	0	0	1.18878	0.17006	1.60466
0.02855	0	0	0	0	2.56299	1.09204	0	0	0.29102	1.24244	0	0	0	1.18872	0.17005	1.59463
0.02721	0	0	0	0	2.5564	1.08853	0	0	0.2912	1.24178	0	0	0	1.18867	0.17004	1.58361
0.02574	0	0	0	0	2.54917	1.08467	0	0	0.29139	1.24107	0	0	0	1.1886	0.17003	1.57152
0.02411	0	0	0	0	2.54126	1.08042	0	0	0.29162	1.2403	0	0	0	1.18851	0.17002	1.55822
0.02233	0	0	0	0	2.53254	1.07577	0	0	0.29185	1.23943	0	0	0	1.18844	0.17001	1.54365
0.02038	0	0	0	0	2.52297	1.07067	0	0	0.29211	1.23849	0	0	0	1.18835	0.16999	1.52766
0.01824	0	0	0	0	2.51247	1.06506	0	0	0.29239	1.23745	0	0	0	1.18826	0.16998	1.5101
0.01588	0	0	0	0	2.50098	1.05889	0	0	0.29272	1.23633	0	0	0	1.18814	0.16996	1.4908
0.0133	0	0	0	0	2.48832	1.05214	0	0	0.29306	1.23508	0	0	0	1.18803	0.16994	1.46965
0.01046	0	0	0	0	2.47443	1.04474	0	0	0.29343	1.23371	0	0	0	1.18791	0.16993	1.44644
0.00735	0	0	0	0	2.45918	1.0366	0	0	0.29385	1.23221	0	0	0	1.18778	0.16991	1.42095
0.00393	0	0	0	0	2.44249	1.02766	0	0	0.29431	1.23057	0	0	0	1.18762	0.16988	1.39297
0.00005	0	0	0	0	2.42277	1.01763	0	0	0.29477	1.22873	0	0	0	1.188	0.16994	1.36148
0	0	0	0	0	2.39784	1.01497	0	0	0.29314	1.22301	0	0	0	1.18838	0.17015	1.33822
0	0	0	0	0	2.3709	1.01214	0	0	0.29136	1.21674	0	0	0	1.18854	0.17035	1.3129
0	0	0	0	0	2.34123	1.00903	0	0	0.28942	1.20985	0	0	0	1.18873	0.17056	1.28507
0	0	0	0	0	2.31198	1.00004	0	0	0.28597	1.20483	0	0	0	1.18858	0.17064	1.24739
0	0	0	0	0	2.28107	0.98753	0	0	0.28153	1.20054	0	0	0	1.18845	0.17068	1.20251
0	0	0	0	0	2.2472	0.97382	0	0	0.27668	1.19583	0	0	0	1.18828	0.17072	1.15332
0	0	0	0	0	2.21003	0.95877	0	0	0.27135	1.19066	0	0	0	1.18809	0.17076	1.09932
0	0	0	0	0	2.16922	0.94225	0	0	0.26549	1.18499	0	0	0	1.18789	0.17081	1.04005
0	0	0	0	0	2.12438	0.92411	0	0	0.25906	1.17876	0	0	0	1.18769	0.17086	0.97495
0	0	0	0	0	2.08561	0.90734	0	0	0.25402	1.17234	0	0	0	1.18458	0.17029	0.91436
0	0	0	0	0	2.06502	0.89708	0	0	0.2532	1.16596	0	0	0	1.17546	0.16836	0.87379
0	0	0	0	0	2.04252	0.88579	0	0	0.2523	1.15898	0	0	0	1.16542	0.16623	0.82924
0	0	0	0	0	2.01782	0.87339	0	0	0.25132	1.15131	0	0	0	1.1544	0.1639	0.78035
0	0	0	0	0	1.98977	0.85977	0	0	0.25023	1.14298	0	0	0	1.14297	0.1614	0.72688
0	0	0	0	0	1.9648	0.84448	0	0	0.24911	1.13239	0	0	0	1.12877	0.15861	0.67005
0	0	0	0	0	1.94649	0.82652	0	0	0.24799	1.11765	0	0	0	1.11318	0.15579	0.61358
0	0	0	0	0	1.91949	0.80932	0	0	0.24663	1.10196	0	0	0	1.1005	0.15325	0.55643
0	0	0	0	0	1.87535	0.79646	0	0	0.24491	1.08529	0	0	0	1.09504	0.15159	0.50345
0	0	0	0	0	1.82698	0.78228	0	0	0.24301	1.06704	0	0	0	1.08909	0.14977	0.44535
0	0	0	0	0	1.77389	0.76672	0	0	0.24093	1.04702	0	0	0	1.08256	0.14776	0.38158
0	0	0	0	0	1.71555	0.74971	0	0	0.23866	1.02498	0	0	0	1.07536	0.14557	0.31155
0	0	0	0	0	1.6516	0.73097	0	0	0.23615	1.00086	0	0	0	1.06749	0.14316	0.23473
0	0	0	0	0	1.58133	0.71048	0	0	0.23341	0.97432	0	0	0	1.05882	0.14051	0.15038
0	0	0	0	0	1.50364	0.68837	0	0	0.23045	0.94495	0	0	0	1.0492	0.13763	0.05756
0	0	0	0	0	1.44443	0.6566	0	0	0.22684	0.92192	0	0	0	1.03882	0.13366	0
0	0	0	0	0	1.41474	0.61278	0	0	0.22259	0.90875	0	0	0	1.027	0.12821	0
0	0	0	0	0	1.38211	0.56476	0	0	0.21793	0.89425	0	0	0	1.014	0.12224	0
0	0	0	0	0	1.34634	0.51198	0	0	0.2128	0.87839	0	0	0	0.99976	0.11568	0
0	0	0	0	0	1.30702	0.454	0	0	0.20718	0.86101	0	0	0	0.98422	0.10848	0
0	0	0	0	0	1.25603	0.40972	0	0	0.20469	0.84014	0	0	0	0.93587	0.09942	0
0	0	0	0	0	1.19886	0.36342	0	0	0.20244	0.81718	0	0	0	0.87889	0.08932	0
0	0	0	0	0	1.17819	0.34008	0	0	0.20088	0.78046	0	0	0	0.81085	0.07808	0
0	0	0	0	0	1.18408	0.33292	0	0	0.19979	0.73253	0	0	0	0.73288	0.06565	0

