**Int. J. Nonlinear Anal. Appl. 14 (2023) 4, 151–160**
**ISSN: 2008-6822 (electronic)**
http://dx.doi.org/10.22075/ijnaa.2023.29567.4192

# New odd Chen Fréchet distributions: Properties and applications

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*(Communicated by Ehsan Kozegar)*

**Abstract**

To introduce a new four-parameter lifetime distribution that will be more flexible in modelling real lifetime data over the existing common lifetime distributions. The new four-parameter lifetime distribution is generated by using the odd Chen generator of distributions. In this method, the probability density function and cumulative distribution function of Fréchet distributions are used as a base distribution for odd Chen Fréchet distributions. The probability density function and cumulative distribution function of the Fréchet distributions are substituted in the odd Chen generator of the distributions model to get the new and more flexible lifetime distribution for modelling real-life data. The authors reveal that the hazard rate of the odd Chen Fréchet distributions is increasing. They also found that the odd Chen generator of distributions gives a much close fit than the Fréchet Distribution (FD), Weibull distribution(WD), and exponential distribution (ED). In this study, a novel probability distribution is introduced. odd Chen generator of distributions is capable of modelling upside-down bathtub-shaped hazard rates. The model is appropriate to fit the asymmetrical data that are not correctly fitted by other distributions. The said distribution can be applied to different fields like insurance, earthquake data for analysis, reliability etc.

Keywords: odd Chen generator of distributions, Reliability Analysis, Moments, Parameter Estimation, Fréchet, MLE
2020 MSC: 60E05

## 1 Introduction

Recently, there has been an increased interest among statisticians to progress new extended distributions to be more capable for exhibiting data in different areas such as lifetime investigation, engineering, economics, finance, demography, actuarial, living, and medical sciences. The Fréchet distribution is an imperative distribution developed within the extreme value theory. It has applications in life testing, floods, horse racing, precipitation, queues in supermarkets, sea whitecaps, and wind speeds. Further evidence about the Fréchet distribution and its applications can be explored in [6], Aiming a more flexible Fréchet distribution, for many years geometricians have been developing various extensions and modified forms of the Fréchet distribution, with different number of parameters. For specimen, the exponentiated Fréchet due to [12], the beta Fréchet due to [11] and [4], the transmuted Fréchet due to [8], the gamma extended Fréchet due to [5, 12], the Marshall–Olkin Fréchet due to [7], the Kumaraswamy Fréchet due to [10], the transmuted Marshall–Olkin Fréchet due to [1], the Kumaraswamy Marshall–Olkin Fréchet due to [3], the

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*Received:* December 2022 *Accepted:* February 2023

Kumaraswamy transmuted Marshall–Olkin Fr due to [13], the Weibull Fréchet due to [2] and the beta exponential Fréchet due to [9]. The probability density function (PDF) and cumulative distribution function (CDF) of the Fréchet distribution are given by (for $x \geq 0$)

$$f\left(x, \gamma, \theta\ \right) = \frac{\gamma}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1} e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\ ; \quad x, k, \gamma > 0 \tag{1.1}$$

$$F\left(x\right) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}}; \quad x, k, \gamma > 0 \tag{1.2}$$

respectively, where $\alpha > 0$ is a scale parameter and $\beta > 0$ is a shape parameter.

## 2 Odd Chen Fréchet distributions

Let $x$ be a random variable which is distributed as odd Chen Fréchet distributions. The survival function of the (CDF) and (PDF) is given:

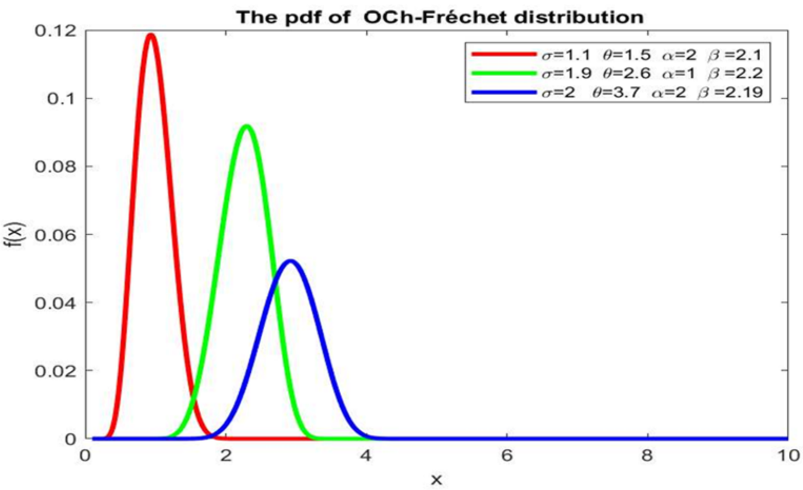
$$f(x, \alpha, \beta, \tau) = \frac{\alpha\beta G(x,\tau)^{\beta-1} g(x,\tau)}{[1-G(x,\tau)]^{\beta+1}} \left[e^{\left[\frac{G(x,\tau)}{1-G(x,\tau)}\right]^{\beta}}\right] e^{-\alpha\left[e^{\left[\frac{G(x,\tau)}{1-G(x,\tau)}\right]^{\beta}}-1\right]} \tag{2.1}$$

