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State estimation of T and Chen chaotic dynamics using sliding mode control

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Abstract

This paper deals with the estimation of the synchronization of two identical systems such as T chaotic systems and Chen chaotic systems. In addition to that, it describes the estimation of the synchronization of one more non-identical system which combines Chen and T systems. The sliding mode control method has been implemented to synchronize two identical systems such as T systems and Chen systems, also the method can be used to synchronize the non-identical chaotic systems, viz. Chen and T systems. The numerical simulation part has been carried out via MATLAB software which strengthens our derived results.

Keywords: Chaos synchronization, Sliding Mode control, Chen system, T system
2020 MSC: 34D06, 34H05, 3428

1 Introduction

Sliding mode control (SMC) is a nonlinear control technique featuring remarkable properties of accuracy, robustness, and easy tuning and implementation. SMS systems are designed to drive the system states onto a particular surface in the state space, named sliding surface. Once the sliding surface is reached, sliding mode control keeps the states on the close neighbourhood of the sliding surface. Hence the sliding mode control is a two part controller design. The first part involves the design of a sliding surface so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law that will make the switching surface attractive to the system state alternative control method for uncertain strict-feedback chaotic systems without using backstepping technique is presented. alternative control method for uncertain strict-feedback chaotic systems without using backstepping technique [5, 1, 3**?** , 4]. There are two main advantages of sliding mode control. First is that the dynamic behaviour of the system may be tailored by the particular choice of the sliding function [6, 7, 8, 13]. Secondly, the closed loop response becomes totally insensitive to some particular uncertainties. This principle extends to model parameter uncertainties, disturbance and non-linearity that are bounded. From a practical point of view SMC allows for controlling nonlinear processes subject to external disturbances and heavy model uncertainties.

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Dynamic sliding mode controller operates as a controller system for controlling the magnetic field strength of the engine mount coil. Controlling the magnetic field strength leads to change the magneto rheological liquid properties and thereby the generated force by the liquid. This controller system is simulated and shown that such a regulator plays the own incredible part in further developing vehicle ride solace so that it can eliminate 60 percentage of the motor's vibration. Three nonlinear normal differential conditions to depict the cooperation among adolescent prey, grown-up prey and hunter populaces. The model is dissected by involving straight steadiness investigation to get the circumstances for which our model shows strength around the conceivable harmony focuses. State estimation-primarily based adaptive manipulate hassle for a category of unsure structures concern to time-postpone and outside disturbance is investigated within sliding mode framework [9, 10, 11, 12, 14]. A simplified kingdom observer with none inputs is designed to reconstruct the unmeasured nation variables, from which a unique linear switching floor is provided. The framework of Lyapunov balance idea and linear matrix inequality method, the system achieves the proper performance for the closed-loop device. A dynamic compensator is designed to enhance the performance of the closed-loop gadget in sliding mode, and its parameter is acquired from a linear matrix inequality (LMI). Simulation outcomes for the widely recognized Chua's circuit and Lorenz chaotic system are furnished to illustrate the effectiveness of the proposed scheme [15, 16, 17, 18]. A sliding mode control for a category of fractional uncertain chaotic systems under perturbations of parameters are taken in which, based on Lyapunov balance concept, theoretically verify that the controller is effective, and the designed control scheme can move towards the system's uncertainty to assure the property of asymptotical stability within the presence of parameter fluctuations. The control problem of synchronization is addressed with a combination of backstepping with sliding mode control provided the certain of uncertainty [19, 2].

In this article, Chen and T systems are considered and its state estimation is discussed separately. In addition, the estimation of combined systems of T and Chen chaotic are analyzed. Chaotic systems can either be dissipative or conservative. It may appear irregular but they are generated by deterministic rules. The chaotic systems can be synchronized to each other, in this two or more chaotic systems can be coupled with each other. The chaotic systems can be controlled. It contains an infinite number of unstable periodic solutions of arbitrary period.

The paper is organized as follows, In section 2, the problem statement is defined, In section 3, describes the proposed methodology by sliding mode control. In section 4, the identical chaotic T systems synchronization is explained with numerical simulation. In section 5, the synchronization of identical Chen chaotic system and its numerical simulation is described. In section 6, the two different chaotic systems synchronization is explained via *Chen* and T chaotic systems and its numerical simulation is furnished. In section 7, the conclusion part is described.

2 Sliding Mode Control Problem Formation

This segment elaborate, the problem statement for the synchronization of identical and different chaotic systems and it is established that sliding mode control is a suitable tool around the synchronization of the dynamics.

Consider the *plant* chaotic dynamics

$$\dot{x} = Px + \eta(x) \tag{2.1}$$

where $x \in R^n$ is the state vector of the plant chaotic dynamics. a is the $n \times n$ vector of the system parameters and $\eta(x)$ is the nonlinearity of the chaotic dynamics.

