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A hybrid model based on Siemens and data envelopment analysis to solve the time-cost trade-off problem considering multi factors

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Abstract

One of the most common problems in the context of project management is the project delay issue. Delay in projects has been caused by many factors, and most of them can be controlled by proper management. One of the most important measures which reflect projects success is its scheduling performance. The capabilities and limitations of an organization for advancing projects are under continuous changes and transformations. Meanwhile, environmental and technological changes over time provide the basis for changes in organizational strategy. Today, the world is moving toward planning and execution of projects by considering factors such as time and cost from the beginning, and these factors are considered simultaneously. In the management of any project, cost, time, risk and quality are critical and essential factors. All of these factors should be in their best condition, so the project can be handled in the best possible way. This paper is looking for a method that includes quality, risk and cost measures in the time-cost tradeoff problem. The "resources" aspect is effective for each of these factors. These resources consist of human resources, machines and financial resources. There are various methods for establishing a tradeoff between time and cost. In this article, a heuristic algorithm based on data envelopment analysis had been developed for multi-criteria time-cost tradeoff. The results of applying this heuristic method to a numerical case study have been reported.

Keywords: Time Cost Trade-off (TCT), Heuristic Algorithm, Risk, Quality, Crashing, Data Envelopment Analysis (DEA), Siemens Algorithm
2020 MSC: 90C08

1 Introduction

In the traditional approaches for discovering the most cost-effective combination which lead to shorter durations of project activities, almost all of the possible combinations are scanned and counted directly. The efficient or optimal combinations are selected from the list of all possible combinations. This approach is feasible for small networks which have a few number of critical paths, or in networks where the opportunity of reducing the duration of all activities is unavailable and consequently a few activities can be considered for the cost-effective duration reduction. For larger

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networks and those networks which have a significant number of critical paths, examining all options requires heavy computations. Even in the case of using computers, the cost of running tasks on them might become very high and economically infeasible. Algorithms can eliminate a considerable number of combinations which are not available in the final solution. By initial elimination, the volume of computational tasks is reduced significantly. Siemens algorithm [29] is an efficient option for the time-cost trade-off problem. This type of heuristic methods only focuses on time and cost, and other aspects of the project such as quality and risk are neglected. By considering these points, this research proposes a heuristic algorithm based on data envelopment analysis. The proposed algorithm tries to establish a tradeoff between time and cost, and focuses on optimization of project's risk and quality.

2 Literature

This section discusses the result published in previous studies about time-cost tradeoff. Chung wei feng et al [13] published their study in an international civil engineering journal. They tried to apply a probabilistic approach and solved the problem through simulation. In Robinson's study, the time value of money was considered. The researchers argued that since projects are done in long term, "interest rate" is the factor which can change the basis of decision about crashing. This study showed the crashing of all activities should be at an equal level, and in such circumstances the impact of "overhead costs" is eliminated [25]. Sou and sen lou [19] published their study in an international project control journal. They investigated the impact of "uncertainties" and "inaccurate data" in the time- cost tradeoff problem, by means of genetic algorithm and fuzzy theory. Li and Cao [20] from Hong Kong's Polytechnique university, combined genetic algorithm and machine learning methods and proposed a technique called MLGAS. They claimed this technique provides better solutions, if there is a nonlinear relationship between costs and activities. In 1994, Burns published an article and proposed a hybrid model of LP/IP (Linear programming/integer programming) for crashing problem. His model was on the basis of linear and integer programming. He tried to minimize "direct crashing costs" in order to achieve the required time (duration). This model could include both types of activities, those that didn't require time crashing and those which should definitely be crashed [7]. From the available studies, Chung wei feng and Burns from Illinois university proposed an algorithm based on genetic algorithm. They also developed a computational program which could determine the efficiency of algorithm. It is the most comprehensive research performed based on Genetic algorithm for solving time-cost tradeoff problem. Khosroshai and Kaka [18] initiated a research at the University of Liverpool. Their focus on the problem led to the conclusion that there is a nonlinear relationship between project cost and project time. Meanwhile, other researchers such as Jin Chaoguang et al proposed models and solutions. Their results were based on a "nonlinear" relation between cost and finishing time of activity, which were easily programmed and solved. Saman [9] published an article and evaluated the impact of crashing on project execution quality. He proposed a linear programming model for making a trade-off between cost, time and quality. In this model, a lower bound (Threshold) of quality is considered for each activity and additional constraints are similar to the classical models mentioned. It should be noted that the concept of "quality slope /gradient" introduced in this article is obtained by dividing the difference between normal and crashing levels of quality by the level of crashing for each activity [26]. In 2010 Nikoomaram, H. et al. defined a new cost function fot TCT problem which consider the time value of money for each activity in crashing process [22].

Many researchers used simulation technique, and a few of them are introduced in this section. In 1963, Van slyke [31] was the first one who applied Monte Carlo simulation for "time crashing problem". He had many contributions; "more accurate estimation of project duration", "flexibility in choosing activity duration distribution function" and "ability to estimate the level of path criticality" are some of his most significant conclusions. Hapke et al modified PERT technique in 1966, and introduced a new technique called GERT. This technique allows activities to follow different distributions. In addition, project finish time can be calculated by Monte Carlo simulation [15]. In 1987, Ameen proposed "cost slope" and "CAPERTSIM" technique. His method was used for decision-making under uncertain conditions and focused on time-cost tradeoff and relations between these variables in the "time crashing problem". This study defined "cost slope" as the ratio of generated cost to crashing level. A preliminary probabilistic model is provided for "crashing problem" by referring to duration of each activity and through a simple simulation [4].