0	0	0	0	0	1.20475	0.31843	0	0	0.19785	0.68601	0	0	0	0.64487	0.05143	0
0	0	0	0	0	1.23139	0.3014	0	0	0.19526	0.62929	0	0	0	0.54705	0.03677	0
0	0	0	0	0	1.25971	0.28311	0	0	0.1924	0.56542	0	0	0	0.43992	0.02093	0
0	0	0	0	0	1.29096	0.26309	0	0	0.18926	0.49528	0	0	0	0.32219	0.00354	0
0	0	0	0	0	1.25125	0.24669	0	0	0.18672	0.4285	0	0	0	0.22787	0	0
0	0	0	0	0	1.18939	0.23018	0	0	0.18415	0.35771	0	0	0	0.13286	0	0
0	0	0	0	0	1.1215	0.21211	0	0	0.18134	0.27999	0	0	0	0.02855	0	0
0	0	0	0	0	1.00269	0.1603	0	0	0.17915	0.20814	0	0	0	0	0	0
0	0	0	0	0	0.87853	0.07909	0	0	0.17551	0.14621	0	0	0	0	0	0
0	0	0	0	0	0.7397	0	0	0	0.17127	0.0778	0	0	0	0	0	0
0	0	0	0	0	0.54546	0	0	0	0.1661	0.00883	0	0	0	0	0	0
0	0	0	0	0	0.33172	0	0	0	0.15845	0	0	0	0	0	0	0
0	0	0	0	0	0.09705	0	0	0	0.14978	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.14026	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.12982	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.11837	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.1058	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.092	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.07685	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.06023	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.04199	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.02197	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

