Int. J. Nonlinear Anal. Appl. 14 (2023) 3, 103–112
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.28314.3858

The effect of weight control and weighing method of efficiency in data envelopment analysis

Monavar Karbalaie Alillou[a], Behrouz Daneshian[b,*], Farzin Modarres Khiyabani[a], Farhad Hosseinzadeh Lotfi[c]

[a] Department of Mathematics, Tabriz Branch, Islamic Azad University, Tabriz, Iran

[b] Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran

[c] Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

(Communicated by Ehsan Kozegar)

Abstract

Data Envelopment Analysis (DEA) determines the efficiency of decision-making units. Weight restrictions in a model with weight restrictions (WR) are considered to determine the efficiency of units, depending on the importance of indicators (inputs and outputs). Since weight plays an important role in the efficiency and ranking of options, in this paper we examine the effect of the type of weighting method of indices in the calculation of the efficiency of decision-making units. It should be noted that change is not applied in decision-making units but in the weighting method in order to understand the effect of different weighting methods in the calculation of efficiency: that is, the efficiency of a unit is calculated with a variety of weighting methods and the impact of the type of weighting method on the indicators is evaluated in the calculation of the efficiency of that unit. In this study, we showed that the efficiency of each unit is affected by weighting methods and that the efficiency of each unit at each change in the weighting method assigns a different value to itself.

Keywords: efficiency, Data Envelopment Analysis, weighting constraints, weight restrictions, CCR
2020 MSC: 90C08

1 Introduction

Charnes et al. [2] introduced Data Envelopment Analysis (DEA) to assess the relative efficiency of a congruent group of decision-maker units (DMUs) such as schools, hospitals, or banks. Index weight plays an important role in calculating efficiency and ranking options. Different methods have been used to determine the weight of indicators, some of which are mentioned here. Hosseinzadeh Lotfi et al. [5] published an article on the return to scale (RTS) problem under weight restrictions in DEA. In this paper, the problem of return to scale under weight restrictions is discussed with reference to DEA. Liu and Peng [9] ranked units on the DEA boundary using a common set of weights. Bian and Yang [1] presented a comprehensive efficiency measure for assessing the resources and environment of Chinese provinces by developing the DEA-Shannon entropy method. Soleimani et al. [14] published an article on return to scale and elasticity scale in the presence of weight restrictions and alternative responses: their work presented

*Corresponding author
Email addresses: first.author@email.address (Monavar Karbalaie Alillou), behrouzdaneshian20@gmail.com (Behrouz Daneshian), first.author@email.address (Farzin Modarres Khiyabani), first.author@email.address (Farhad Hosseinzadeh Lotfi)

Received: July 2022 *Accepted:* September 2022

new results. Lee et al. [8] found the relative weights of indicators by the AHP method and used the DEA model to measure the relative efficiency of hydrogen energy technologies. Wu et al. [16] used Shannon entropy to determine cross-efficiency weights. Chiang et al. [3] calculated efficiency ratio by determining a common set of weights.

Silva and Milioni [13] published an article titled "Adjusted spherical frontier mode (ASFM-LP) with weight restrictions": in this paper, the ASFM-LP model provides a parameter analysis model for resource allocation, commonly referred to as inputs. Lai et al. [7] used the AHP technique to calculate the weight of input and output indicators: they evaluated the efficiency of airports by using the DEA/AR model. Kumar et al. [6] used the AHP technique in order to calculate the weight of input and output indicators: they calculated the relative efficiency of the telecom sector by using the DEA/AHP model. Podinovski and Bouzdine-Chameeva [11] published an article on sustained weight restrictions in DEA. Additionally, Podinovski [10] published an article on optimal weights in DEA models with weight restrictions. Song et al. [15] used Shannon entropy weighing methods to improve efficiency evaluation.

Ennen and Batool [4] presented an article on airport productivity in Pakistan using DEA with weight restrictions. In the present paper, the effects of weighing methods on efficiency calculation with a variety of weighting methods have been investigated using the multiplicative CCR model with weight restrictions. To measure the effect of various weighting methods in the calculation of efficiency, change in not applied in decision-making units but in the weighting method: that is, the efficiency of a unit is calculated with a variety of weighting methods and the effect of the type of weighting method on the indicators in the calculation of the efficiency of the unit is evaluated. Main content are provided in Section 2, while Section 3 presents the proposed method with a numerical example. Section 4, finally, presents the results of these numerical examples.