The observer chaotic dynamics is described by

$$\dot{\hat{x}} = P\hat{x} + \eta(\hat{x}) + u, \tag{2.2}$$

where $\hat{x} \in R^n$ is the state vector of the observer chaotic dynamics and $u \in R^n$ is the controller, that to be designed.

The relation between plants along with observer dynamics is pictured as

$$y = \hat{x} - x. \tag{2.3}$$

Then the relation dynamics is given as

$$\dot{y} = Py + \phi(x, \hat{x}) + u, \tag{2.4}$$

where

$$\phi(x, \hat{x}) = \eta(\hat{x}) - \eta(x). \tag{2.5}$$

The scope of the present work is to square up the controller u such that time $t \to \infty$, which gives the relation dynamics $y \to 0$ for all initial conditions. To solve this problem, the controller u defined as

$$u = -\phi(x, \hat{x}) + Qv, \tag{2.6}$$

where Q is a constant gain vector is chosen such that (P, Q) is controllable. Conflating (4.10) into (2.4), the relation dynamics refines to

$$\dot{y} = Py + Qv \tag{2.7}$$

which is a linear time- invariant control system. Thus, the chaotic Synchronization problem can be transformed to equivalent stabilization problem, stabilizing the zero solution $y = 0$ of the system (2.4) by the sliding mode control.

In sliding mode control, the variable is defined to be

$$s(y) = Ry = r_1 y_1 + r_2 y_2 + \cdots + r_n y_n \tag{2.8}$$

where $R \in R^n$ is constant row vector to be determined. The sliding manifold is outlined by the means of

$$s = \{y \in R^n : s(y) = Ry = 0\}. \tag{2.9}$$

The mandatory condition for the state trajectory y to keep as to the sliding manifold S is

$$s \equiv 0 \text{ and } \dot{s} \equiv R[Py + Qv] = 0. \tag{2.10}$$

Solving (2.10) for v, the equal counter part of the control law is revised as

$$v(t) = -\left(RQ\right)^{-1} RPy(t). \tag{2.11}$$

By substituting (2.11) into (4.10), the closed-loop system dynamics in the overall sliding phase is acquired, which means

$$\dot{y} = [I - Q\left(RQ\right)^{-1} R]Py. \tag{2.12}$$

The row vector R is designated such that the system matrix of the controlled dynamics $\dot{y} = [I - Q\left(RQ\right)^{-1} R]Py$ has all eigen value with negative real parts. Which implies, the dynamics (2.12) is globally asymptotically stable.

t In the end, in order to figure the sliding mode controller for (4.10), the constant plus proportional rate law is applied.

$$\dot{s} = -Qsgn(s) - ks \tag{2.13}$$

where $sgn(s)$ denotes the sign function and the gains $Q > 0, k > 0$ are determined such that the sliding condition is mitigated along with sliding motion will occur.

From equations (2.10) and (2.13), the control $v(t)$ upgraded as

$$v(t) = -(RQ)^{-1}[R(kI + P)y + Qsgn(s)]. \tag{2.14}$$

3 Stability Under Sliding Mode Control

For encompassing the globally and asymptotically synchronized for all initial conditions, the Lyapunov candidate function can be outlined by

$$V(y) = \frac{1}{2}s^2. \tag{3.1}$$

The sliding mode motion is in reference to the equations

$$s(y) = 0 \text{ and } \dot{s}(y) = 0. \tag{3.2}$$

The dynamics in the overall sliding mode is globally asymptotically stable when

$$s(y) \neq 0, V(y) > 0.$$

Differentiating across relation dynamics (4.10) or the equivalent dynamics (2.12), acquired as

$$\dot{V}(y) = s\dot{s} = -ks^2 - Qs^3 sgn(s) < 0. \tag{3.3}$$

Hence, by Lyapunov stability theory, the relation dynamics (2.7) is globally asymptotically stable for all $y(0) \in R^n$.

4 State Estimation of an Equivalent Identical T Systems

In view this, the sliding mode control method is applied for the estimation of two an equivalent T chaotic systems . G. Tigan and D. Opris developed the T chaotic dynamics. It is three dimensional system. Hence, the plant system is figured by the T dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c-a)x_1 - ax_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \tag{4.1}$$

where x_1, x_2, x_3 represents the states of the T system (4.1) and $a, b, c$are parameters. The observer system is also figured as the T dynamics and it take it representation as

$$\begin{aligned} \dot{\hat{x}}_1 &= a(\hat{x}_2 - \hat{x}_1) + u_1 \\ \dot{\hat{x}}_2 &= (c-a)\hat{x}_1 - a\hat{x}_1\hat{x}_3 + u_2 \\ \dot{\hat{x}}_3 &= -b\hat{x}_3 + \hat{x}_1\hat{x}_2 + u_3 \end{aligned} \tag{4.2}$$

where $\hat{x}_1, \hat{x}_2, \hat{x}_3$ are the states of the system (4.2) and $u = (u_1, u_2, u_3)$is the sliding mode controller planned to calculate. When

$$a = 2.1,\ b = 0.6,\ c = 30,$$

the T dynamics (4.1) is chaotic. Figure 1 glorifies the chaotic nature of the T dynamics (4.1).

