Int. J. Nonlinear Anal. Appl. 14 (2023) 8, 129–135
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.24597.3033

Some best proximity point results for generalized cyclic contraction mappings

Thounaojam Stephen[a], Yumnam Rohen[a], Hüseyin Işık[b,*], Laishram Shanjit[a]

[a]Department of Mathematics, National Institute of Technology Manipur, Langol 795004, India

[b]Department of Engineering Science, Bandırma Onyedi Eylül University, Bandırma 10200, Balıkesir, Turkey

(Communicated by Ali Farajzadeh)

Abstract

In this study, we establish some best proximity point results for generalized cyclic contraction mappings in partially ordered metric spaces. We also prove some best proximity point theorems by introducing the T-restriction property and generalized pointwise cyclic contraction mapping. Some illustrations are provided to support our results.

Keywords: Best proximity point, ordered metric space, semi-sharp proximal pair, generalized pointwise cyclic contraction

2020 MSC: 47H09, 47H10

1 Introduction and Preliminaries

Suppose (X, d) is a metric space, and $T : X \to X$ is a mapping. The mapping T is known as a contractive if $d(Ts, Tt) < d(s, t)$, $\forall s, t \in X$ with $s \neq t$, and a contraction if $d(Ts, Tt) \leq kd(s, t)$, $\forall s, t \in X, k \in]0, 1[$. The mapping T is said to be nonexpansive if $k = 1$. Every contraction mapping is a contractive mapping, but the converse may not hold. For example, take $X =]-\infty, 0[\subseteq \mathbb{R}$ and $Ts = e^s - 2$, for all $s \in X$. Using the mean value theorem, we can easily verify that T is a contractive mapping but not a contraction mapping.

If $Tu = u$ for some $u \in X$, then the point u is said to be a fixed point of T. If $Tu \neq u$, then it will be interesting to search a point $s \in X$ so that s is in proximity to Tu. Fan [7] states that if E is a nonempty compact convex subset in a normed linear space X, and $T : E \to X$ is any continuous mapping, then there is $u_0 \in E$ so that

$$\|u_0 - Tu_0\| = \min_{u \in E} \|u - Tu_0\|.$$

In particular, if $T(E) \subseteq E$, then u_0 is a fixed point of T. Let G and H be nonempty bounded subsets in X. Throughout this article, we denote (X, d), a metric space with metric d and we adopt some notations as:

$$\begin{aligned} R(s, H) &:= \sup\{\|s - t\| : t \in H\}, s \in G; \\ G_0 &:= \{s \in G : \ \|s - t\| = dist(G, H), t \in H\}; \\ H_0 &:= \{t \in H : \ \|s - t\| = dist(G, H), s \in G\}, \end{aligned}$$

*Corresponding author

Email addresses: stepthounaojam@gmail.com (Thounaojam Stephen), ymnehor2008@yhaoo.com (Yumnam Rohen), isikhuseyin76@gmail.com (Hüseyin Işık), shanjit2015@yhaoo.co.in (Laishram Shanjit)

Received: December 2021 *Accepted:* February 2023

where $dist(G, H) := \inf\{\|s - t\| : \ s \in G \text{ and } t \in H\}$.

Definition 1.1. [6] The pair (C, D) of nonempty subsets in a Banach space X is said to be a semi-sharp proximal pair iff for each $s \in C$ and $t \in D$, there is at most one element $s' \in D$ and at most one element $t' \in C$ such that

$$d(s, s') = d(t, t') = dist(C, D).$$

Every nonempty closed convex pair (C, D) in a strictly convex Banach space is a semi-sharp proximal pair.

Definition 1.2. [5, 2] Let (K_1, K_2) be a pair of nonempty subsets in (X, d). A mapping $T : K_1 \cup K_2 \to K_1 \cup K_2$ is known as a cyclic contraction if

(i) $T(K_1) \subseteq K_2$ and $T(K_2) \subseteq K_1$;
(ii) for each $(s, t) \in K_1 \times K_2$ and for $0 < \alpha < 1$,

$$d(Ts, Tt) \le \alpha d(s, t) + (1 - \alpha)dist(K_1, K_2).$$

Definition 1.3. [11] A pair (C, D) of nonempty subsets in (X, d) is said to have the projectional property if for every $(s, t) \in C \times D$ with $d(s, t) = dist(C, D)$ and for any sequence $\{s_n\}$ in C and sequence $\{t_n\}$ in D satisfying

$$\lim_{n\to+\infty} d(s_n, t) \to dist(C, D) = \lim_{n\to+\infty} d(s, t_n),$$

implies $\lim_{n\to+\infty} s_n = s$ and $\lim_{n\to+\infty} t_n = t$.