This paper uses multi-criteria decision making in determining the weights of indices: a variety of weighting methods are employed, and weight restrictions added to the multiplicative CCR model. Thus, the efficiency of each unit has been calculated by a variety of we and the effect of weighing methods on efficiency calculations have been investigated.

2 Main content

2.1 Prerequisites

2.1.1 Pair-wise comparison in AHP

The criteria for each level are compared to their corresponding index in AHP in a pair-wise manner at the higher level, and their weights are calculated. In these comparisons, decision makers use verbal judgments in such a way that if index i is compared with index j, the decision maker will say that the importance of i over j is one of the states of Table 1. In other words, these judgments have been converted to quantitative values of 1 to 9 by Saaty [12].

Table 1: Degree of importance for pair-wise comparisons

Degree of importance	Importance or priorities
9	An index has complete priority over another index (completely superior)
7	An index has very great priority over another index
5	An index has great priority over another index
3	An index has slight priority over another index (relatively superior)
1	The two indices has equal priority
2,4,6,8	Priority between the above intervals

Definition of pair-wise matrix. Suppose $A = [a_{ij}]_{n\times n}$ is a $n \times n$ matrix. A is called a pair wise matrix whenever:

$$a_{ji} = \frac{1}{a_{ij}},\ (i,j = 1,...,n),\ \forall i,j a_{ij} \geq 0$$

2.1.2 Relative weighting methods in AHP

1. Eigenvector method:
   We employed the following steps for calculating weights using the eigenvector method:
   (a) Matrix A is built.
   (b) Matrix $(A - \lambda.I)$ is determined.
   (c) Determinant of the matrix $(A - \lambda.I)$ is calculated, is put equal to zero, and the values are calculated.

(d) The largest λ is called $\lambda_{\max}$, which we set to $(A-\lambda_{\max}.I)*W=0$; the values of w_i is calculated using the equation $(A-\lambda_{\max}.I)*W=0$.

2. Least squares method or logarithmic least squares method:
We considered the following problem using the least squares method:

$$MINZ=\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}w_j-w_i)^2$$

$$s.t\ \sum_{i=1}^{n}w_i=1$$

To solve the above problem, we considered the Lagrangian equation as follows:

$$\sum_{i=1}^{n}(a_{il}w_l-w_i)a_{il}-\sum_{j=1}^{n}(a_{li}w_j-w_l)+\lambda=0,\ l=1,...,n$$

By adding $\sum_{i=1}^{n}w_i=1$ to the above problem and solving it, the w_i values were calculated.

3. Approximate methods:
These methods are divided into four categories:
(a) Row sum: In the pair wise matrix, first the sum of elements of each row is calculated to obtain a column vector. Then, this column vector is normalized to unit and the weight vector is obtained.
(b) Column sum: First the sum of elements of each column is calculated to obtain a row vector. Then, the elements of this vector are reversed. Finally, this row vector is normalized to unit and the weight vector is obtained.
(c) Arithmetic mean: We first normalize the elements of each column to unit and then calculate the row mean of the resulting matrix elements to obtain the weight vector.
(d) Geometric mean: First, the geometric mean of the elements of each matrix row is calculated to obtain a column vector. Then, the weight vector is obtained by normalizing the resulting vector to the unit.

2.1.3 Efficiency calculation

In this section, the weighted constraint obtained through each method is separately added to model (2.1) in order to calculate the efficiency for each unit. Then, the resulting model is executed using GAMZ software. After this, the efficiency of that unit is calculated with a variety of weighing methods separately. For example, the weight of indices was found by the least squares weighting method, the resulting weight restriction was added to the model (2.1), and the unit efficiency was calculated with this weighing method. Thereafter, the efficiency of the same unit was calculated with other weighing methods to observe the effect of changing the weighting methods in evaluating the efficiency of that unit.