$$\mathrm{F}(\mathrm{x}, \alpha, \beta, \tau) = 1 - \mathrm{e}^{-\alpha\left[\mathrm{e}^{\left[\frac{\mathrm{G}(\mathrm{x},\tau)}{1-\mathrm{G}(\mathrm{x},\tau)}\right]^{\beta}}-1\right]} \tag{2.2}$$

and by Substituting the equation (1.1) in (2.1) we get the CDF of the new distribution and by deriving the CDF we get the PDF as follows.

$$f\left(x, \alpha, \beta, \gamma, \theta\right) = \frac{\alpha\beta\left[e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right]^{\beta-1} \frac{\gamma}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1}\left(e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right)^{\beta-1}}{\left[1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right]^{\beta+1}} \left[e^{\left[\frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}\right]^{\beta}}\right] e^{-\alpha\left[e^{\left[\frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}\right]^{\beta}}-1\right]}. \tag{2.3}$$

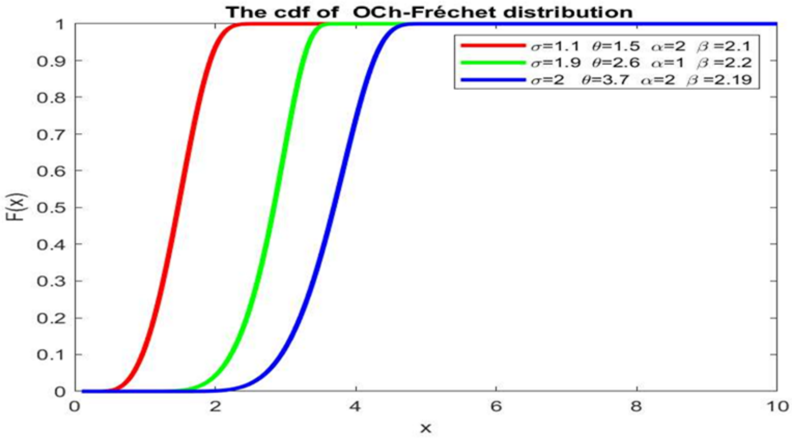
The reasonable shapes of PDF odd Chen Fréchet distribution is as follows:




The CDF of this distribution is

$$F\left(x, \alpha, \beta, \gamma, \theta\right) = 1 - e^{-\alpha\left(e^{\left(\frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}\right)^{\beta}}-1\right)}. \tag{2.4}$$

The reasonable shapes of CDF odd Chen Fréchet distribution




The Survival function of the new distribution is:

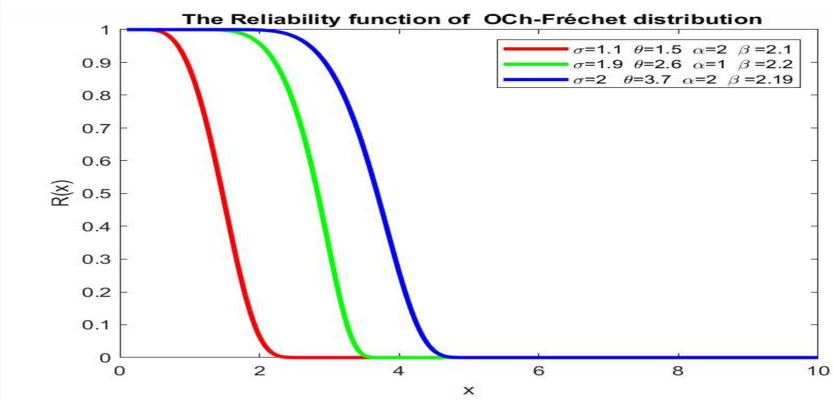
$$S\left(t\right)=1-F\left(t\right)$$

$$S\left(t\right)_{\text{TEF}}=1-F\left(t\right)_{\text{TEF}}$$

and

$$S\left(x,\alpha,\beta,\gamma,\theta\right)=\text{Exp}\left(-\alpha\left(\left(\text{Exp}\left(\mathrm{e}^{-\left(\frac{\theta}{\mathrm{x}}\right)^{\gamma}}-1\right)^{-\beta}\right)-1\right)\right) \tag{2.5}$$