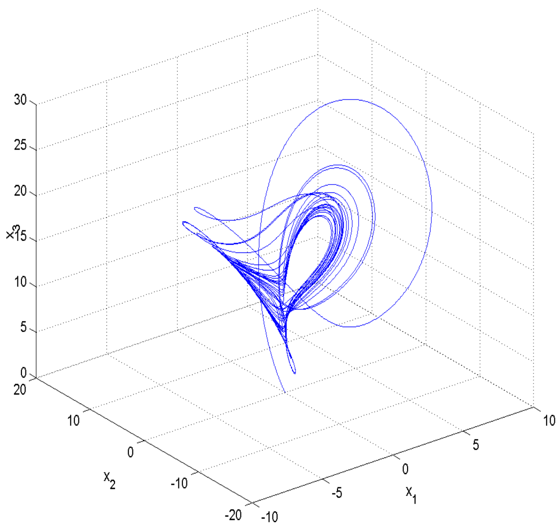

Figure 1: Chaotic Nat Dynamics

The error dynamics $y$is outlined by

$$y_i = \hat{x}_i - x_i,\ (i = 1, 2, 3) \tag{4.3}$$

Then the error dynamics is procured as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (c-a)y_1 - a(\hat{x}_1\hat{x}_3 - x_1x_3) + u_2 \\ \dot{y}_3 &= -by_3 + \hat{x}_1\hat{x}_2 - x_1x_2 + u_3 \end{aligned} \tag{4.4}$$

The matrix representation of the error dynamics (4.4) in defined by

$$\dot{y} = Py + \phi(x, \hat{x}) + u \tag{4.5}$$

where $P = \begin{bmatrix} -a & a & 0 \\ c-a & 0 & 0 \\ 0 & 0 & -b \end{bmatrix}, \phi(x, \hat{x}) = \begin{bmatrix} 0 \\ -a(\hat{x}_1\hat{x}_3 - x_1x_3) \\ \hat{x}_1\hat{x}_2 - x_1x_2 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

The sliding mode controller is described as

$$u = -\phi(x, \hat{x}) + Qv \tag{4.6}$$

where Q is chosen such that (P, Q) is controllable. In this case Q is chosen as

$$Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{4.7}$$

The parameter values are $a = 2.1, b = 0.6, c = 30$. The sliding mode variable is designated as

$$s = Ry = \begin{bmatrix} 5 & 4 & 3 \end{bmatrix} y = 5y_1 + 4y_2 + 3y_3 \tag{4.8}$$

which causes the sliding mode state equation asymptotically stable.

The sliding mode gain k and Q are updated as

$$k = 10, Q = 0.1$$

Using the equation (2.14), $v(t)$updated as

$$v(t) = -12.595y_1 - 4.215y_2 - 2.357y_3 \ . \tag{4.9}$$

where $\phi(x, \hat{x}), Q$ and $v(t)$ are defined as in equations (4.1). The identical T systems (4.1) and (4.2) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller u defined by (4.6).

4.1 Numerical Discussion and Results for Identical *T* system

For the numerical validation, MATLAB ode solver is used to solve identical chaotic T dynamics (4.1) and (4.2) with the sliding mode controller (4.6). The parameters of the T system (4.1) are selected so that the system has chaotic behaviour, *viz.*

$$a = 2.1, \ b = 0.6, \ c = 30.$$

The initial conditions of the plant and observer systems are taken as

$$x_1(0) = 0.1987, x_2(0) = 0.4654, \\ x_3(0) = 0.9213$$

and

$$\hat{x}_1(0) = 0.3409, \hat{x}_2(0) = 0.9198, \\ \hat{x}_3(0) = 0.5198 \ .$$

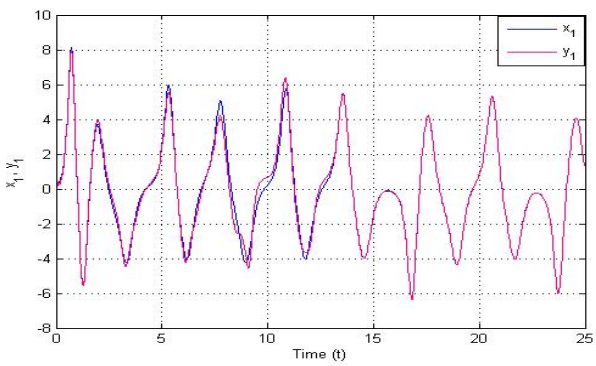

Figure 2: Synchronization of identical T systems between x_1 and y_1

From equation (4.6)

$$u = -\phi(x, \hat{x}) + Qv \tag{4.10}$$

$$v(t) = [-12.595, -4.215 - 2.357] \ .$$

after simplification, Figure 2, 3 and 4 exhibits the estimation of the plant system (4.1) and the observer system (4.2).