Charnes et al. [2] defined the multiplicative CCR model as follows:

$$\begin{aligned}
&\max\sum_{r=1}^{s}u_ry_{ro}\\
&s.t\ \sum_{i=1}^{m}v_ix_{io}=1\\
&\sum_{r=1}^{s}u_ry_{rj}-\sum_{i=1}^{m}v_ix_{ij}\le 0\quad j=1,...,n\\
&v_i\ge 0\quad i=1,...,m\\
&u_r\ge 0\quad r=1,...,s
\end{aligned}\tag{2.1}$$

The CCR method was modified by Charnes et al. [2]: weight variables v_i and u_r were considered to be greater than zero so that the weights were not equal to zero in the above inequality. The weights are taken to be greater than or equal to the positive Non-Archimedean number, ε, to prevent the problem from deviating from the linear state.

3 The proposed method

First, the pair wise comparison matrix of indices (inputs and outputs) was determined according to the opinion of experts. Then, the weight of the input and output indices of each option (bank) was calculated with a variety of weighting methods and the weight restrictions were obtained from each weighing method: these were separately added to Model (2.1) (multiplicative CCR model) and the model executed with the GAMS software. Therefore, the efficiency of each unit was calculated with a variety of weighting methods: for example, the weight of the indices was found by the least squares weighting method, the resulting weight restrictions were added to Model (2.1), and the unit efficiency calculated with this weighing method. Then, the efficiency of the same unit was calculated with other weighing methods to observe the effect of changing the weighting methods in evaluating the efficiency of the unit.

3.1 Steps of the proposed method

1. Finding the matrix of pairwise comparisons of indicators according to the opinion of experts
2. Calculating the weight of the input and output indicators of each option with a variety of weighting methods
3. Add the weight constraint obtained from each method separately to the CCR multiplicative model
4. Execution of the model obtained from step 3 using Qamz software and calculating the efficiency of each unit with various weighting methods
5. Investigating the effect of changing the weighting method in evaluating the efficiency of the units
6. Find the real efficiency of each unit by regression method
7. Calculate the relative error of actual performance with a variety of weighting methods
8. Find the best weighting method

4 Experimental Example

In this example, we examine the effect of the type of index weighting method in calculating the efficiency of 30 branches of one of the banks of Iran by the proposed method. It is indicated that the type of weighting method in calculating the weight of the indexes affects the efficiency of each unit, and the efficiency of each unit at each time changed to see the weighting method assign a different value to itself. We emphasize that the effect of different weighting methods in the calculation of the efficiency causes change not in the decision-making units but in the weighting methods: that is, we calculated the efficiency of each unit with a variety of weighting methods, and assessed the effect of the type of weighing method of indices in the calculation of efficiency of the same unit. The input data (i1= Personnel grade, i2= Paid profit, i3= Deferred claims), of these 30 banks is provided in Table 2, while its output data (O1= Facility, O2 =Total Deposit, O3 =Profit received, O4 =Commission received, O5 =Other resources) is given in Table 3.

Table 2: inputs data

	I1 (Personnel grade)	**I2 (Paid profit)**	**I3 (Deferred claims)**
DMU1	7.14	2973130019	106858226
DMU2	6.55	3828801690	304483790
DMU3	5.23	1133793983	304483790
DMU4	4.87	877247942	85475000
DMU5	12.93	4839219824	7716168688
DMU6	7.2	3188240311	1708846082
DMU7	11.17	6775382776	985942000
DMU8	5.83	472101054	708507168
DMU9	7.61	1830532119	2653205624
DMU10	9.89	7686691834	8492311825
DMU11	12.6	10477302534	2793348419
DMU12	6.77	1933947316	238079274
DMU13	13.41	13055762503	6923511360
DMU14	5.12	16159072224	151735209
DMU15	4.21	2540161237	1014669788
DMU16	6.02	3709111887	656330678
DMU17	3.95	3722905402	4258698837

First, the pair wise comparison matrix of indices (inputs and outputs) of banks was determined according to the opinion of experts. Then, the weight of the input and output indices of each option (bank) was calculated with a variety of weighting methods and the weight restrictions were obtained from each weighing method: these were separately

DMU18	9.38	13783341943	28962912103
DMU19	12.6	4284521024	2675677600
DMU20	4.15	2478706280	17715906789
DMU21	19.62	5768938003	486555808
DMU22	6.16	6969387436	2945344182
DMU23	9.36	8720381004	677530596
DMU24	9.21	3960509398	140135858
DMU25	18.78	13027785897	17434609276
DMU26	9.98	2780303621	1535413020
DMU27	6.89	2804974193	39191304
DMU28	24.3	15169239239	6685666213
DMU29	14.62	23411109793	55540232900
DMU30	11.03	5996255760	2248521132