The reasonable shapes Survival odd Chen Fréchet distribution:

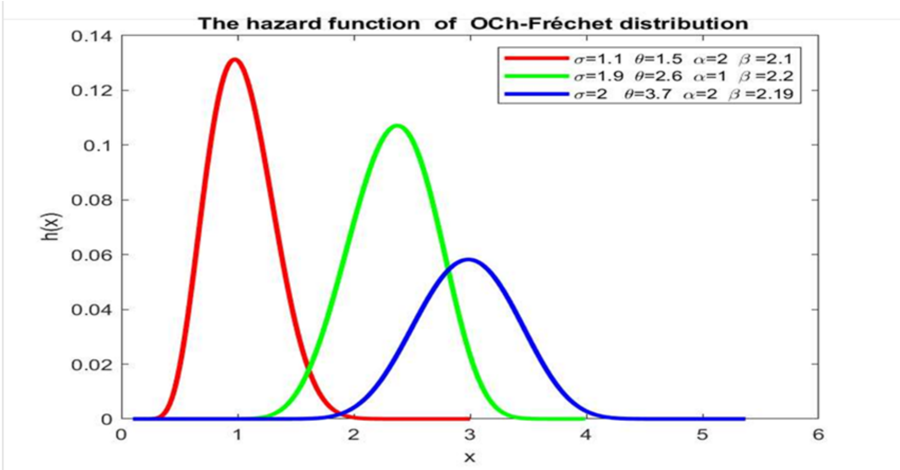



For any random variable $X$ which follows odd Chen Fréchet distribution, its hazard function is given as:

$$h\left(x,\alpha,\beta,\gamma,\theta\right)=\frac{f\left(x,\alpha,\beta,\gamma,\theta\right)}{s\left(x,\alpha,\beta,\gamma,\theta\right)}$$

$$h\left(x,\alpha,\beta,\gamma,\theta\right)\frac{\frac{\alpha\beta\left(e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right)^{\beta-1}\frac{\gamma}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1}e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{\left(1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right)^{\beta+1}}e^{\left(\frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}\right)^{\beta}}e^{-\alpha\left(e^{\left(\frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}\right)^{\beta}}-1\right)}}{\text{Exp}\left(-\alpha\left(\left(\text{Exp}\left[e^{-\left(\frac{\theta}{x}\right)^{\gamma}}-1\right]^{-\beta}\right)-1\right)\right)}. \tag{2.6}$$

The reasonable shapes hazard function odd Chen Fréchet distribution:




## 3 Statistical Properties

In this section, some of the properties of the odd Chen Fréchet distribution are discussed:

### 3.1 Quantile function

The quantile function or inverse cumulative distribution function returns the value $t$ such that:

$$t = Q\left(u\right) = F^{-1}\left(u\right), \quad 0 < u < 1,$$

$$u = \left( -e^{-\alpha\left( e^{\left( \frac{e^{-\left(\frac{\theta}{x}\right)^{\gamma}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\gamma}}} \right)^{\beta}} -1 \right)} \right)$$

and

$$x = \theta\left(1 + \left(-log\left(\frac{-\alpha log\left(1-u\right)}{\alpha}\right)\right)^{\frac{-1}{\beta}}\right)^{\frac{-1}{\gamma}}. \tag{3.1}$$

### 3.2 Moments

Let $x$ denote the random variable follows odd Chen Fréchet distribution then $r^{th}$ order moment about origin of $\mu_r$ is:

$$\mu_{\mathrm{r}}' = \mathrm{E}\left(\mathrm{x}^{\mathrm{r}}\right) = \int_0^{\infty} \mathrm{x}^{\mathrm{r}} \mathrm{f}(\mathrm{x}, \alpha, \beta, \gamma, \theta) \cdot \mathrm{dx}$$

$$E\left(x^r\right) = \int_0^{\infty} x^r \left( \begin{array}{c} \sum_{i=1}^{\infty} \sum_{j=0}^{i} \sum_{k,m=0}^{\infty} \frac{(-1)^{i+j}\alpha^i (i-j)^k \Gamma k\beta+m}{i!k!m!\Gamma k\beta} \\ \binom{i}{j}\frac{\gamma(k\beta+m)}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1} e^{-\left(\frac{\theta}{x}\right)^{\gamma}} \left[e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right]^{k\beta+m-1} \end{array} \right) \mathrm{dx} \tag{3.2}$$