Shahriari proposed a two-objective mathematical model for balancing compressing the project time with activities delay to prepare a suitable tool for decision makers caught in available facilities and due to the time of projects. Also drawing the scheduling problem to real world conditions by considering nonlinear objective function and the time value of money are considered. The presented problem was solved using NSGA-II, and the effect of time compressing reports on the non-dominant set [27].

Gen and Cheng proposed a model based on genetic algorithm. This model aimed at minimizing the direct cost of all crashing durations on PERT network [14]. In 1997, Chau et al believed that mathematical models such as linear programming and integer programming are not efficient for solving time crashing problems from "computation time" perspective and are infeasible for large-scale problems. They developed a Genetic algorithm-based model where resource constraints are included for each of the activities [8]. In 1999, Li and Cao proposed a technique called MLGAS by combining Genetic algorithm and machine learning methods. They claimed if the relations between costs and activities were nonlinear, the solutions provided by this technique is higher than previous techniques. But this technique also solely focused on direct cost minimization. In 2000, Sou Sen-Lou [20] by assumping fuzzy duration for activities, tried to minimize the direct cost of crashing based on Genetic algorithm (FGA) which realized the required goal [19].

In 2017 Shahriari extended a new VIKOR method as a compromise ranking approach to solve multiple criteria decision-making (MCDM) problems through intuitionistic fuzzy analysis. Using compromise method in MCDM problems contributes to the selection of an alternative as close as possible to the positive ideal solution and far away from the negative ideal solution, concurrently is very useful approach when we are considering multiple factors in crashing and TCT problem [28].

Deterministic Time Cost Trade-off problem (DTCTP) had been examined by different methods which gave exact solutions. But none of these exact methods provide solutions to largescale DTCTP problem. So researcherts get inspired to use other methods. The study of Aken in [2] is an example of heuristic methods based on Lagrangian relaxation inside AOA network. In [21], Liu et al. used Genetic algorithm for solving DTCTP. Penget et al. [24] also used Genetic algorithm for solving DTCTP. Almaghrabi and Combrowski [12] added reward and penalty to DTCTP's objective function. Ane et al [5] introduced activities crashing. Van Slyke in 1963 was the first one who used Monte Carlo simulation for Crashing problems. He had many achievements such as more accurate estimation of projects duration, flexibility for choosing activities duration distribution functions and ability of measuring path criticality [31].

In general, time-cost tradeoff problem is a NP-hard problem. Researchers focused on using meta-heuristic algorithm for solving them in recent years. In the same context, Ballesteros-Perez et al [6], Agdas et al [1] used the Genetic algorithm for solving time-cost tradeoff problem. Alavipour and Arditi used a hybrid algorithm based on Genetic method and linear programming [3]. Eirgash and Togan used the teaching-learning based optimization algorithm [30].

In general, time-cost tradeoff problem had many reflections in recent years and the study of Jeunet and Orm [17] is one of them. They considered the time, cost and quality tradeoff for part time and work. The studies of Cui et al. [11], Hasyyati et al. [16], Novianto et al. [23] are some of other researchers performed in recent years.

3 Problem Statement

Project completion calculations are based on this assumption that the necessary activities of project, especially those activities located at critical path are finished at normal durations. In many instances, it is necessary to finish a project earlier that duration estimated on the network. In such circumstances, accelerating activities is one of solutions for crashing project duration. In order to reduce the duration of an activity, the allocated resources should be increased or changes should be made at technical procedures in order to reduce the duration. In other words, in order to perform an activity in a duration shorter than normal duration, it is necessary to increase the level of resources such as staff, equipments and machines. More expensive and more powerful equipments should be used, or changes should be made in technical procedures. Crashing activities duration to shorter durations has its own costs. On the other hand, shorter duration for completing the project leads to less costs for employers and contrtactors. Working capital would have a higher turnover and for new product development projects where there is competition between various producers, faster introduction of a new product might allocate a significant share of market demand to that product.

In time-cost tradeoff problem, activity cost slope is the amount of extra direct costs which is generated by reducing activity duration for one time unit. Since direct cost – time diagram is not linear, activity cost slope cannot be constant for different times (The exact value of activity cost slope is the derivation of cost at corresponding time). Since drawing the exact diagram for direct cost of activities is impossible and any diagram is approximate, it is useful to represent cost diagrams with "linear approximation" similar to Figure 1. Such an approach make cost estimation much easier.

Equation (3.1) shows the procedure for estimating cost slope. Cost slope is negative since time crashing lead to more costs. Meanwhile, since the absolute value of slope cost is used in calculations, the absolute symbol is added to the formula:

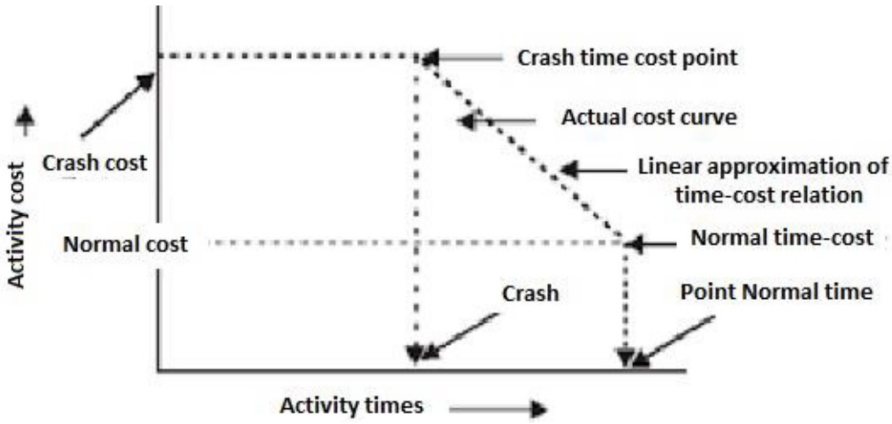

Figure 1: Activity cost slope

$$C = \left| \frac{C_f - C_n}{D_f - D_n} \right| \tag{3.1}$$

In this equation, C_f is Crashing cost, C_n is normal cost, D_f is Crashing duration and D_n is Normal duration.