Table 3: outputs data

	O1 (Facility)	**O2 (Total Deposit)**	**O3 (Profit received)**	**O4 (Commission received)**	**O5 (Other resources)**
DMU1	90504304878	30376829549	4522740900	108756781	15416750
DMU2	161920262494	32174061999	9221629290	156922747	70970000
DMU3	51746989236	21369294589	2233950758	64336996	79625000
DMU4	29495575267	17242783569	2236306947	91095828	42380000
DMU5	114671285098	112323373303	9068971211	254981597	139891965
DMU6	44417322288	50662977928	3039268874	492892036	1793364120
DMU7	47467684724	78559215044	2080344159	166298846	78095000
DMU8	57863179290	10916788965	1775227490	115656953	42090000
DMU9	18056815228	54063972550	1315620133	109410609	560427728
DMU10	397657868216	74776888338	5835683004	525388447	2908964909
DMU11	239603119215	135140978344	9482348129	166570539	279350000
DMU12	13600845684	25971685399	933400866	51487498	35745000
DMU13	78969374178	161673336420	8598302839	579006243	2929678688
DMU14	13798127749	163469692347	1045491766	40117689	30000000
DMU15	39437772477	32887597065	1638052939	41631242	15866640
DMU16	26239143285	62926569585	1922051809	65002291	402163800
DMU17	52527538112	43705020308	3500862579	85628882	178656152
DMU18	51283454594	92906646567	614078472	3714573160	2407617904
DMU19	111251270395	61225924301	8297348892	1043783028	5269499831
DMU20	43135726159	29835481457	6759904341	259732334	1047512000
DMU21	178198658997	92387572661	5725128530	717447204	722555845
DMU22	137430777076	91455231824	7149036756	163464742	17580000
DMU23	241570202824	126129179916	11952958774	407261824	6300200
DMU24	59593191488	78125111745	2352574767	288566533	981342480
DMU25	376539317995	149474528152	23825373284	381532753	530791482
DMU26	98162656426	45371451173	4842274378	140620659	36943159
DMU27	37961337118	38056679195	2757334379	194802223	1259351462
DMU28	310694461869	281686402800	23854145146	5033810697	34782640506
DMU29	55540232900	293102637826	11839916149	645533183	2513531683
DMU30	63127550626	77973595630	2578202292	199231848	411600124

added to Model (2.1) (multiplicative CCR model) and the model executed with the GAMS software. Therefore, the efficiency of each unit was calculated with a variety of weighting methods: for example, the weight of the indices was found by the least squares weighting method, the resulting weight restrictions were added to Model (2.1), and the unit efficiency calculated with this weighing method. Then, the efficiency of the same unit was calculated with other weighing methods to observe the effect of changing the weighting methods in evaluating the efficiency of the unit.

The index pair comparison matrix (inputs and outputs) of banks according to expert opinions is shown in Table 4.

Weight of the indices has been calculated with different weighting methods and the weight restriction of each weighing method calculated: for each unit, the weight restriction of each method was added separately to the model (2.1). The resulting model was implemented using the GAMS software.

The weights obtained from different weighing methods are as follows:

Table 4: pair comparison matrix

	Personnel grade	**Paid profit**	**Deferred claims**	**Facility**	**Total deposed**	**Profit received**	**Commission received**	**Other resources**
Personnel grade	1	8	8	3	7	3	3	4
Paid profit	1.8	1	8	1.6	1.5	1.6	1.7	1.7
Deferred claims	1.8	1.8	1	1.8	1.7	1.6	1.7	1.7
Facility	1.3	6	8	1	1.5	1.3	1.4	1.5
Total deposit	1.7	5	7	5	1	1.3	1.5	1.4
Profit received	1.3	6	6	3	3	1	1.7	1.5
Commission received	1.3	7	7	4	5	7	1	1.5
Other resources	1.4	7	7	5	4	5	5	1

Weights obtained by the sum row method and the weight restrictions obtained from it:

$$v_1 \geq 3.72v_2$$
$$v_2 \geq 5.04v_3$$
$$u_5 \geq 1.09u_4$$
$$u_4 \geq 1.60u_3$$
$$u_3 \geq 1.4u_2$$
$$u_2 \geq 1.17u_1$$