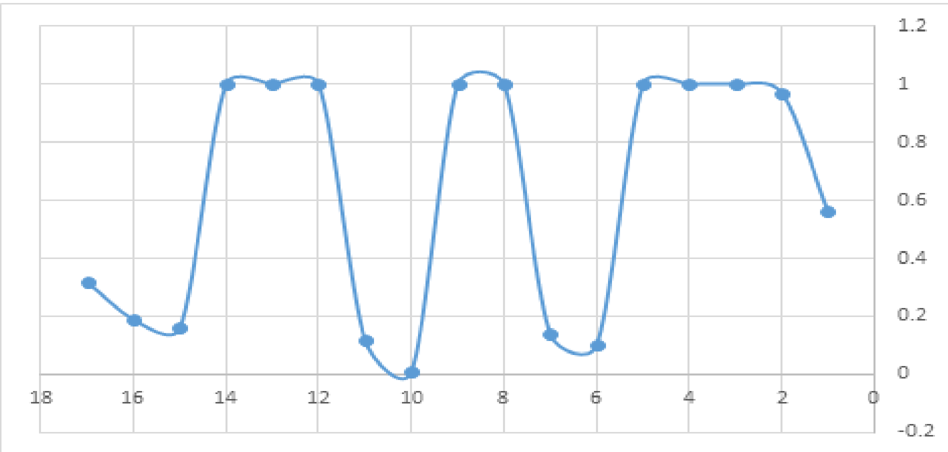

Figure 1: represents the ratio of the number of zeros within the interpreted variables

Table (2) It represents the advanced estimations of the Lasso regression for the explanatory variables

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}
0.02561	0	0	0	0	1.16191	0.22406	0	0	0.05941	0.23491	0	0	0	0.38989	0.02938	0.3838
0.02559	0	0	0	0	1.16183	0.224	0	0	0.05942	0.23489	0	0	0	0.38984	0.02938	0.38364
0.02556	0	0	0	0	1.16174	0.22394	0	0	0.05942	0.23487	0	0	0	0.38977	0.02938	0.38343
0.02553	0	0	0	0	1.16163	0.22387	0	0	0.05942	0.23485	0	0	0	0.38971	0.02938	0.38322
0.02551	0	0	0	0	1.16152	0.2238	0	0	0.05942	0.23483	0	0	0	0.38964	0.02938	0.383

1- The ability of the Lasso regression to provide good estimations for the research data.

2- The ability of the Lasso regression to sort the influential explanatory variables and isolate them from the rest.

3- The ability of a lasso regression to separate the influencing variables from the inactive ones.