Project Quality Management is another important aspect of project management. Project Quality Management consists of processes and activities of execution unit. This unit defines responsibilities, qualitative goals and policies which satisfy the project requirements. Project Quality Management implements quality management system in a desirable manner through procedures, policies and continuous process improvement activities. Activities crashing can lead higher cost and also to lower project quality.

Quality index actually reflects the addition of the manner of doing an activity to the estimation process. The quality of doing some activities might reduce during Crashing, and this situation can be a good reason for adding this index to the estimation process. The concept of quality slope is defined according to equation (3.2):

$$Q = \left| \frac{Q_f - Q_n}{D_f - D_n} \right| \tag{3.2}$$

In this equation, Q_f is quality of crashed activity and Q_n is normal quality. In fact the division of quality reduction to Crashing time is considered as Quality slope. With this index, we can show that for each day of Crashing, how much of the quality of Crashed activity is reduced.

Besides Quality management, project risk management is an important issue for time-cost tradeoff. Project risk management is the systematic execution of managerial policies, procedures and guidelines in order to perform tasks such as risk concept confirmation, analysis, evaluation, confrontation, monitoring and connection.

Risk of any activity consists of any unknown probable events or conditions which in case of occurrence have negative or positive consequences on project objectives. Each of these events or conditions have specific causes and recognizable results and outcomes. The consequences of these events have a direct impact on pre-defined time,cost and quality of project. The causes of these events aren't limited to project's internal environment. According to the essence of project and execution methods, many of factors in surrounding environment have an impact on execution and objectives of project. Events with external causes are usually uncontrollable. Of course some internal events such as lack of managers knowledge, and inefficiency of executive equipment and facilities are also uncontrollable. The important point is the fact that before risk identification, it is not possible to estimate its positive or negative impacts on project's objectives. It is solely one of the most critical factor for execution of projects in unreliable and uncertain conditions. After identification and analysis of risks, it would possible to make plans and guidances for them. On the other hand, the unknown risks of project are unmanagle, even with reliance on experiences of managers from similar previous projects, or by applying contingency approaches and techniques. Risk of any project is directly related to sum of risks of each of its activities. So the risk of any activity can be an important measure of Crashing. The reason is that Crashing usually increase of performing an activity.

4 A heuristic method for time-cost tradeoff

This research tries to develop a hybrid methodology based on Siemens algorithm and Data envelopment analysis which can solve time-cost tradeoff problem in a multi-criteria manner. In the following paragraphs, some explanations are given about Siemens algorithm and Data envelopment analysis.

Siemens algorithm is mainly used for solving time-cost tradeoff problem. If the project have pre-defined completion date which is written in the contract, and this date is earlier than date which is calculated based on usual CPM estimation according to normal activities durations, then Siemens method can discover a desirable combination which reduces activities durations. Under new requirements the project can be completed at due date and also the extra cost of time crashing would be the lowest possible value [31].

In Siemens algorithm, crashing costs is the only criterion considered for activities crashing. This research aims to propose a multi-criteria method for solving time-cost tradeoff that considers cost, quality and risk of activities crashing simultaneously. This procedure is done by means of data envelopment analysis.

Date envelopment analysis is a powerful mathematical tool which uses linear programming for discovering the relative efficiency in a set of homogeneous decision-making units (DMUs). A DMU is efficient if none of other DMUs can produce the same output by using the same or lower levels of inputs. In fact, Charnes, Cooper and Rhodes proposed the first known DEA model called CCR model. DEA operates in a way that there is no need to consider any of assumptions about the internal interactions of DMU. In other words, DEA handles DMU like a black box and only considers the input and output consumed by each DMU. This description is almost always sufficient and effective. For example if the goal of analysis is the identification of inefficient DMUs and evaluating their inefficiency level, then "black box" is an appropriate approach. This procedure aims to measure the efficiency of technology transfer methods. Top management of organizations always wanted to evaluate the performance of organizations units, and it will continue to be an important issue in the future. Traditional evaluation methods which were solely based on financial measures lost their weight in modern management and top managers or strategic managers don't accept them. In such a context, a new perspective is created toward performance evaluation. Now all of the available resources can be evaluated independently. They should be evaluated in real-time in the chosen duration. Naturally some of activities are quantitative and some other are qualitative. Therefore the method chosen for measuring these performances is very important. The results of measurements help to determine what resources have been wasted by organization during fulfilling its goals. Because of this reason, the researchers defined performance evaluation as measurement of activities about a specific issue or idea. This technique use Date envelopment analysis (DEA) for evaluating the efficiency of decision making units. It is used for evaluation of relative performance. The basis and methodology of this technique are represented parametrically in the following section. Suppose each decision-making unit has multiple inputs and outputs. The efficiency is achieved by dividing its output to its input. The division is not possible because of having more than one input and output. Therefore some weights should be attributed to inputs and outputs of the DMU. With the help of these weights, inputs and outputs are converted to single values.