Weights obtained by the sum column method and the weight constrains obtained from it:

$$v_1 \geq 15.2v_2$$
$$v_2 \geq 1.3v_3$$
$$u_5 \geq 1.6u_4$$
$$u_4 \geq 1.7u_3$$
$$u_3 \geq 1.2u_2$$
$$u_2 \geq 1.03u_1$$

Weights obtained by geometric mean and the resulting weight restrictions:

$$v_1 \geq 15.2v_2$$
$$v_2 \geq 1.3v_3$$
$$u_5 \geq 1.6u_4$$
$$u_4 \geq 1.7u_3$$
$$u_3 \geq 1.2u_2$$
$$u_2 \geq 1.03u_1$$

Weights obtained by means of the arithmetic mean and the weight restrictions obtained:

$$v_1 \geq 8.1v_2$$
$$v_2 \geq 2.2v_3$$
$$u_5 \geq 1.3u_4$$
$$u_4 \geq 1.8u_3$$
$$u_3 \geq 1.2u_2$$
$$u_2 \geq 1.2u_1$$

Weights obtained by the eigenvector method and the weight restrictions obtained:

$$v_1 \geq 11v_2$$
$$v_2 \geq 1.9v_3$$
$$u_5 \geq 1.4u_4$$
$$u_4 \geq 2u_3$$
$$u_3 \geq 1.2u_2$$
$$u_2 \geq 1.3u_1$$

Weights obtained by least squared method and the weight restrictions obtained through it:

$$v_1 \geq 13.8v_2$$
$$v_2 \geq 1.3v_3$$
$$u_5 \geq 1.3u_4$$
$$u_4 \geq 1.8u_3$$
$$u_3 \geq 1.9u_2$$
$$u_2 \geq 1.2u_1$$

The weight restriction obtained through each method for each unit was separately to Model (2.1). The model so executed calculates the efficiency of each unit with a variety of weighing methods separately. The effect of the type of weighing method in calculating the efficiency of that unit was also evaluated.

Therefore, according to the procedure described above, we added the weighted constraint obtained for each unit from the least squares method o the multiplicative model (2.1). The model was executed with the GAMS software: efficiency obtained by this weighing method is shown in the second column of Table 5. Then, the weight restriction obtained from the eigenvector method was added to the multiplicative model (2.1) and the model executed with GAMS software: the efficiency of this weighting method can be observed in the third column of Table 5. Similarly, the weight restriction obtained from the sum row method was added to the multiplicative model (2.1) and the model run with GAMS software: the efficiency obtained by this weighing method can be observed in the fourth column of Table 5. The weight restriction obtained from the sum column method was added to the multiplicative model (2.1) and the model executed with GAMS software: the efficiency obtained by this weighting method can be observed in the fifth column of Table 5. The weight restriction obtained from the arithmetic mean method has been added to the multiplicative model (2.1) and the model run with GAMS software: the efficiency of this weighting method can be observed in the sixth column of Table 5. Finally, the weight restriction obtained from the geometric mean method was added to the multiplicative model (2.1) and the model run with the GAMS software: the efficiency determined by this weighting method is given in the seventh column of Table 5. And then we see efficiency determined by without weight limit of the units in the eighth column of the table.

Table 5: determined efficiencies

	Efficiency by least squares weighing method	**Efficiency by eigen vector weighing method**	**Efficiency by sum row weighing method**	**Efficiency by sum column weighing method**	**Efficiency by arithmetic mean weighing method**	**Efficiency by geometric mean weighing method**	**Efficiency without weight limit**
DMU1	0.35	0.23	0.35	0.25	0.26	0.23	1

In Table 5, it can be observed that the efficiency of all DMUs is exactly affected by weighting methods and that the efficiency of the DMUs changes by changing the weighting methods. In other words, the type of weighting method in the calculation of the weight of the indices affects the efficiency of each unit: the efficiency of each DMU changes by changing the weighting method. For example, the efficiency of DMU18 with only the sum row weighing method was one and has a value less than other weighting methods: the efficiency of the rest of the DMUs also varies with change in the weighting methods. The statistics dispersion indicators clearly show the efficiency dispersion due to the variety of weighting methods in each unit: for example, efficiency variance in DMU18 is 0.05 and the efficiency variance in DMU10 is 0.04. Therefore, it can be clearly seen that the amount of efficiency per change in each DMU in the weighting method assigns different values to itself. This implies the effect of the type of weighting method used to measure the efficiency of each unit.