$$\mathbf{c} = \left( \begin{array}{c} \sum_{i=1}^{\infty} \sum_{j=0}^{i} \sum_{k,m=0}^{\infty} \frac{(-1)^{i+j}\alpha^i (i-j)^k \Gamma k\beta+m}{i!k!m!\Gamma k\beta} \\ \binom{i}{j}\frac{\gamma(k\beta+m)}{\theta} \end{array} \right)$$

$$E\left(x^r\right) = c\int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{r}} \left(\frac{\theta}{\mathbf{x}}\right)^{\gamma+\mathbf{1}} \mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^{\gamma}} \left[\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^{\gamma}}\right]^{\mathbf{k}\beta+\mathbf{m}-\mathbf{1}} \mathbf{.dx}$$

and let $u=\left(\frac{\theta}{x}\right)^{\gamma}$, $\frac{\theta}{x}=u^{-\gamma},\ x=\theta u^{\gamma},\ dx=\theta\gamma u^{\gamma-1}\mathrm{du}$, then

$$\begin{aligned}E\left(x^{r}\right)=&c\int_{0}^{\infty}\left(\theta u^{\gamma}\right)^{r}uu^{-\gamma}e^{-u}\left[e^{-u}\right]^{k\beta+m-1}\cdot\theta\gamma u^{\gamma-1}\mathrm{du}\\=&c\theta^{r+1}\gamma\int_{0}^{\infty}\left(\mathrm{u}^{\gamma}\right)^{\mathrm{r}}\mathrm{u}^{1-\gamma}\mathrm{e}^{-\mathrm{u}}\left[\mathrm{e}^{-\mathrm{u}}\right]^{\mathrm{k}\beta+\mathrm{m}-1}\cdot\mathrm{u}^{\gamma-1}\mathrm{du}\\=&c\theta^{r+1}\gamma\int_{0}^{\infty}\left(u^{\gamma}\right)^{r}\ \left(e^{-u}\right)\left[e^{-u}\right]^{k\beta+m-1}.\ \mathrm{du}\\=&c\theta^{r+1}\gamma\int_{0}^{\infty}\left(u^{\gamma}\right)^{r}\ \left[e^{-u}\right]^{k\beta+m}.\ \mathrm{du}\\=&c\theta^{r+1}\gamma\left[\frac{\Gamma 1+r\gamma}{(m+k\beta)^{1+r\gamma}}\right].\end{aligned}$$

$$E\left(x^{r}\right)=\bar{\mu}_{r}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\frac{\gamma^{2}(k\beta+m)}{\theta}\left(\theta^{r+1}\right)\left(\frac{\Gamma 1+r\gamma}{(m+k\beta)^{1+r\gamma}}\right)\end{array}\right)\tag{3.3}$$

$$\mu_{1}^{'}=Ex=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^{2}\left(k\beta+m\right)\left(\theta^{1}\right)\left[\frac{\Gamma 1+\ \gamma}{(m+k\beta)^{1+\ \gamma}}\right]\end{array}\right)$$

$$\mu_{2}^{'}=Ex^{2}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^{2}\left(k\beta+m\right)\left(\theta^{2}\right)\left[\frac{\Gamma 1+2\gamma}{(m+k\beta)^{1+2\gamma}}\right]\end{array}\right)$$

$$\mu_{3}^{'}=Ex^{3}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^{2}\left(k\beta+m\right)\left(\theta^{3}\right)\left[\frac{\Gamma 1+3\gamma}{(m+k\beta)^{1+3\gamma}}\right]\end{array}\right)$$

$$\mu_{4}^{'}=Ex^{4}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^{2}\left(k\beta+m\right)\left(\theta^{4}\right)\left[\frac{\Gamma 1+4\gamma}{(m+k\beta)^{1+4\gamma}}\right]\end{array}\right).$$

### 3.3 Moments about the mean

Let $x$ denote the random variable follows odd Chen Fréchet distribution then moments about the mean order moment about origin of $\mu_r$ is:

$$\mu_{r}=E\left(x-\mu\right)^{r}=\int_{0}^{\infty}\left(x-\mu\right)^{r}\ f\left(x,\alpha,\beta,\gamma,\theta\right)\mathrm{dx}\tag{3.4}$$

$$=\int_{0}^{\infty}\left(x-\mu\right)^{r}\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\frac{\gamma(k\beta+m)}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1}e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\ \left(e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right)^{k\beta+m-1}\end{array}\right)\mathrm{dx}\tag{3.5}$$

$$c=\left(\begin{array}{c}\sum_{i=1}^{\infty}\ \sum_{j=0}^{i}\ \sum_{k,m=0}^{\infty}\ \frac{(-1)^{i+j}\alpha^{i}(i-j)^{k}\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\frac{\gamma(k\beta+m)}{\theta}\end{array}\right).$$