For doing so, lets consider $U = (u_1, ..., u_s)$ and $V = (v_1, ..., v_m)$ as weight vectors of input and output respectively. Therefore DMU_j uses $\sum_{i=1}^{m} v_i x_{ij}$ Input and produces output $\sum_{r=1}^{s} u_r y_{rj}$. Therefore the efficiency of DMU_p is calculated by solving problem (4.1).

$$\max \frac{\sum_{i=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \tag{4.1}$$

$$s.t \quad \frac{\sum_{i=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \leq 1, \ j = 1, ..., n$$

$$u_r, v_i \geq 0, \ r = 1, ..., s, \ i = 1, ..., m$$

Until this moment, various methods have been proposed for measuring the efficiency of organizations. But DEA is better than all of them in terms of data classification and analysis. It allows the efficiency changes over time and has no assumption about efficiency threshold. Therefore it is used more than other techniques for performance evaluation. It is a useful technique for comparison of units based on efficiency.

After these explanations about Siemens algorithm and Data envelopment analysis, now is the time for introducing the methodology of this research. This methodology is used for providing a heuristic activity time crashing technique. It works by simultaneous consideration of three criteria which are cost slope, quality and project risk. Figure 2 shows the steps of the proposed method.

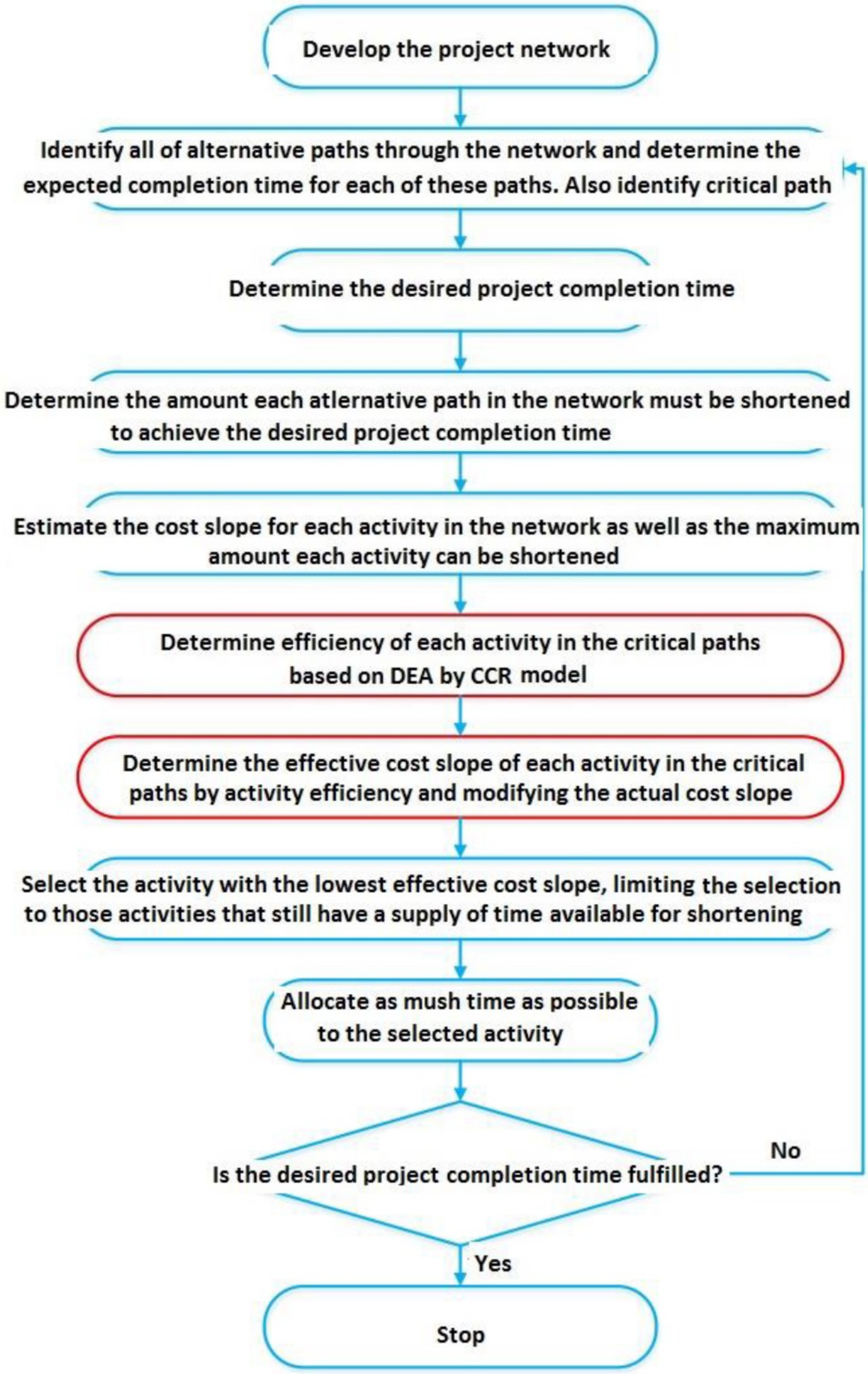

Figure 2: Proposed algorithm flowchart for multi-criteria time-cost tradeoff

As it can be seen in the flowchart of figure 2, DEA is used in step 6 for determining the efficiency of activities on the critical path. DEA is a linear programming technique and its main goal is comparison and evaluation of efficiency of a certain number of similar decision-making units which have different received inputs and generated outputs. Charnes et al [10] proposed the preliminary general DEA model called CCR which can measure the relative technical efficiency of different units. Their study applied mathematical programming optimization method to estimate the efficiency of systems with multiple inputs/outputs. Table 1 shows the multiplicative and envelope CCR models for inputs and outputs.