In order to determine the best weighing method for each DMU, the following procedures were performed:

DMU2	0.67	0.43	0.64	0.47	0.47	0.43	1
DMU3	0.28	0.18	0.28	0.20	0.21	0.18	1
DMU4	0.20	0.17	0.25	0.18	0.19	0.17	1
DMU5	0.31	0.27	0.40	0.29	0.30	0.27	1
DMU6	0.30	0.33	0.35	0.33	0.34	0.33	0.77
DMU7	0.16	0.14	0.20	0.15	0.15	0.14	0.67
DMU8	0.27	0.15	0.23	0.17	0.17	0.15	1
DMU9	0.13	0.15	0.21	0.16	0.16	0.15	1
DMU10	1	0.53	0.70	0.60	0.58	0.53	1
DMU11	0.52	0.34	0.50	0.38	0.38	0.34	0.85
DMU12	0.08	0.08	0.12	0.09	0.09	0.08	0.66
DMU13	0.31	0.36	0.45	0.37	0.38	0.36	0.83
DMU14	0.27	0.38	0.57	040	0.42	0.37	1
DMU15	0.28	0.20	0.29	0.22	0.22	0.20	0.71
DMU16	0.20	0.21	0.28	0.22	0.22	0.20	0.98
DMU17	0.42	0.35	0.50	0.38	0.38	0.35	0.79
DMU18	0.54	0.90	1	0.85	0.93	0.90	1
DMU19	0.40	0.43	0.50	0.43	0.45	0.43	0.95
DMU20	0.46	0.52	0.68	0.54	0.55	0.52	1
DMU21	0.28	0.20	0.28	0.22	0.22	0.20	1
DMU22	0.64	0.47	0.69	0.52	0.52	0.47	0.98
DMU23	0.73	0.52	0.75	0.56	0.57	0.52	1
DMU24	0.25	0.23	0.30	0.24	0.25	0.23	1
DMU25	0.57	0.41	0.60	0.45	0.45	0.41	0.93
DMU26	0.29	0.19	0.30	0.21	0.22	0.19	0.87
DMU27	0.24	0.24	0.30	0.24	0.25	0.24	1
DMU28	0.82	1	1	1	1	1	1
DMU29	0.32	0.45	0.61	0.47	0.48	0.45	1
DMU30	0.20	0.17	0.23	0.18	0.19	0.17	0.68

For each DMU, the relative error of the actual performance was obtained through the following relationship:

$$\frac{|\text{efficiency of } DMU_i \text{ by the weighting method} j - \text{actual efficiency of } DMU_i|}{\text{actual efficiency of } DMU_i} \qquad i = 1, ..., 30, \ j = 1, ..., 6$$

These errors were calculated for each DMU with a variety of weighting methods. The weighting method in which least error is observed is the best way to weigh the DMU.

Weighting method 1: Least squares weighing method

Weighting method 2: Eigenvector weighing method

Weighting method 3: Sum row weighting method

Weighting method 4: Sum column weighting method

Weighting method 5: Arithmetic mean weighting method

Weighting method 6: Geometric mean weighing method

For example, for DMU_4, this is calculated as:

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method1} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.2 - 0.98|}{0.98} = 0.79$$

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method2} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.17 - 0.98|}{0.98} = 0.83$$

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method3} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.25 - 0.98|}{0.98} = 0.74$$

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method4} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.23 - 0.98|}{0.98} = 0.76$$

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method5} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.19 - 0.98|}{0.98} = 0.81$$

$$\frac{|\text{efficiency of } DMU_4 \text{ by the weighting method6} - \text{actual efficiency of } DMU_4|}{\text{actual efficiency of } DMU_4} = \frac{|0.17 - 0.98|}{0.98} = 0.83$$

It can be observed that for DMU_4, the weighting method has the least error, and the best weighting method for DMU_4 is method 3. For other DMUs, we also calculated these errors and observed that on average, method 3 (total line weighting method) is a better method than other methods.