Letting $u=\left(\frac{\theta}{x}\right)^{\gamma},\frac{\theta}{x}=u^{-\gamma},\quad x=\theta u^{\gamma},dx=\theta\gamma\ u^{\gamma-1}\mathrm{du}$ imply

$$\begin{aligned}E\left(x-\mu\right)^{r}=&C\int_{0}^{\infty}\left(x-\mu\right)^{r}\ \left(\left(\frac{\theta}{x}\right)^{\gamma+1}e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\ \left(e^{-\left(\frac{\theta}{x}\right)^{\gamma}}\right)^{k\beta+m-1}\right)\mathrm{dx}\\=&C\int_{0}^{\infty}\left(\theta u^{\gamma}-\mu\right)^{r}\ \left(\left(u\right)\left(u^{-\gamma}\right)e^{-u}\left[e^{-u}\right]^{k\beta+m-1}\right)\theta\gamma\ u^{\gamma-1}\ du\\=&\gamma C\left(\theta^{r+1}\right)\int_{0}^{\infty}\left(u^{\gamma}-\frac{\mu}{\theta}\right)^{r}e^{-u}\left(e^{-u}\right)^{k\beta+m-1}\ \ du\end{aligned}$$

$$=\gamma C\left(\theta^{r+1}\right)\int_0^\infty \sum_{j=0}^{r}\binom{r}{j}(u^\gamma)^j\left(-\frac{\mu}{\theta}\right)^{r-j}\left(e^{-u}\left(e^{-u}\right)^{k\beta+m-1}\right)du$$

$$=\gamma C\left(\theta^{r+1}\right)\sum_{j=0}^{r}\binom{r}{j}\left(-\frac{\mu}{\theta}\right)^{r-j}\int_0^\infty (u^\gamma)^j\left(e^{-u}\left(e^{-u}\right)^{k\beta+m-1}\right)du$$

$$=\gamma C\left(\theta^{r+1}\right)\sum_{j=0}^{r}\binom{r}{j}\left(-\frac{\mu}{\theta}\right)^{r-j}\frac{\Gamma 1+rj}{(k\beta+m)^{1+rj}}$$

$$=\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\binom{i}{j}\frac{\gamma(k\beta+m)\gamma\left(\theta^{r+1}\right)}{\theta}\sum_{j=0}^{r}\binom{r}{j}\left(-\frac{\mu}{\theta}\right)^{r-j}\frac{\Gamma 1+rj}{(k\beta+m)^{1+rj}} \tag{3.6}$$

$$E(x-\mu)^2=\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left(\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right)$$

$$\sigma^2=\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left[\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right] \tag{3.7}$$

$$\text{StandardDeviation}=\sigma=\sqrt{\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left[\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right]\end{array}\right)} \tag{3.8}$$

$$\mu_3=\mathbf{E}(\mathbf{x}-\mu)^{\mathbf{3}}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{3}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^3)\left[\binom{3}{j}(\theta^3)\left(-\frac{\mu}{\theta}\right)^{3-j}\frac{\Gamma 1+3j}{(k\beta+m)^{1+3j}}\right]\end{array}\right)$$

$$\mathbf{E}(\mathbf{x}-\mu)^{\mathbf{4}}=\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{4}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^4)\left[\binom{4}{j}(\theta^4)\left(-\frac{\mu}{\theta}\right)^{4-j}\frac{\Gamma 1+4j}{(k\beta+m)^{1+4j}}\right]\end{array}\right).$$

### 3.4 Coefficient of Variation

The coefficient of variation for odd Chen Fréchet distribution is given by:

$$C\cdot V=\frac{\sigma}{\mu'_1}\times 100\%=\frac{\sqrt{\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{2}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left[\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right]\end{array}\right)}}{\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^1)\left(\frac{\Gamma 1+\ \gamma}{(m+k\beta)^{1+\ \gamma}}\right)\end{array}\right)}. \tag{3.9}$$

### 3.5 Coefficient of Skewness

Coefficient of skewness for odd Chen Fréchet distribution is given by:

$$S.K=\frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}=\frac{\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{3}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^3)\left[\binom{3}{j}(\theta^3)\left(-\frac{\mu}{\theta}\right)^{3-j}\frac{\Gamma 1+3j}{(k\beta+m)^{1+3j}}\right]\end{array}\right)}{\left(\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left[\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right]\end{array}\right)\right)^{\frac{3}{2}}}. \tag{3.10}$$