2 Generalized Cyclic Contractions on Ordered Metric Spaces

More results about the existence of best proximity points for contraction and contractive, one can see Rossafi et al. [13], Karapinar [10] and Gabeleh et al. [8]. Motivated by Definition 1.2 of Eldred et al. [5] and Thafai et al. [2], Karapinar [9] introduced the following definition.

Definition 2.1. [9] Let (K_1, K_2) be a pair of nonempty subsets in (X, d). A mapping T from $K_1 \cup K_2$ into itself is known as a generalized cyclic contraction if

(i) $T(K_1) \subseteq K_2$ and $T(K_2) \subseteq K_1$;
(ii) for each $(s, t) \in K_1 \times K_2$ and for $0 < \alpha < 1$,

$$d(Ts, Tt) \le \frac{\alpha}{3}[d(s, t) + d(Ts, s) + d(Tt, t)] + (1 - \alpha)dist(K_1, K_2).$$

Motivated by Abkar and Gabeleh [1], Nieto and Rodríguez-López [12], we present some new interesting results in this section.

Lemma 2.2. Let (C, D) be a pair of nonempty subsets in (X, d), and C be complete in X. Let $(C, \preceq)$ be a partially ordered set and T be a cyclic mapping from $C \cup D$ into itself, $\alpha \in]0, 1[$ so that

$$d(Ts', T^2s) \le \frac{\alpha}{3}[d(s', Ts) + d(Ts', s') + d(T^2s, Ts)] + (1 - \alpha)dist(C, D),$$

and T^2 be nondecreasing on C with $s \preceq s'$ for all $(s, s') \in C \times C$. If $s_0 \in C$ with $s_0 \preceq T^2s_0$, then $\lim_{n\to+\infty} d(s_n, s_{n+1}) = dist(C, D)$, where $s_{n+1} := Ts_n$.

Proof . Since $s_0 \preceq T^2s_0$ and T^2 is nondecreasing on C, we have

$$s_0 \preceq T^2s_0 \quad \preceq \quad \cdots \preceq T^{2n}s_0 \preceq \ldots.$$

Now,

$$d(s_{n+2}, s_{n+1}) \le \frac{\alpha}{3}[d(s_{n+1}, s_n) + d(s_{n+2}, s_{n+1}) + d(s_{n+1}, s_n)] + (1 - \alpha)dist(C, D),$$

which is equivalent to

$$d(s_{n+2}, s_{n+1}) \quad \leq \quad \frac{2\alpha}{3-\alpha} d(s_{n+1}, s_n) + \frac{3(1-\alpha)}{3-\alpha} dist(C, D),$$

for $n = 0, 1, 2, \ldots$. Let $R_n := d(s_{n+1}, s_n)$. Then,

$$R_{2n} \quad \leq \quad \left(\frac{2\alpha}{3-\alpha}\right)^{2n} d(Ts_0, s_0) + \left[\left(\frac{2\alpha}{3-\alpha}\right)^{2n-1} + \cdots + \left(\frac{2\alpha}{3-\alpha}\right)^{2} + \left(\frac{2\alpha}{3-\alpha}\right)\right] \frac{3(1-\alpha)}{3-\alpha} dist(C, D),$$

such that $R_{2n} \to dist(C, D)$ as $n \to +\infty$. Similarly,

$$R_{2n+1} \quad \leq \quad \left(\frac{2\alpha}{3-\alpha}\right)^{2n+1} d(Ts_0, s_0) + \left[\left(\frac{2\alpha}{3-\alpha}\right)^{2n} + \left(\frac{2\alpha}{3-\alpha}\right)^{2n-1} + \cdots + \left(\frac{2\alpha}{3-\alpha}\right)\right] \frac{3(1-\alpha)}{3-\alpha} dist(C, D),$$

such that $\lim_{n\to+\infty} R_{2n+1} = dist(C, D)$. Thus, we conclude that $\lim_{n\to+\infty} R_n = dist(C, D)$. □

We now state the following two theorems.