Figure 5 indicates the error between the plant and observer systems (4.1) and (4.2).

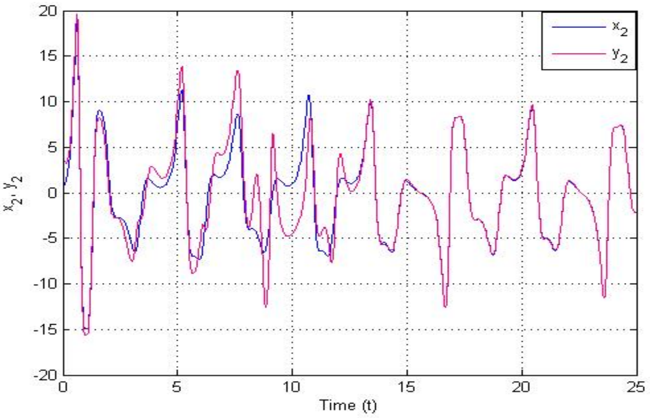

Figure 3: Synchronization of identical T systems between x_2 and y_2

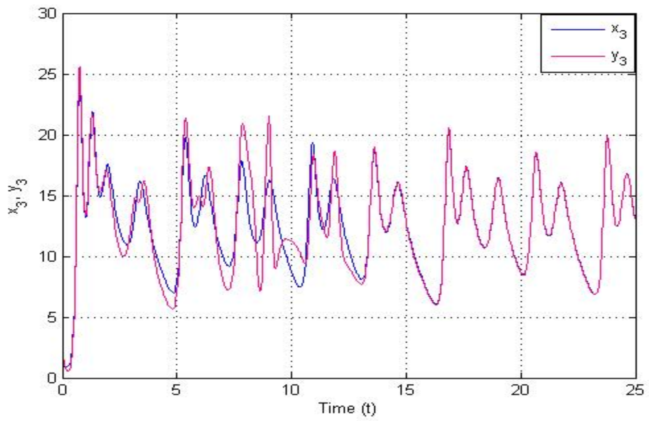

Figure 4: Synchronization of identical T systems between x_3 and y_3

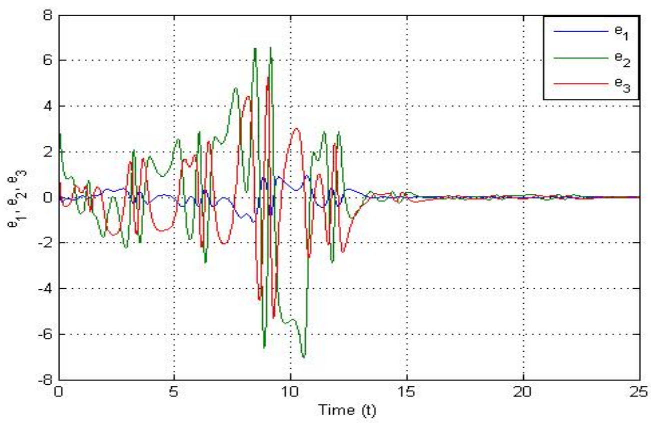

Figure 5: Error between two identical T systems

5 State Estimation of Identical Chen Chaotic Systems

This section explained about how sliding mode control work for identical Chen chaotic systems . The sientists G. Chen and T. Ueta (1999) introduced the three dimensional chaotic Chen system.

Thus, the plant system is outlined by the Chen dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= (\gamma - \alpha)x_1 - x_1x_3 + \gamma x_2 \\ \dot{x}_3 &= -\beta x_3 + x_1x_2 \end{aligned} \tag{5.1}$$

where x_1, x_2, x_3 are the states of the system (5.1) and α, β, γ are parameters.

The observer dynamics is also sketched by the Chen dynamics and it is represents by

$$\begin{aligned} \dot{\hat{x}}_1 &= \alpha(\hat{x}_2 - \hat{x}_1) + u_1 \\ \dot{\hat{x}}_2 &= (\gamma - \alpha)\hat{x}_1 - \hat{x}_1\hat{x}_3 + \gamma\hat{x}_2 + u_2 \\ \dot{\hat{x}}_3 &= -\beta\hat{x}_3 + \hat{x}_1\hat{x}_2 + u_3 \end{aligned} \tag{5.2}$$

where $\hat{x}_1, \hat{x}_2, \hat{x}_3$ are the states of the system (5.3) and $u = (u_1, u_2, u_3)$is the sliding mode controller to be designed. When $\alpha = 35, \beta = 3, \gamma = 28$, the Chen dynamics (5.1) is chaotic.