0.02547	0	0	0	0	1.1614	0.22372	0	0	0.05942	0.23481	0	0	0	0.38956	0.02937	0.38274
0.02543	0	0	0	0	1.16127	0.22362	0	0	0.05943	0.23478	0	0	0	0.38947	0.02937	0.38245
0.0254	0	0	0	0	1.16112	0.22353	0	0	0.05943	0.23475	0	0	0	0.38937	0.02937	0.38215
0.02535	0	0	0	0	1.16096	0.22342	0	0	0.05943	0.23472	0	0	0	0.38927	0.02937	0.38181
0.02531	0	0	0	0	1.16079	0.22331	0	0	0.05943	0.23468	0	0	0	0.38916	0.02936	0.38145
0.02525	0	0	0	0	1.16059	0.22317	0	0	0.05944	0.23464	0	0	0	0.38903	0.02936	0.38103
0.02519	0	0	0	0	1.16038	0.22303	0	0	0.05944	0.2346	0	0	0	0.38889	0.02936	0.38058
0.02513	0	0	0	0	1.16014	0.22287	0	0	0.05944	0.23455	0	0	0	0.38874	0.02935	0.38009
0.02506	0	0	0	0	1.15988	0.2227	0	0	0.05945	0.2345	0	0	0	0.38857	0.02935	0.37955
0.02499	0	0	0	0	1.15961	0.22251	0	0	0.05945	0.23445	0	0	0	0.38838	0.02934	0.37896
0.0249	0	0	0	0	1.15929	0.2223	0	0	0.05946	0.23439	0	0	0	0.38818	0.02934	0.3783
0.02481	0	0	0	0	1.15896	0.22207	0	0	0.05946	0.23432	0	0	0	0.38796	0.02933	0.37759
0.02471	0	0	0	0	1.15859	0.22182	0	0	0.05947	0.23425	0	0	0	0.38771	0.02932	0.37679
0.0246	0	0	0	0	1.15817	0.22154	0	0	0.05948	0.23416	0	0	0	0.38744	0.02932	0.37592
0.02448	0	0	0	0	1.15772	0.22123	0	0	0.05949	0.23408	0	0	0	0.38715	0.02931	0.37498
0.02434	0	0	0	0	1.15723	0.22089	0	0	0.05949	0.23398	0	0	0	0.38682	0.0293	0.37393
0.0242	0	0	0	0	1.15668	0.22053	0	0	0.0595	0.23387	0	0	0	0.38647	0.02929	0.37279
0.02404	0	0	0	0	1.15608	0.22013	0	0	0.05951	0.23375	0	0	0	0.38608	0.02928	0.37153
0.02386	0	0	0	0	1.15543	0.21968	0	0	0.05952	0.23362	0	0	0	0.38565	0.02927	0.37015
0.02366	0	0	0	0	1.1547	0.21919	0	0	0.05954	0.23348	0	0	0	0.38519	0.02926	0.36863
0.02345	0	0	0	0	1.15391	0.21865	0	0	0.05955	0.23333	0	0	0	0.38468	0.02924	0.36697
0.02321	0	0	0	0	1.15304	0.21806	0	0	0.05957	0.23316	0	0	0	0.38411	0.02923	0.36513
0.02295	0	0	0	0	1.15208	0.21741	0	0	0.05958	0.23297	0	0	0	0.38349	0.02921	0.36312
0.02267	0	0	0	0	1.15102	0.2167	0	0	0.0596	0.23277	0	0	0	0.38282	0.0292	0.36092
0.02236	0	0	0	0	1.14986	0.21592	0	0	0.05962	0.23255	0	0	0	0.38208	0.02918	0.3585
0.02202	0	0	0	0	1.14858	0.21507	0	0	0.05964	0.2323	0	0	0	0.38127	0.02916	0.35584
0.02164	0	0	0	0	1.14717	0.21413	0	0	0.05966	0.23203	0	0	0	0.38038	0.02914	0.35293
0.02123	0	0	0	0	1.14562	0.21309	0	0	0.05969	0.23174	0	0	0	0.3794	0.02911	0.34972
0.02078	0	0	0	0	1.14392	0.21196	0	0	0.05972	0.23142	0	0	0	0.37833	0.02909	0.3462
0.02028	0	0	0	0	1.14205	0.21071	0	0	0.05975	0.23106	0	0	0	0.37716	0.02906	0.34234
0.01974	0	0	0	0	1.13999	0.20935	0	0	0.05978	0.23068	0	0	0	0.37588	0.02902	0.3381
0.01914	0	0	0	0	1.13769	0.20785	0	0	0.05982	0.23025	0	0	0	0.37449	0.02899	0.33346
0.01849	0	0	0	0	1.13516	0.20621	0	0	0.05985	0.22979	0	0	0	0.37297	0.02896	0.32836
0.01777	0	0	0	0	1.1324	0.2044	0	0	0.0599	0.22929	0	0	0	0.37129	0.02892	0.32275
0.01699	0	0	0	0	1.12934	0.20242	0	0	0.05995	0.22873	0	0	0	0.36945	0.02887	0.31659
0.01612	0	0	0	0	1.12599	0.20025	0	0	0.06	0.22813	0	0	0	0.36744	0.02883	0.30982
0.01517	0	0	0	0	1.12231	0.19786	0	0	0.06006	0.22747	0	0	0	0.36523	0.02877	0.30238
0.01412	0	0	0	0	1.11824	0.19524	0	0	0.06013	0.22675	0	0	0	0.36281	0.02872	0.29421
0.01297	0	0	0	0	1.11377	0.19237	0	0	0.06021	0.22596	0	0	0	0.36016	0.02865	0.28523
0.01172	0	0	0	0	1.10883	0.18921	0	0	0.0603	0.22511	0	0	0	0.35726	0.02859	0.27537
0.01033	0	0	0	0	1.10339	0.18575	0	0	0.06039	0.22418	0	0	0	0.35409	0.02851	0.26453
0.00882	0	0	0	0	1.09737	0.18196	0	0	0.0605	0.22316	0	0	0	0.35062	0.02843	0.25261
0.00715	0	0	0	0	1.09074	0.17779	0	0	0.06062	0.22206	0	0	0	0.34683	0.02834	0.23952
0.00532	0	0	0	0	1.0834	0.17322	0	0	0.06076	0.22086	0	0	0	0.3427	0.02825	0.22513
0.00332	0	0	0	0	1.07528	0.16821	0	0	0.06091	0.21956	0	0	0	0.33818	0.02814	0.20931
0.00112	0	0	0	0	1.0662	0.16269	0	0	0.06107	0.21813	0	0	0	0.3333	0.02804	0.19192
0.00028	0	0	0	0	1.06267	0.16096	0	0	0.0615	0.21683	0	0	0	0.32763	0.02738	0.17941
0	0	0	0	0	1.05736	0.16015	0	0	0.06168	0.21505	0	0	0	0.32205	0.0267	0.16703
0	0	0	0	0	1.04973	0.15961	0	0	0.06163	0.21281	0	0	0	0.31648	0.02604	0.15379
0	0	0	0	0	1.04139	0.15896	0	0	0.06157	0.21046	0	0	0	0.31051	0.02534	0.13949
0	0	0	0	0	1.02736	0.15961	0	0	0.06132	0.20874	0	0	0	0.30745	0.02499	0.12843
0	0	0	0	0	1.01197	0.16031	0	0	0.06106	0.20688	0	0	0	0.30414	0.02461	0.11633
0	0	0	0	0	0.99509	0.16107	0	0	0.06077	0.20486	0	0	0	0.30059	0.02421	0.10309
0	0	0	0	0	0.97656	0.1619	0	0	0.06044	0.20267	0	0	0	0.29677	0.02379	0.08859
0	0	0	0	0	0.95622	0.1628	0	0	0.06009	0.20029	0	0	0	0.29267	0.02335	0.0727
0	0	0	0	0	0.9339	0.16376	0	0	0.05969	0.19772	0	0	0	0.28829	0.02288	0.05533