Theorem 2.3. Let M and N be nonempty closed subsets in (X, d). Let $(M, \preceq)$ be a partially ordered set. Consider M has the following property:

$$\text{if } \{s_n\} \text{ is non-decreasing and } \lim_{n\to+\infty} s_n = s \in M, \text{ then } s_n \preceq s, \quad \forall n. \tag{2.1}$$

Let $T : M \cup N \to M \cup N$ be a cyclic mapping, T be continuous, T^2 be nondecreasing on M and

$$d(Ts', T^2 s) \leq \frac{\alpha}{3}[d(s', Ts) + d(Ts', s') + d(T^2 s, Ts)] + (1-\alpha) dist(M, N),$$

for some $\alpha \in (0, 1)$ and $\forall (s, s') \in M \times M$ with $s \preceq s'$. Let $s_0 \in M$ with $s_0 \preceq T^2 s_0$, and define $s_{n+1} := Ts_n$. If $\{s_{2n}\}$ has a convergent subsequence in M, then there is $u \in M$ so that $d(u, Tu) = dist(M, N)$.

Proof . By Lemma 2.2, $u \in L$ such that $d(u, S_{2n_k-1}) \to dist(M, N)$. Since T is continuous and T^2 is non-decreasing, it is easy to see that $\{S_{2n}\}$ has a convergent subsequence $\{S_{2n_k}\}$ such that $S_{2n_k} \to u$ and $d(u, Tu) = dist(M, N)$. □

Theorem 2.4. Let (M, N) be nonempty convex closed pair of subsets in a Banach space B and $(M, \preceq)$ be a partially ordered set. Let $T : M \cup N \to M \cup N$ be a cyclic mapping, T^2 be nondecreasing on M and

$$\|Ts' - T^2 s\| \quad \leq \quad \frac{\alpha}{3}[\|s' - Ts\| + \|Ts' - s'\| + \|T^2 s - Ts\|] + (1-\alpha) dist(M, N),$$

for some $\alpha \in (0, 1)$ and $(s, s') \in M \times M$ with $s \preceq s'$. Let $s_0 \in M$ with $s_0 \preceq T^2 s_0$, and define $s_{n+1} := Ts_n$. If M is bounded and T is weakly continuous on M, then there is $u \in M$ so that $d(u, Tu) = dist(M, N)$.

Proof . By Lemma 2.2, $||S_{2n} - TS_{2n}|| \to dist(A, B)$. The rest of the proof is similar to Theorem 4.2 of [1] □

Definition 2.5. [14] A pair (C, D) of nonempty subsets in (X, d) is said to have the property UC iff the following holds:

If $\{s_n\}$ and $\{w_n\}$ are sequences in C, and $\{t_n\}$ is a sequence in D such that

$$\lim_{n\to+\infty} d(s_n, t_n) = dist(C, D) = \lim_{n\to+\infty} d(w_n, t_n),$$

implies $\lim_{n\to+\infty} d(s_n, w_n) = 0$.

Lemma 2.6. [14] Let (C, D) be a pair of nonempty subsets in (X, d), which satisfies the property UC. Let $\{s_n\} \subset C$ and $\{t_n\} \subset D$ such that

$$\lim_{m\to+\infty} \sup_{m\leq n} d(s_m, t_n) = dist(C, D) \text{ or } \lim_{n\to+\infty} \sup_{m\geq n} d(s_m, t_n) = dist(C, D).$$

Then $\{s_n\}$ is a Cauchy sequence.

We now prove the following theorem.

Theorem 2.7. Let $(X, d, \preceq)$ be a partially ordered metric space and (P, Q) a pair of nonempty subsets with the property UC. Let P be complete which satisfes the condition (2.1) and T a cyclic mapping from $P \cup Q$ into $P \cup Q$ so that T and T^2 are nondecreasing on P. Further,

$$\begin{aligned} d(Tu', T^2u) &\leq \frac{\alpha}{3}[d(u', Tu) + d(Tu', u') + d(T^2u, Tu)] + (1-\alpha)dist(P, Q), \\ d(Tv', T^2v) &\leq \frac{\alpha}{3}[d(v', Tv) + d(Tv', v') + d(T^2v, Tv)] + (1-\alpha)dist(P, Q), \end{aligned}$$

for some $\alpha \in]0, 1[$ and for all $(u, u') \in P \times P$, $(v, v') \in Q \times Q$ with $u \preceq u'$, $v \preceq v'$. If $u_0 \in P$, $u_0 \preceq T^2u_0$, and $u_{n+1} := Tu_n$, then there is $s \in P$ such that $d(s, Ts) = dist(P, Q)$ and $\lim_{n\to+\infty} u_{2n} = s$.