Figure 6 depicts the chaotic behavior of the Chen dynamics (5.1).

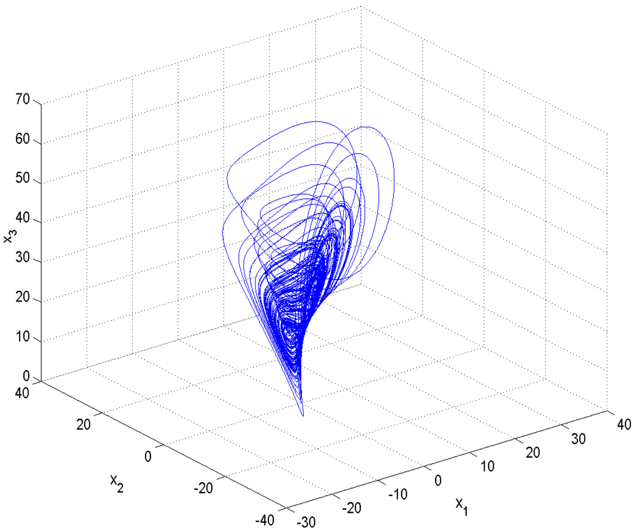

Figure 6: Chaotic Behaviour of the Chen System

The error y_i is defined by

$$y_i = \hat{x}_i - x_i, \ (i = 1, 2, 3) \tag{5.3}$$

Hence, the error dynamics is obtained by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (\gamma - \alpha)y_1 + \gamma y_2 - \hat{x}_1\hat{x}_3 + x_1x_3 + u_2 \\ \dot{y}_3 &= -\beta y_3 + \hat{x}_1\hat{x}_2 - x_1x_2 + u_3 \end{aligned} \tag{5.4}$$

The matrix representation of the error system 5.4 is

$$\dot{y} = Py + \phi(x, \hat{x}) + u \tag{5.5}$$

where

$$P = \begin{bmatrix} -\alpha & \alpha & 0 \\ \gamma-\alpha & \gamma & 0 \\ 0 & 0 & -\beta \end{bmatrix},$$

$$\phi(x,\hat{x}) = \begin{bmatrix} 0 \\ -\hat{x}_1\hat{x}_3 + x_1x_3 \\ \hat{x}_1\hat{x}_2 - x_1x_2 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

The sliding mode controller is designed by

$$u = -\phi(x,\hat{x}) + Qv \tag{5.6}$$

where Q is chosen such that (P, Q) is controllable. In this case Q is chosen as

$$Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{5.7}$$

The parameter values are

$$\alpha = 35, \beta = 3, \gamma = 28$$

The sliding mode variable is selected as

$$\begin{aligned} s &= Ry = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \\ y &= -y_1 + y_2 + y_3 \end{aligned} \tag{5.8}$$

which makes the sliding mode state equation asymptotically stable. The sliding mode gain k and Q are updated as

$$k = 10, Q = 1.$$

Using the equation (2.14), $v(t)$raised as

$$v(t) = -11y_1 - 4y_2 - 7y_3, \tag{5.9}$$

where $\phi(x,\hat{x}), Q$ and $v(t)$ are defined as in equations (5.1) The identical *Chen* systems (5.1) and (5.3) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller u defined by (5.1).

5.1 Numerical Discussion and Results for Identical Chen Chaotic System

For the numerical validation, MATLAB ode solver is used to solve the identical Chen chaotic dynamics (5.1) and (5.3) with the sliding mode controller (4.1).

The parameters of the Chen system (5.1) are selected so that the system has chaotic behaviour, *viz.* $\alpha = 35, \beta = 3, \gamma = 28$. The initial conditions of the plant and observer systems are taken as

$$x_1(0) = 7, x_2(0) = 15, x_3(0) = 4$$

and

$$\hat{x}_1(0) = 12, \hat{x}_2(0) = 25, \hat{x}_3(0) = 20.$$

from equation (5.4)

$$u = -\phi(x,\hat{x}) + Qv$$

$$v(t) = [-11 - 4 - 7].$$

after simplification. Figure 7, 8 and 9 explained the state estimation of the plant system (5.1) and observer system (5.3). Figure 10 displays the error between the plant system (5.1) and observer system (5.3).