In this study, Inputs of CCR model for efficiency estimation are activity cost slope (C_j), activity risk increase slope for one unit Crashing (R_j) and activity quality reduction slope for one unit Crashing (Q_j). In addition, the only applied output is the difference between normal activity time and crashing activity time. According to these

Table 1: Multiplicative and envelope CCR models for inputs and outputs

Envelope CCR input model	**Multiplicative CCR input model**
$\min \theta p - \varepsilon \left[\sum_i^{\cdot} s_i^- - \sum_i^{\cdot} s_i^+ \right]$	$\max \sum_r u_r y_{rp} = \theta p$
s.t	s.t
$\sum_j \lambda_j x_{ij} + s_i^- = \theta pxip \; \forall i$	$\sum_i v_i x_{ip} = 1$
$\sum_j \lambda_j y_{rj} - s_r^+ = yrp \; \forall r$	$\sum_r u_y y_{rj} - \sum_i v_i x_{ij} \leq 0 \; \forall j$
$\lambda_j, s_i^-, s_r^+ \geq 0, \theta p \; free$	$u_r, v_i \geq \epsilon$
Envelope CCR output model	**Multiplicative CCR output model**
$\max \phi p + \varepsilon \left[\sum_i^{\cdot} s_i^- - \sum_i^{\cdot} s_i^+ \right]$	$\min \sum_r v_i x_{ip} = \phi p$
s.t	s.t
$\sum_j \lambda_j x_{ij} + s_i^- = xip \; \forall i$	$\sum_r u_r y_{rp} = 1$
$\sum_j \lambda_j y_{rj} - s_r^+ = \phi pyrp \; \forall r$	$\sum_i v_i x_{ij} - \sum_r u_r y_{rj} \geq 0 \; \forall j$
$\lambda_j, s_i^-, s_r^+ \geq 0, \theta, \phi p \; free$	$u_r, v_i \geq \epsilon$

considerations, the inputs (x_{ij}) and outputs (y_{rj}) used in this research are determined. Finally the efficiency of every activity on critical path (E_j) is determined.

In the next step, the effective cost slope (EC_j) is determined through equation (4.2).

$$EC_j = \frac{C_j}{N_j E_j} \tag{4.2}$$

where N_j is the number of critical paths that j activity is on them and haven't crashed enough.

These considerations give us the ability to examine the impact of quality reduction and risk increase in the activity crashing process which should be added to crashing cost.

5 Results analysis

In this section, the proposed methodology is implemented on two numerical case studies. The AON (activity on node) network of the first project is shown in figure 3. In this figure, the normal duration (inside parenthesis) and the number of each activity are shown. 1st and 8th activities are virtual. They determine the start and end of project. The red path shows the critical path of project. Table 2 contains other activity related information. This project is programmed to be completed in 140 time units. We aim to reduce the completion time to 110 units by applying the proposed methodology. Table 3 shows the results of 7 consecutive iterations. Figure 4 shows the AON network after "activities crashing". According to this figure, all of remained paths after crashing show project's critical path.

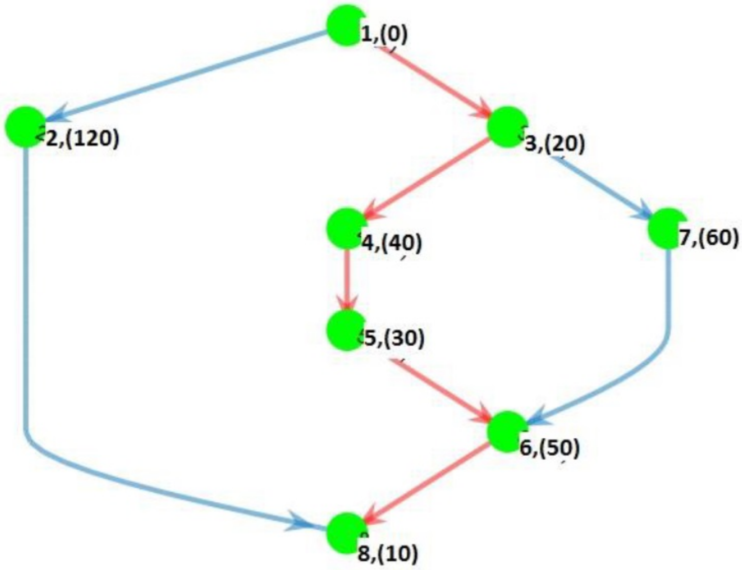

Figure 3: AON network of project for the first case study

Table 2: Information of each activity for the first case study

Activity	D_f	D_n	C_f	C_n	**C**	**Q**	**R**
1	0	0	0	0	0	0	0
2	100	120	14000	12000	100	1	2
3	15	20	2800	1800	200	2	1
4	30	40	22000	16000	600	4	1
5	20	30	2000	1400	60	3	2
6	40	50	4800	3600	120	2	1
7	45	60	18000	13500	300	3	2
8	0	0	0	0	0	0	0