Proof . Since $u_0 \preceq T^2u_0$, and T and T^2 are nondecreasing on P. Clearly, $u_0 \preceq T^2u_0 \preceq \cdots \preceq T^{2n}u_0 \preceq \ldots$ and $Tu_0 \preceq T^3u_0 \preceq \cdots \preceq T^{2n+1}u_0 \preceq \ldots$, for $n = 0, 1, 2, \ldots$. By Lemma 2.2, we have $\lim_{n\to+\infty} d(u_{2n+2}, u_{2n+1}) = dist(P, Q)$ and $\lim_{n\to+\infty} d(u_{2n}, u_{2n+1}) = dist(P, Q)$. Since (P, Q) has the property UC, $\lim_{n\to+\infty} d(u_{2n}, u_{2n+2}) = 0$. We need to claim that the sequence $\{u_n\}$ is Cauchy. Let $\varepsilon > 0$ and choose $m_0 \in \mathbb{N}$ so that

$$d^*(T^{2m}u_0, T^{2m+1}u_0) < \varepsilon, d^*(T^{2m+2}u_0, T^{2m+1}u_0) < \varepsilon,$$

and $d^*(T^{2m}u_0, T^{2m+2}u_0) < \varepsilon$ for each $m \geq m_0$, where $d^*(s, t) = d(s, t) - dist(P, Q)$ for $(s, t) \in P \times P$. Since

$$d^*(T^{2m}u_0, T^{2m+1}u_0) < \varepsilon \text{ and } d^*(T^{2m+2}u_0, T^{2m+1}u_0) < \varepsilon,$$

for all $m \geq m_0$, we have $d^*(T^{2m}u_0, T^{2n+1}u_0) < \varepsilon$. Thus, $\lim_{m\to+\infty} \sup_{n\geq m} d^*(T^{2m}u_0, T^{2n+1}u_0) = 0$. Using Lemma 2.6, $\{u_{2n}\}$ is Cauchy, and since P is complete, we have $\lim_{n\to+\infty} u_{2n} = u \in P$. Since $u_0 \preceq T^2u_0 \preceq \cdots \preceq T^{2n}u_0 \preceq \ldots$, we conclude that $u_{2n} \preceq u$. Therefore,

$$\begin{aligned} d(u, Tu) &= \lim_{n\to+\infty} d(T^{2n}u_0, Tu) \\ &= \lim_{n\to+\infty} d(Tu, T^2(T^{2n-2}u_0)) \\ &\leq \lim_{n\to+\infty} \Big[\frac{\alpha}{3}\{d(u_0, T^{2n-2}u_0) + d(T^2(T^{2n-2}u_0), T(T^{2n-2}u_0)) + d(Tu, u)\} + (1-\alpha)dist(P, Q)\Big] \\ &= dist(P, Q). \end{aligned}$$

Again, $dist(P, Q) \leq d(u, Tu)$. Hence, $d(u, Tu) = dist(P, Q)$ and $\lim_{n\to+\infty} T^{2n}u_0 = u$. □

Example 2.8. Let $X = (\mathbb{R}, d)$, be a metric space with $d(u, v) = |u - v|$ for all $u, v \in \mathbb{R}$. Let $C = (-\infty, 0]$ and $D = [0, +\infty)$ be subsets in X. Define a partial order "$\preceq$" on X as

$$u_1 \preceq u_2 \Leftrightarrow u_1 \leq u_2, \text{ for } u_1, u_2 \in X.$$

Define $T : C \cup D \to C \cup D$ by

$$Tu = \begin{cases} -\frac{u}{2}, & \text{if } u \in C, \\ -u, & \text{if } u \in D. \end{cases}$$

Clearly, for $\alpha = \frac{1}{9}$, T satisfies all the assumptions of Theorem 2.7 and $d(0, T0) = dist(C, D)$. Now, if $u_0 \in C$, and $u_{n+1} = Tu_n$, then $u_0 \preceq T^2u_0$, and $\{u_{2n}\}$ is nondecreasing and $\lim_{n\to+\infty} u_{2n} = 0$.