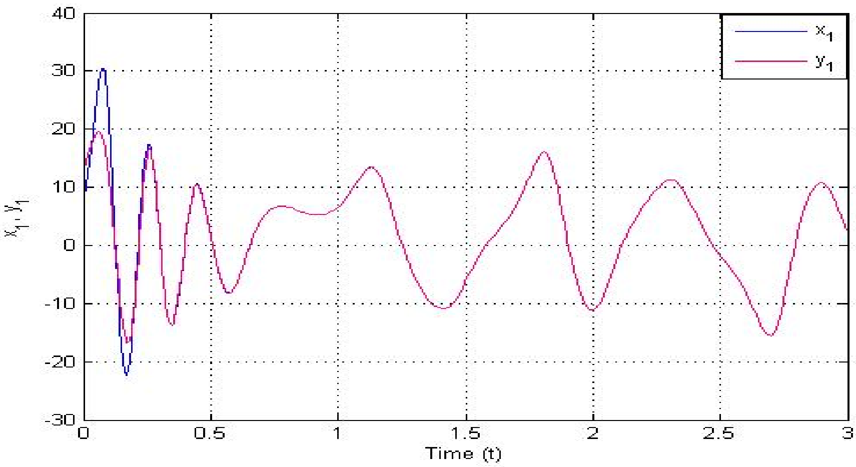

Figure 7: Synchronization of identical Chen Systems between x_1 and y_1

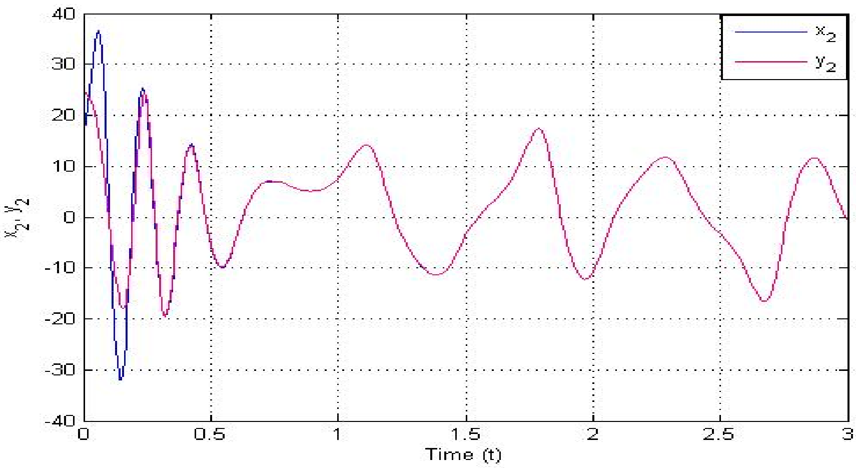

Figure 8: Synchronization of identical Chen Systems between x_2 and y_2

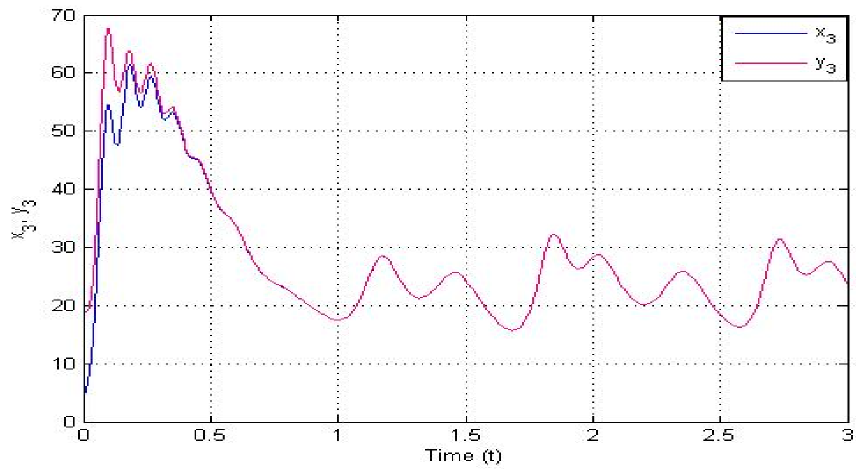

Figure 9: Synchronization of identical Chen Systems between x_3 and y_3

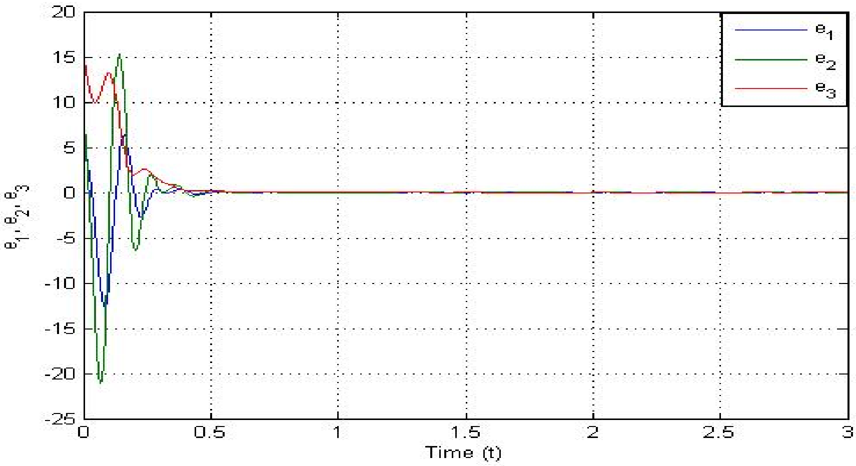

Figure 10: Error between two identical *Chen* systems

6 Estimation of T and Chen Chaotic Dynamics

The sliding control method for the estimation of Chen system, and T system are briefly explained in this part. Here, the Chen chaotic dynamics as the plant system and T dynamics as the observer system.