### 3.6 Coefficient of Kurtosis

The coefficient of Kurtosis of for truncated exponential Fréchet Distribution is given by:

$$C.K=\frac{E(t-\mu)^4}{\sigma^4}=\frac{\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{4}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^4)\left[\binom{4}{j}(\theta^4)\left(-\frac{\mu}{\theta}\right)^{4-j}\frac{\Gamma 1+4j}{(k\beta+m)^{1+4j}}\right]\end{array}\right)}{\left(\left(\begin{array}{c}\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k,m=0}^{\infty}\sum_{j=0}^{2}\frac{(-1)^{i+j}\alpha^i(i-j)^k\Gamma k\beta+m}{i!k!m!\Gamma k\beta}\\ \binom{i}{j}\gamma^2(k\beta+m)(\theta^r)\left[\binom{2}{j}(\theta^2)\left(-\frac{\mu}{\theta}\right)^{2-j}\frac{\Gamma 1+2j}{(k\beta+m)^{1+2}}\right]\end{array}\right)\right)^2}. \tag{3.11}$$

# 4 Moment Generating Function

Let $X$ be random variable follows odd Chen Fréchet distribution, then the moment generating function (mfg) of x is obtained as:

$$\begin{aligned}
M_X(t) =& E(e^{-xt}) = \int_0^\infty e^{-xt}\ f(x,\alpha,\beta,\gamma,\theta)dx \\
=& \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots + \frac{(tx)^r}{r!}\right) f(x,\alpha,\beta,\gamma,\theta)\ dx \\
=& \int_0^\infty \frac{t^r}{r!} x^r f(x,\alpha,\beta,\gamma,\theta)\ dx \\
=& \sum_{r=0}^\infty \frac{t^r}{r!} \mu'_r \\
=& \sum_{r=0}^\infty \frac{t^r}{r!} \sum_{i=1}^\infty \sum_{j=0}^i \sum_{k,m=0}^\infty \frac{(-1)^{i+j}\alpha^i (i-j)^k \Gamma k\beta + m}{i!k!m!\Gamma k\beta} \binom{i}{j} \gamma^2 (k\beta + m)(\theta^r) \left[\frac{\Gamma 1 + r\gamma}{(m+k\beta)^{1+r\gamma}}\right]. \qquad (4.1)
\end{aligned}$$

Similarly, the characteristic function of odd Chen Fréchet distribution, can be obtained as:

$$M_X(ti) = \sum_{r=0}^\infty \frac{\mathrm{ti}^r}{r!} \sum_{i=1}^\infty \sum_{j=0}^i \sum_{k,m=0}^\infty \frac{(-1)^{i+j}\alpha^i (i-j)^k \Gamma k\beta + m}{i!k!m!\Gamma k\beta} \binom{i}{j} \gamma^2 (k\beta + m)(\theta^r) \left[\frac{\Gamma 1 + r\gamma}{(m+k\beta)^{1+r\gamma}}\right] \qquad (4.2)$$

# 5 Parameter estimation

Let $x_1, x_2, x_3, x_4, \ldots, x_n$ be a random sample of size n from odd Chen Fréchet distribution. The likelihood function, $L$ of odd Chen Fréchet distribution is given by:

$$L = \prod_{i=1}^n \left( \frac{\left(\alpha\beta\frac{\gamma}{\theta}\left(\frac{\theta}{x}\right)^{\gamma+1} e^{-\left(\frac{\theta}{x}\right)^\gamma}\right)\left[e^{-\left(\frac{\theta}{x}\right)^\gamma}\right]^{\beta-1}}{\left[1 - e^{-\left(\frac{\theta}{x}\right)^\gamma}\right]^{\beta+1}} \left(e^{\left[1-e^{-\left(\frac{\theta}{x}\right)^\gamma}\right]^{-\left(\frac{\theta}{x}\right)^\gamma}}\right]^\beta\right) e^{-\alpha\left[\mathbf{e}^{\left[\frac{\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}{\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}\right]^\beta} \mathbf{-1}\right]} \right)$$

$$\log L = \sum_{\mathbf{i=1}}^{\mathbf{n}} \log\left(\alpha^{\mathbf{n}}\beta^{\mathbf{n}}\frac{\gamma^n}{\theta^n}\left(\frac{\left[\left(\frac{\theta}{xi}\right)^{\gamma+1} e^{-\left(\frac{\theta}{xi}\right)^\gamma}\right]\left[e^{-\left(\frac{\theta}{x}\right)^\gamma}\right]^{\beta-1}}{\left[\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}\right]^{\beta+\mathbf{1}}}\left[\mathbf{e}^{\left[\frac{\mathbf{e}^{-\left(\frac{\theta}{\mathbf{xi}}\right)^\gamma}}{\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}\right]^\beta}\right]\mathbf{e}^{-\alpha\left[\mathbf{e}^{\left[\frac{\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}{\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}\right]^\beta}\mathbf{-1}\right]}\right)\right) = 0$$