0	0	0	0	0	0.90938	0.16482	0	0	0.05926	0.19493	0	0	0	0.28362	0.0224	0.03629
0	0	0	0	0	0.88243	0.16596	0	0	0.05879	0.19191	0	0	0	0.27864	0.0219	0.01544
0	0	0	0	0	0.85462	0.16525	0	0	0.05818	0.19023	0	0	0	0.27552	0.02136	0
0	0	0	0	0	0.83192	0.16016	0	0	0.05734	0.19127	0	0	0	0.27568	0.02065	0
0	0	0	0	0	0.81426	0.15475	0	0	0.05641	0.19103	0	0	0	0.27311	0.01977	0
0	0	0	0	0	0.79525	0.14878	0	0	0.05539	0.19079	0	0	0	0.27038	0.01884	0
0	0	0	0	0	0.77456	0.14299	0	0	0.0545	0.19087	0	0	0	0.26571	0.01769	0
0	0	0	0	0	0.7517	0.13853	0	0	0.05407	0.19165	0	0	0	0.25639	0.01607	0
0	0	0	0	0	0.7273	0.13359	0	0	0.05359	0.19244	0	0	0	0.24622	0.01436	0
0	0	0	0	0	0.70131	0.12809	0	0	0.05307	0.1932	0	0	0	0.23514	0.01257	0
0	0	0	0	0	0.67369	0.12197	0	0	0.05249	0.19393	0	0	0	0.22307	0.0107	0
0	0	0	0	0	0.64585	0.11541	0	0	0.05186	0.19069	0	0	0	0.20952	0.009	0
0	0	0	0	0	0.61592	0.10831	0	0	0.05117	0.18658	0	0	0	0.19458	0.00728	0
0	0	0	0	0	0.58268	0.10123	0	0	0.05046	0.18097	0	0	0	0.17755	0.00566	0
0	0	0	0	0	0.56733	0.09851	0	0	0.0501	0.1712	0	0	0	0.16128	0.00371	0
0	0	0	0	0	0.55317	0.09586	0	0	0.04973	0.16028	0	0	0	0.14356	0.00159	0
0	0	0	0	0	0.53405	0.09328	0	0	0.04935	0.14864	0	0	0	0.12545	0	0
0	0	0	0	0	0.50294	0.09138	0	0	0.04902	0.13669	0	0	0	0.10904	0	0
0	0	0	0	0	0.47051	0.08923	0	0	0.04865	0.12378	0	0	0	0.09119	0	0
0	0	0	0	0	0.43682	0.08677	0	0	0.04822	0.10983	0	0	0	0.07177	0	0
0	0	0	0	0	0.40199	0.08397	0	0	0.04772	0.09478	0	0	0	0.05064	0	0
0	0	0	0	0	0.36612	0.08077	0	0	0.04715	0.07853	0	0	0	0.02764	0	0
0	0	0	0	0	0.32936	0.07711	0	0	0.04649	0.06099	0	0	0	0.0026	0	0
0	0	0	0	0	0.28251	0.06005	0	0	0.04607	0.04404	0	0	0	0	0	0
0	0	0	0	0	0.23302	0.03999	0	0	0.04561	0.02596	0	0	0	0	0	0
0	0	0	0	0	0.18634	0.01396	0	0	0.04476	0.00883	0	0	0	0	0	0
0	0	0	0	0	0.13331	0	0	0	0.04327	0	0	0	0	0	0	0
0	0	0	0	0	0.06978	0	0	0	0.04137	0	0	0	0	0	0	0
0	0	0	0	0	0.00364	0	0	0	0.03927	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.03686	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.0342	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.03129	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.02808	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.02454	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.02062	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.01629	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.01146	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.00607	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