Table 3: Crashing of first case study project with the proposed methodology

Iteration = 1 Critical paths by Make span = 140 : 1 3 4 5 6 8 Selected Activity = 5, Crash Amount = 10 Crashing Cost = 600, Project risk = 20, Quality Reduction = 30
Iteration = 2 Critical paths by Make span = 130 : 1 3 4 5 6 8 1 3 7 6 8 Selected Activity = 6, Crash Amount = 10 Crashing Cost = 1800, Project risk = 30, Quality Reduction = 50
Iteration = 3 Critical paths by Make span = 120 : 1 2 8 1 3 4 5 6 8 1 3 7 6 8 Selected Activity = 2, Crash Amount = 10 Crashing Cost = 2800, Project risk = 50, Quality Reduction = 60

Iteration = 4
Critical paths by Make span = 120 :
1 3 4 5 6 8
1 3 7 6 8
Selected Activity = 3, Crash Amount = 5
Crashing Cost = 3800, Project risk = 55, Quality Reduction = 70

Iteration = 5
Critical paths by Make span = 115 :
1 3 4 5 6 8
1 3 7 6 8
Selected Activity = 7, Crash Amount = 5
Crashing Cost = 5300, Project risk = 65, Quality Reduction = 85

Iteration = 6
Critical paths by Make span = 115 :
1 3 4 5 6 8
Selected Activity = 4, Crash Amount = 5
Crashing Cost = 8300, Project risk = 70, Quality Reduction = 105

Iteration = 7
Critical paths by Make span = 110 :
1 2 8
1 3 4 5 6 8
1 3 7 6 8
Selected Activity = 2, Crash Amount = 0
Crashing Cost = 8300, Project risk = 70, Quality Reduction = 105

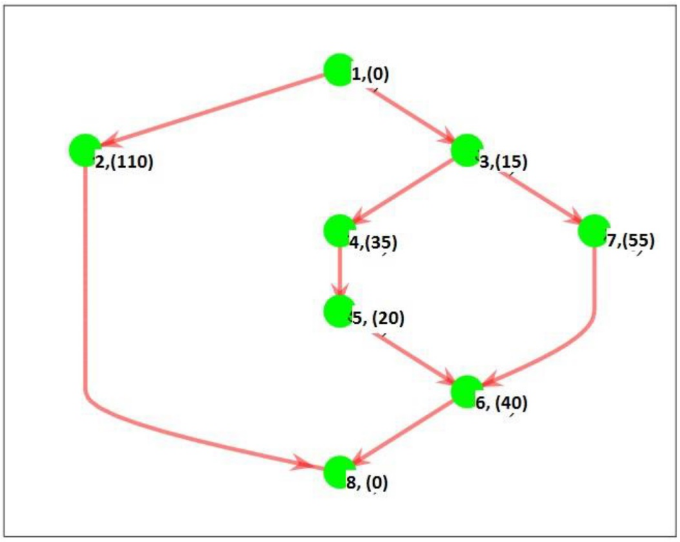

Figure 4: AON network of project for the first case study after crashing with the proposed methodology

The second case study is a project with 44 activities (activities 1 and 44 are virtual activities which determine the start and end of project). The AON network of this project is shown in figure 5. The red line shows the critical path. Table 4 contains other information about each of activities. This project should be completed in 171 time units. By implementing the proposed methodology on this project, we aim to reduce the completion time to 120 time units. The algorithm of proposed methodology reaches "stop criterion" after 71 iterations. Therefore project risk reaches 227 and quality reduction is equal to 210. Table 5 shows the duration of each activity after activity crashing in second case study. Figure 7 shows the levels of activity crashing after implementation of proposed methodology.

Table 4: Information of each activity in the second case study

Act	D_f	D_n	C_f	C_n	**C**	**Q**	**R**	**Act**	D_f	D_n	C_f	C_n	**C**	**Q**	**R**
2	3	8	2000	500	300	3	5	23	4	9	4050	1900	430	3	5
3	3	6	2050	850	400	3	1	24	6	9	6020	500	1840	1	5
4	3	5	2480	480	1000	4	5	25	6	8	1900	360	770	1	4
5	10	15	1830	380	290	4	4	26	2	5	2070	870	400	2	4
6	4	8	2100	300	450	4	1	27	5	7	4000	500	1750	5	4
7	11	12	1890	690	1200	2	2	28	2	6	15020	1180	3460	2	2
8	5	9	320	200	30	4	3	29	6	15	64600	7000	6400	5	4
9	4	11	25000	1200	3400	4	5	30	5	12	3710	700	430	2	1
10	5	6	770	230	540	1	5	31	2	4	470	370	50	5	4
11	6	8	615	175	220	1	1	32	4	8	7720	920	1700	2	1
12	3	7	6420	2300	1030	3	5	33	14	19	16800	5000	2360	1	2
13	12	19	19980	5700	2040	5	5	34	3	5	1110	940	85	2	1
14	3	5	3150	550	1300	2	3	35	7	12	27550	5300	4450	4	1
15	2	4	525	125	200	3	5	36	5	7	5600	3400	1100	3	5
16	11	13	3520	660	1430	2	1	37	9	11	3580	300	1640	2	4
17	6	9	2535	885	550	4	3	38	4	8	8690	770	1980	5	2
18	4	7	12660	3000	3220	2	5	39	6	11	2190	690	300	3	5
19	6	9	610	250	120	3	4	40	1	5	6520	3000	880	3	1
20	10	13	9980	980	3000	4	5	41	6	8	2940	2200	370	5	3
21	5	7	3440	1440	1000	5	4	42	4	9	12600	1100	2300	2	2
22	8	11	3970		1230	5	1	43	3	8	3850	600	650	4	4