5 Conclusion

Since weights play an important role in the ranking and efficiency of the options, we have examined the effect of various weighting methods in the calculation of efficiency. In this paper, we find the weight of the indices with a variety of weighting methods and add the weight restriction obtained from each weighing method separately to the classical multiplicative CCR model. We performed the resulting model with the GAMS software and calculated the efficiency of each unit by each change in the weighing method. We found that the efficiency of each unit is affected by weighting methods and that the efficiency of each unit at each changes in the weighting method assigns a different value to itself: these points towards the effect of the type of weighting method used to calculate the efficiency of each unit. We also concluded that on average, method 3 (total line weighting method) is a better method than other methods. In a future study, it is suggested that the effect of weight control and weighting method on the efficiency of units in the field of fuzzy numbers be investigated.

References

[1] Y. Bian and F. Yang, *Resource and environment efficiency analysis of provinces in China: A DEA approach based on Shannon's entropy*, Energy Policy **38** (2010), no. 4, 1909–1917.

[2] A. Charnes, W.W. Cooper and E. Rhodes, *Measuring the efficiency of decision making units*, Eur. J. Operat. Res. **2** (1978), no. 6, 429–444.

[3] C.I. Chiang, M.J. Hwang and Y.H. Liu, *Determining a common set of weights in a DEA problem using a separation vector*, Math. Comput. Model. **54** (2011), no. 9–10, 2464–2470.

[4] D. Ennen and I. Batool, *Airport efficiency in Pakistan-A data envelopment analysis with weight restrictions*, J. Air Transport Manag. **69** (2018), 205–212.

[5] F. Hosseinzadeh Lotfi, G.R. Jahanshahloo and M. Esmaeili, *An alternative approach in the estimation of returns to scale under weight restrictions*, Appl. Math. Comput. **189** (2007), no. 1, 719–724.

[6] A. Kumar, R. Shankar and R.M. Debnath, *Analyzing customer preference and measuring relative efficiency in telecom sector: A hybrid fuzzy AHP/DEA study*, Telemetr. Inf. **32** (2015), no. 3, 447–462.

[7] P.L. Lai, A.P. Potter, M. Beynon and A. Beresford, *Evaluating the efficiency performance of airports an integrated AHP/DEA/AR technique*, Transport Policy **42** (2015), 75–85.

[8] S.K. Lee, G. Mogi, Z. Li, K.S. Hui, S.K. Lee, K.N. Hui and J.W. Kim, *Measuring the relative efficiency of hydrogen energy technologies for implementing the hydrogen economy: An integrated fuzzy AHP/DEA approach*, Int. J. Hydrogen Energy **36** (2011), no. 20, 12655–12663.

[9] F.H.F. Liu and H.H. Peng, *Ranking of units on the DEA frontier with common weights*, Comput. Oper. Res. **35** (2008), no. 5, 1624–1637.

[10] V.V. Podinovski, *Optimal weights in DEA models with weight restrictions*, Eur. J. Oper. Res. **254** (2016), no. 3, 916–924.

[11] V.V. Podinovski and T. Bouzdine-Chameeva, *Consistent weight restrictions in data envelopment analysis*, Eur. J. Oper. Res. **244** (2015), no. 1, 201–209.

[12] T.L. Saaty, *The analytic hierarchy process*, McGraw Hill, New York, 1980.

[13] R.C. Silva and A.Z. Milioni, *The adjusted spherical frontier model with weight restrictions*, European J. Operat. Res. **220** (2012), no. 3, 729–735.

[14] M. Soleimani-Damaneh, G.R. Jahanshahloo, S. Mehrabian and M. Hasannasab, *Returns to scale and scale elasticity in the presence of weight restrictions and alternative solutions*, Knowledge-Based Syst. **23** (2010), no. 2, 86–93.

[15] M. Song, Q. Zhu, J. Peng and E.D.S. Gonzalez, *Improving the evaluation of cross efficiencies: A method based on Shannon entropy weight*, Comput. Indust. Eng. **112** (2017), 99–106.

[16] J. Wu, J. Sun, L. Liang and Y. Zha, *Determination of weights for ultimate cross efficiency using Shannon entropy*, Expert Syst. Appl. **38** (2011), no. 5, 5162–5165.