4- Logistic regression can be used to compare with the results.

5- Poisson regression can be used to estimate model parameters and compare with results.

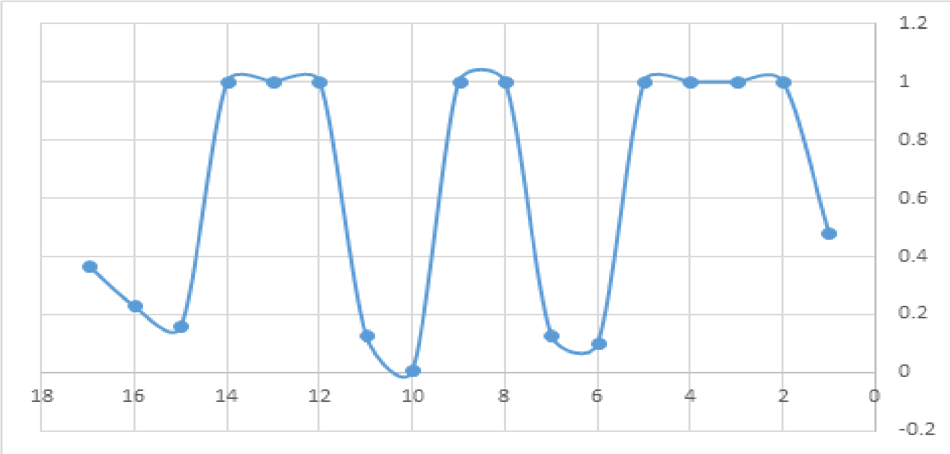

Figure 2: represents the ratio of the number of zeros within the advanced variables

Table (3) represents the $(\varphi.\omega.\epsilon.\theta.\emptyset.\phi.\alpha)$ values

The estimated parameter	The value
φ	-65.4631
ω	24.46822
ϵ	8160.518
θ	1020.641
$\emptyset$	0.011874
α	0.076329

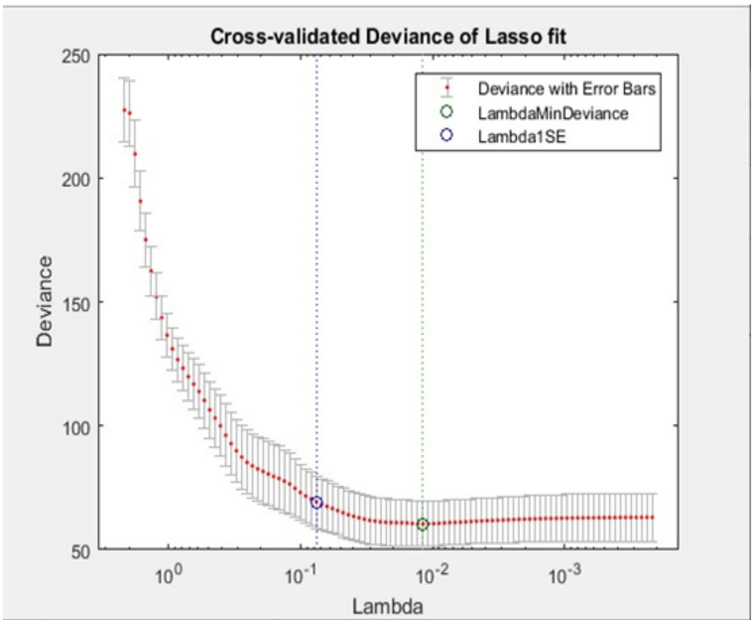

Figure 3: represents the Lambda values

References

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