The plant system is outlined by the *Chen* chaotic dynamics and it is described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= (\gamma - \alpha)x_1 - x_1x_3 + \gamma x_2 \\ \dot{x}_3 &= x_1x_2 - \beta x_3 \end{aligned} \tag{6.1}$$

where x_1, x_2, x_3 are the states of the system (6.1) and α, β, γ are the parameters of the dynamics (6.1).

The slave system is described by the T dynamics as

$$\begin{aligned} \dot{\hat{x}}_1 &= a(\hat{x}_2 - \hat{x}_1) + u_1 \\ \dot{\hat{x}}_2 &= (c - a)\hat{x}_1 - a\hat{x}_1\hat{x}_3 + u_2 \\ \dot{\hat{x}}_3 &= -b\hat{x}_3 + \hat{x}_1\hat{x}_2 + u_3 \end{aligned} \tag{6.2}$$

where $\hat{x}_1, \hat{x}_2, \hat{x}_3$ are the states of the system (6.2) and $u = (u_1, u_2, u_3)$is the sliding mode controller to be designed.

The error y is defined by

$$y_i = \hat{x}_i - x_i, \ \ (i = 1, 2, 3) \tag{6.3}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + (a - \alpha)(x_2 - x_1) + u_1 \\ \dot{y}_2 &= (c - a)y_1 + (c + \alpha - a - \gamma)x_1 \\ &\quad - \gamma x_2 - a\hat{x}_1\hat{x}_3 + x_1x_3 + u_2 \\ \dot{y}_3 &= -by_3 + (\beta - b)x_3 + \hat{x}_1\hat{x}_2 - x_1x_2 + u_3. \end{aligned} \tag{6.4}$$

The matrix representation of the error dynamics (6.4) is

$$\dot{y} = Py + \phi(x, \hat{x}) + u \tag{6.5}$$

$$P = \begin{bmatrix} -a & a & 0 \\ c - a & 0 & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

where

$$\phi(x, \hat{x}) = \begin{bmatrix} (a - \alpha)(x_2 - x_1) \\ (c + \alpha - a - \gamma)x_1 - \gamma x_2 - a\hat{x}_1\hat{x}_3 + x_1x_3 \\ -(\beta - b)x_3 - \hat{x}_1\hat{x}_2 + x_1x_2 \end{bmatrix}.$$

The sliding mode controller is designed by

$$u = -\phi(x, \hat{x}) + Qv \tag{6.6}$$

where Q is chosen such that (P, Q) is controllable. In this case Q is chosen as

$$Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{6.7}$$

The parameter values are

$$a = 2.1, b = 0.6, c = 30.$$

The sliding mode variable is selected as

$$\begin{aligned} s = Ry &= \begin{bmatrix} 5 & 4 & 7 \end{bmatrix} y \\ &= 5y_1 + 4y_2 + 7y_3 \end{aligned} \tag{6.8}$$

which makes the sliding mode state equation asymptotically stable.

The sliding mode gain k_q and Q are updated as

$$k = 10, Q = 1$$

Using the equation (6.8), $v(t)$ updated as

$$v(t) = -8.4437y_1 - 2.1563y_2 \; -3.1125y_3 \;\; . \tag{6.9}$$

where $\phi(x, \hat{x}), Q$ and $v(t)$ are defined as in equations (6.1). The *Chen* and T systems (6.1) and (6.2) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller u defined by (6.1).

6.1 A Numerical Discussion and Results for Estimation of T and Chen Chaotic Dynamics

For the numerical validation, MATLAB ode solver is used to solve the systems of differential equations (6.1) and (6.2) with the sliding mode controller (4.8). The parameters of the Chen system (6.1) are selected so that the system has chaotic behaviour, *viz.* $\alpha = 35$, $\beta = 3$, $\gamma = 28$. The parameters of the T system (6.2) are selected so that the system has chaotic behaviour, *viz.*

$$a = 2.1, \; b = 0.6, \; c = 30.$$

The initial conditions of the plant and observer systems are taken as

$$\begin{aligned} &x_1(0) = 40.1987, x_2(0) = 20.4654, \\ &x_3(0) = 80.9213 \end{aligned}$$

and

$$\begin{aligned} &\hat{x}_1(0) = 10.3409, \hat{x}_2(0) = 60.9198, \\ &\hat{x}_3(0) = 90.5198. \end{aligned}$$

The sliding mode controller,

$$u = -\phi(x, \hat{x}) + Qv$$

$$v(t) = [-8.4437 - 2.1563 - 3.1125] \;\; .$$

when synchronize the system (6.1) and (6.2), it holds the following results via following figures. Figure 11, 12 and 13 shows the Synchronization of the Chen system (6.1) and the T system (6.2). Figure 14 shows error between Chen system (6.1) and the T system (6.2).