$$= \left( \begin{array}{c} \log\alpha^n + \log\beta^n + \log\gamma^n - \log\theta^n + (\gamma+1)\sum_{\mathbf{i=1}}^{\mathbf{n}} \log\left(\frac{\theta}{xi}\right) - \sum_{\mathbf{i=1}}^{\mathbf{n}} \left(\frac{\theta}{xi}\right)^\gamma + \sum_{\mathbf{i=1}}^{\mathbf{n}} \log\left[e^{-\left(\frac{\theta}{x}\right)^\gamma}\right]^{\beta-1} \\ -\sum_{\mathbf{i=1}}^{\mathbf{n}} \log\left[\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}\right]^{\beta+\mathbf{1}} + \sum_{\mathbf{i=1}}^{\mathbf{n}}\left[\frac{\mathbf{e}^{-\left(\frac{\theta}{\mathbf{xi}}\right)^\gamma}}{\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}\right]^\beta \sum_{\mathbf{i=1}}^{\mathbf{n}} \alpha\left[\mathbf{e}^{\left[\frac{\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}{\mathbf{1}-\mathbf{e}^{-\left(\frac{\theta}{\mathbf{x}}\right)^\gamma}}\right]^\beta} - \mathbf{1}\right] \end{array} \right) \qquad (5.1)$$

$$\frac{\partial \log L}{\mathrm{d}\gamma} = \frac{n(-1+e^{\left(\frac{\theta}{x}\right)^\gamma} + (-1+e^{\left(\frac{\theta}{x}\right)^\gamma}(1+(\frac{1}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}})^\beta(-1+e^{(\frac{1}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}})^\beta}\alpha)\beta))(\frac{\theta}{x})^\gamma)Log[\frac{\theta}{x}]}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}} = 0 \qquad (5.2)$$

$$\frac{\partial \log L}{\mathrm{d}\beta} = \sum_{i=1}^n n\left(\frac{1}{\beta} - (\frac{1}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}})^\beta(-1+e^{(\frac{1}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}})^\beta}\alpha)Log[\frac{1}{-1+e^{\left(\frac{\theta}{x}\right)^\gamma}}]\right) = 0 \qquad (5.3)$$

$$\frac{\mathrm{dLnL}}{\mathrm{d}\theta}=\sum_{i=1}^{n}\frac{n\gamma(-1+e^{(\frac{\theta}{x})^{\gamma}}+(-1+e^{(\frac{\theta}{x})^{\gamma}}(1+(\frac{1}{-1+e^{(\frac{\theta}{x})^{\gamma}}})^{\beta}(-1+e^{(\frac{1}{-1+e^{(\frac{\theta}{x})^{\gamma}}})^{\beta}}\alpha)\beta))(\frac{\theta}{x})^{\gamma})}{(-1+e^{(\frac{\theta}{x})^{\gamma}})\theta}=0 \tag{5.4}$$

$$\frac{\mathrm{dLnL}}{\mathrm{d}\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}-e^{(\frac{e^{-(\frac{\theta}{x})^{\gamma}}}{1-e^{-(\frac{\theta}{x})^{\gamma}}})^{\beta}}. \tag{5.5}$$

The maximum likelihood estimates $(\hat{k},\hat{\gamma},\hat{\theta},\hat{\alpha})$ equations $\frac{dLogL}{d\alpha}=0, \frac{dLogL}{d\theta}=0, \frac{dLogL}{d\beta}=0, \frac{dLogL}{d\gamma}=0$,The Equation (5.2),(5.3) and Equation (5.4) and (5.5) cannot be solved as they both are in closed forms. So we compute the parameters of the Transmuted Survival Exponential Pareto Distribution.

## 6 Application of odd Chen Fréchet distribution

The flexibility and performance of odd Chen Fréchet distribution are evaluated on competing models viz Fréchet Distribution (FD), Weibull Pareto Distribution (WPD), Weibull distribution(WD), Exponential distribution (ED) .Here, the distribution is fitted to data set for the number of hours the patients people with Mayo Clinic Press were in hospital before death for AL Hussein Educational Hospital in Karbala, for sample size (n=110) (see table 1), the performance of the distribution was compared with Fréchet Distribution distribution (FD), Weibull Pareto Distribution (WPD), Weibull distribution(WD), Exponential distribution (ED) .for the data set using Akaike Information Criterion (AIC), Akaike Bayesian Criterion Corrected (BIC). Information Criterion (AIC),Distribution with the lowest AIC, AICC considered the most flexible and superior distribution for a given data. The results are presented in the tables (2).