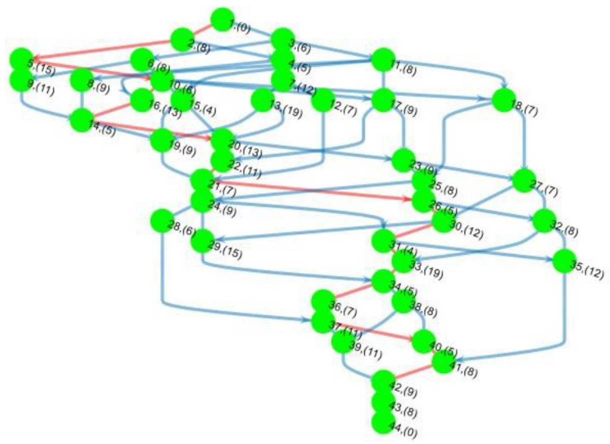

Figure 5: AON network of project for the second case study

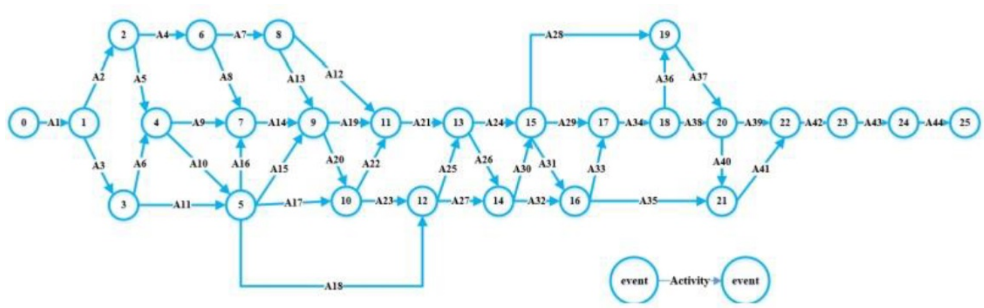

Figure 6: AON network of project for the second case study

Table 5: Duration of each activity after crashing in the second case study

Act	**d**	**Act**	**d**
2	3	23	5
3	6	24	7
4	3	25	8
5	10	26	2
6	6	27	7
7	11	28	6
8	9	29	15
9	11	30	5
10	5	31	2
11	8	32	7
12	7	33	14
13	17	34	3
14	5	35	12
15	4	36	5
16	11	37	9
17	9	38	8
18	7	39	6
19	9	40	1
20	13	41	6
21	6	42	9
22	8	43	3

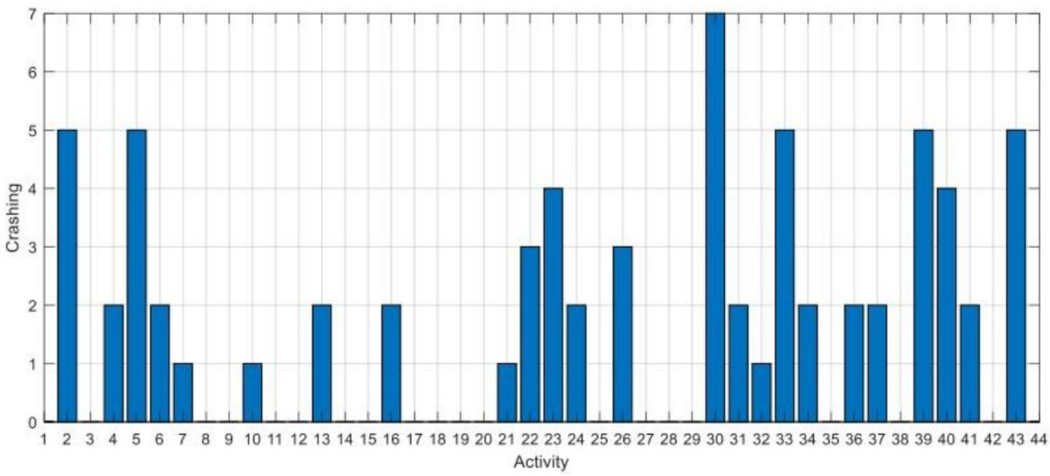

Figure 7: Activities crashing based on the proposed method in the second case study

6 Conclusion

As it has been shown, the proposed methodology of this research features a new method for estimation of effective cost slope. The final outcome of the proposed methodology is a time-cost tradeoff diagram which can be used to make decision about project. Any decision should be made according to financial and temporal considerations. By using data envelopment analysis in estimating effective cost slope, it would be possible to add other criteria such as crashing risk and quality reduction beside crashing cost in the activity crashing process. This algorithm lead to correct and accurate results if the initial estimations of project duration and cost are accurate.

In the future studies, if there are certain budgets for specific durations of project, it is possible to add scheduled budget through appropriate boundaries as the main constraints of the study. In this case the crashing budget would be dependent on project events.

References

[1] D. Agdas, D.J. Warne, J. Osio-Norgaard and F.J. Masters, *Utility of genetic algorithms for solving large-scale construction time-cost trade-off problems*, J. Comput. Civil Eng. **32** (2018), no. 1, 04017072.

[2] C. Akkan, *A Lagrangian heuristic for the discrete time–cost tradeoff problem for activity-on-arc project networks*, Working Paper, Koc University, Istanbul, 1998.

[3] S.R. Alavipour and D. Arditi, *Time-cost tradeoff analysis with minimized project financing cost*, Automat. Construct. **98** (2019), 110–121.

[4] D.A. Ameen, *A computer assisted PERT simulation*, J. Syst. Manag. **38** (1987), no. 4, 6.