7 Conclusions

The sliding mode control method has been used to control the T and *Chen* chaotic systems in this study. This method has the advantage of being a systematic procedure for synchronizing chaotic systems with no derivative in the controller. Two non-identical chaotic systems (T and *Chen*) have been used to test the sliding control design. The efficiency of the proposed control strategies for chaotic systems has been demonstrated and validated using numerical simulations.

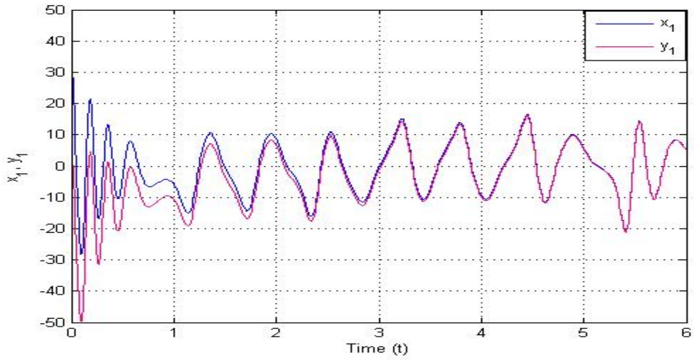

Figure 11: Synchronization of non-identical Chen and T Systems between x_1 and y_1

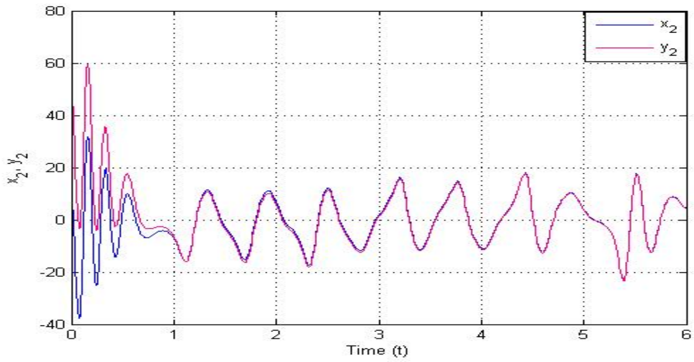

Figure 12: Synchronization of non-identical Chen and T Systems between x_2 and y_2

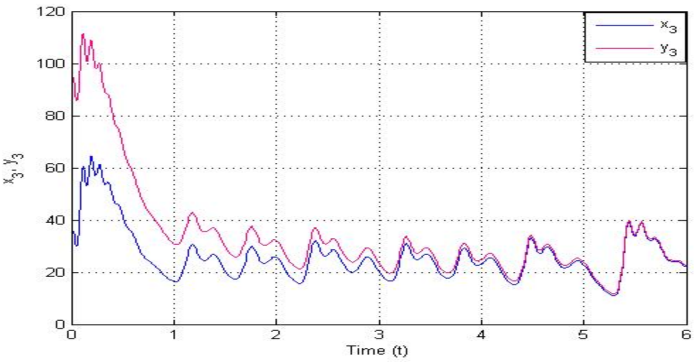

Figure 13: Synchronization of non-identical Chen and T Systems between x_3 and y_3

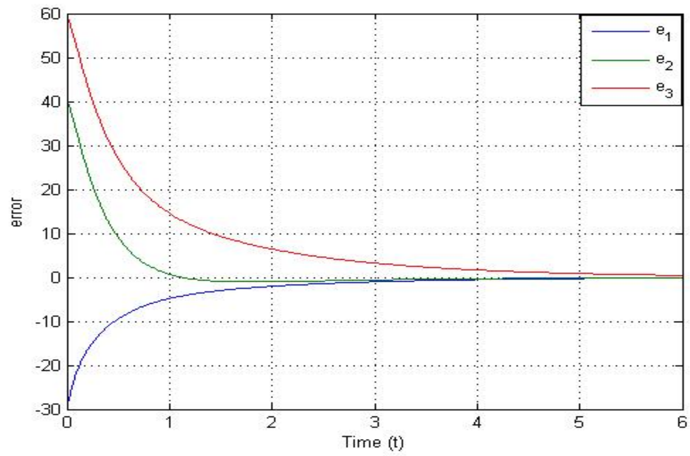

Figure 14: Error between Chen and T systems.

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