Table 1: Data set for the number of hours patients were in hospital before death

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.14 | 0.39 | 1.25 | 1.45 | 1.65 | 2.1 | 2.69 | 3 | 3.5 | 4.1 | 4.8 |
| 0.14 | 0.39 | 1.25 | 1.45 | 1.7 | 2.15 | 2.7 | 3.1 | 3.6 | 4.3 | 4.8 |
| 0.21 | 0.4 | 1.27 | 1.5 | 1.73 | 2.2 | 2.75 | 3.15 | 3.6 | 4.4 | 4.9 |
| 0.28 | 0.4 | 1.3 | 1.5 | 1.75 | 2.4 | 2.8 | 3.15 | 3.6 | 4.45 | 4.95 |
| 0.28 | 0.42 | 1.3 | 1.55 | 1.77 | 2.4 | 2.8 | 3.17 | 3.75 | 4.45 | 5 |
| 0.28 | 0.42 | 1.35 | 1.55 | 1.8 | 2.45 | 2.9 | 3.2 | 3.8 | 4.5 | 5.1 |
| 0.3 | 0.45 | 1.38 | 1.58 | 1.85 | 2.5 | 2.95 | 3.2 | 3.9 | 4.55 | 5.15 |
| 0.32 | 1 | 1.39 | 1.6 | 1.9 | 2.6 | 2.95 | 3.25 | 4 | 4.6 | 5.2 |
| 0.35 | 1 | 1.4 | 1.6 | 2 | 2.65 | 2.97 | 3.3 | 4 | 4.65 | 5.25 |
| 0.35 | 1.2 | 1.4 | 1.65 | 2 | 2.67 | 3 | 3.4 | 4 | 4.75 | 5.5 |

To choose the best model within the set of models that was compared with the new distribution, the best is the model corresponding to the lowest value for Akaike Information Criterion (AIC) and Akaike Information Correct ($\mathrm{AIC_c}$) (see tabul 2.) , the general formula for (AIC) ,($\mathrm{AIC_c}$) and (BIC) are:

$$AIC = -2\log\left(\frac{\hat{\theta}_{MLE}}{x}\right)+2K \tag{6.1}$$

where

$\log\left(\frac{\hat{\theta}_{MLE}}{x}\right)$: value of the logarithm maximum likelihood function.

$K$: Estimated number of parameters and

$$\mathrm{AIC_c} = AIC+\frac{2K\,(K+1)}{N-K-1} \tag{6.2}$$

where

AIC: Akaike Information Criterion.

$K$: Estimated number of parameters.

$N$: sample size.

$$\text{BIC} = -2\log\left(\hat{\theta}_{MLE}\right) + K\log(N) \tag{6.3}$$

where

BIC: Bayesian Information Criterion.

$K$: Estimated number of parameters.

$N$: sample size.

Table 2: ML Estimates and AIC , $\text{AIC}_\text{C}$,BIC, and comparison Truncated Exponential Fréchet Distribution ion with Fréchet Distribution (FD), Weibull Pareto Distribution (WPD), Weibull distribution(WD), Exponential distribution (ED)

| Distributions | MLE | -2log | AIC | $\text{AIC}_\text{C}$ | BIC |
|---|---|---|---|---|---|
| odd Chen Fréchet distribution | $\hat{\alpha} = 0.01$ $\hat{\theta} = 0.02$ $\hat{\gamma} = 0.34$ $\hat{\beta} = 0.95$ | 192.702 | 393.404 | 393.7787 | 404.2068 |
| Fréchet distribution | $\hat{\gamma} = 3.45$ $\hat{\beta} = 0.563$ | 357.562 | 719.124 | 719.2342 | 724.5251 |
| Weibull distribution | $\hat{\alpha} = 0.4234$ $\hat{\theta} = 0.2841$ | 421.154 | 825.154 | 825.609 | 805.340 |
| Exponential | $\hat{\theta} = 0.14956$ | 834.413 | 836.413 | 836.509 | 852.094 |

## 7 Conclusion

In this paper, a novel probability distribution is introduced. The new distribution is a odd Chen Fréchet distribution,. Some of the properties are derived and discussed like moments, reliability analysis, and hazard rate. The method of maximum likelihood estimation is used for determining the parameters. The performance of the new model is determined by fitting to real-life data using the goodness of fit criteria such as AIC, AICC and BIC. this leads not to reflect the hypothesis of the notice (the appropriateness of the real data for probability distributions under study It is found that odd Chen Fréchet distribution gives a better fit to the data set as compared Fréchet Distribution (FD), Weibull distribution(WD), Exponential distribution (ED) Further, odd Chen Fréchet distribution can be applied to various areas. Truncated Exponential Fréchet Distribution and distribution may suitable for most of the lifetime data and provides better outcomes than other well-known distribution.

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