[5] T. Ann and S.S. Erenguc, *The resource constrained project scheduling problem with multiple crashable modes: a heuristic procedure*, Eur. J. Oper. Res. **107** (1998), no. 2, 250–259.

[6] P. Ballesteros-Perez, K.M. Elamrousy and M.C. González-Cruz, *Non-linear time-cost trade-off models of activity crashing: application to construction scheduling and project compression with fast-tracking*, Automat. Construct. **97** (2019), 229–240.

[7] S.A. Burns, L. Liu and C.W. Feng, *The LP/IP hybrid method for construction time – cost trade off analysis*, Construct. Manag. Econ. J. **14** (1994), no. 3, 265–276.

[8] D.K.H. Chau, W.T. Chan and K. Govindan, *A time-cost trade-off model with resource consideration using genetic algorithm*, Civil Eng. Syst. **14** (1997), 291–311.

[9] J. Chao-Guang, J. Zhuo-Shang, L.I.N. Yan, Z. Yuan-Min and H. Zhen-Dong, *Research on the fully fuzzy time-cost trade-off based on genetic algorithms*, J. Marine Sci. Applic. **4** (2005), no. 3, 18–23.

[10] A. Charnes, W.W. Cooper and E. Rhodes, *Measuring the efficiency of decision making units*, Eur. J. Oper. Res. **2** (1978), no. 6, 429–444.

[11] W. Cui, T. Ma and L. Caracoglia, *Time-cost "trade-off" analysis for wind-induced inhabitability of tall buildings equipped with tuned mass dampers*, J. Wind Eng. Industr. Aerodyn. **207** (2020), p. 104394.

[12] S.E. Elmaghraby and J. Kamburowski, T*he analysis of activity network under generalized precedence relations*, Manag. Sci. **38** (1992), no. 9, 1245–1263.

[13] C.W. Feng, L. Liu and S.A. Burns, *Using genetic algorithms to solve construction time-cost trade-off problems*, J. Comput. Civil Engin. **11** (1996), no. 3, 184–189.

[14] M. Gen and R. Cheng, *Genetic algorithms and engineering design*, John Wily and Sons, New York, 1997.

[15] M. Hapke, A. Jaszkiewicz and R. Slowinski, *Fuzzy project scheduling system for software development*, Fuzzy Sets Syst. **67** (1994), no. 1, 101–117.

[16] S.N. Hasyyati, I.A. Puspita and W. Tripiawan, *Project acceleration of outside plant-fiber optic (OSP-FO) project in PT. XYZ using time cost trade off (TCTO) method by adding overtime hours*, IOP Conf. Ser.: Mater. Sci. Eng. **852** (2020), no. 1, 012103.

[17] J. Jeunet and M.B. Orm, *Optimizing temporary work and overtime in the time cost quality trade-off problem*, Eur. J. Oper. Res. **284** (2020), no. 2, 743–761.

[18] F. Khosrowshahi and A.P. Kaka, *The mathematics of cost-duration trade-off curves*, CIB W92 Procure. Syst. Symp.-Inf. Commun. Construct. Procurement, 2000.

[19] S.S. Leu, A.T. Chen and C.H. Yang, *A GA-based fuzzy optimal model for construction time–cost trade-off*, Int. J. Project Manag. **19** (2001), no. 1, 47–58.

[20] H. Li, J.N. Cao and P.E.D. Love, *Using machine learning and GA to solve time-cost trade-off problems*, J. Construct. Eng. Manag. **125** (1999), no. 5, 347–353.

[21] S.X. Liu, M.G. Wang, L.X. Tang and Y.Y. Nie, *Genetic algorithm for the discrete time/cost trade-off problem in project network*, J. Northeastern Univ. Natural Sci. **21** (2000), no. 3, 257–259.

[22] H. Nikoomaram, F.H. Lotfi, J. Jassbi and M.R. Shahriari, *A new mathematical model for time cost trade-off problem with budget limitation based on time value of money*, Appl. Math. Sci. **4** (2010), no. 63, 3107–3119.

[23] R. Novianto, A. Bastari and A. Rahman, *Time-cost trade-off analysis on Jetty construction project (case study: Indonesian navy Jetty construction project)*, STTAL Postgrad.Int. Conf., **4** (2020), no. 1.

[24] W. Peng and C. Wang, *A multi-mode resource-constrained discrete time–cost tradeoff problem and its genetic algorithm based solution*, Int. J. Project Manag. **27** (2009), 600–609.

[25] D. Robinson, *A dynamic programming solution to cost-time trade-off for CPM*, Manag. Sci. **22** (1965), 158–166.

[26] M. Saman, *Crashing in PERT networks*, M.Sc. Thesis, University of Jordan, Amman-Jordan, 1991.

[27] M. Shahriari, *Multi-objective optimization of discrete time–cost tradeoff problem in project networks using non-dominated sorting genetic algorithm*, J. Industr. Eng. Int. **12** (2016), no. 2, 159–169.

[28] M. Shahriari, *Soft computing based on a modified MCDM approach under intuitionistic fuzzy sets*, Iran. J. Fuzzy Syste. **14** (2017), no. 1, 23–41.

[29] N. Siemens, *A simple CPM time-cost tradeoff algorithm*, Manag. Sci. 17 (1971), no. 6, B–354.

[30] V. Toğan and M.A. Eirgash, *Time-cost trade-off optimization of construction projects using teaching learning based optimization*, KSCE J. Civil Eng. **23** (2019), no. 1, 10–20.

[31] R.M. Van Slyke, *Monte Carlo methods and the PERT problem*, Oper. Res. **33** (1963), 141–143.