3 Generalized Pointwise Cyclic Contractions

We introduce the following definition.

Definition 3.1. Let (G, H) be a pair of nonempty bounded subsets in (X, d) and T a cyclic mapping from $G \cup H$ into itself. We say that the pair (G, H) has T-restriction property if

$$\max\{d(s, Ts), d(t, Tt)\} \leq d(s, t), \quad \text{for each } (s, t) \in G \times H.$$

Eldred [4] introduced proximal normal structure and proved the best proximity point theorems (see Theorems 2.1 and 2.2 of [4]). Anuradha and Veeramani [3] introduced proximal pointwise contraction and proved some interesting best proximity point results in a reflexive Banach space. Kosuru and Veeramani [11] introduced pointwise cyclic contraction, a generalization of proximal pointwise contraction, and proved some best proximity point theorems (see Theorems 4.1, 4.5 and 4.6 of [11]). Motivated by the definition of generalized cyclic contraction for [9] and definition of pointwise cyclic contraction for [11], we introduce the following definition.

Definition 3.2. Let (G, H) be a pair of nonempty subsets in (X, d). A mapping T from $G \cup H$ into itself is known as a generalized pointwise cyclic contraction if

(i) $T(G) \subseteq H$ and $T(H) \subseteq G$;

(ii) for any $(s, t) \in G \times H$, there is $\alpha(s), \alpha(t) \in]0, 1[$ so that

$$d(Ts, Tw) \leq \frac{\alpha(s)}{3}[d(s, w) + d(s, Ts) + d(Tw, w)] + (1 - \alpha(s))dist(G, H), \ \forall w \in H,$$
$$d(Tt, Tw) \leq \frac{\alpha(t)}{3}[d(t, w) + d(t, Tt) + d(Tw, w)] + (1 - \alpha(t))dist(G, H), \ \forall \ w \in G.$$

In the above definitions, if the pair (G, H) has T-restriction property, then $d(Ts, Tt) \leq d(s, t)$ for $s \in G, \ t \in H$ (relatively nonexpansive). Furthermore, if $dist(G, H) < d(s, t)$, then $d(Ts, Tt) < d(s, t)$. The following example verifies that even though T is a generalized pointwise cyclic contraction, if (G, H) does not have T-restriction property, then T is not a relatively nonexpansive mapping.

Example 3.3. Let $X = (R^2, \|.\|_1)$, and $G = \{(s, t) : s \in [0, 1], t \in [0, 1]\}$, $H = \{(s, 0) : 2 \leq s \leq 3\}$. Define $T : H \to G$ by

$$T(s, t) = (1, 0), \ \forall (s, t) \in H,$$

and $T : G \to H$ by

$$T(s,t) = \begin{cases} (2, 0), & \text{if} \quad 0 \leq s \leq 1, t = 0, \\ \left(2 + \frac{m(m-1)}{(m+1)^3}, 0\right), & \text{if} \quad 0 \leq s \leq 1, \frac{1}{m+1} < t \leq \frac{1}{m}. \end{cases}$$

Clearly, mapping T from $G \cup H$ into itself is cyclic, and by taking particular points $(1, \frac{1}{2}) \in G, (2, 0) \in H$ and $m = 2$, it is easy to see that the pair (G, H) does not have T-restriction property. Fix $(s, t) \in C$ with $t \neq 0$. Then, there is $m_0 \in \mathbb{N}$ so that $\frac{1}{m_0+1} < u \leq \frac{1}{m_0}$. Now, for any $(0, w) \in H$,

$$\begin{aligned} \|T(s,t) - T(0,w)\| &= \left\| \left(2 + \frac{m_0(m_0-1)}{(m_0+1)^3}, 0\right) - (1, 0) \right\| \\ &\leq 1 + \frac{m_0(m_0-1)}{(m_0+1)^3} \\ &= \left(1 - 3\frac{m_0(m_0-1)}{(m_0+1)^2}\right) + \frac{m_0(m_0-1)}{(m_0+1)^2}\left[\frac{1}{m_0+1} + 3\right] \\ &\leq \frac{m_0(m_0-1)}{(m_0+1)^2}[\|T(s,t) - T(w,0)\| + \|T(s,t) - T(w,0)\| \\ &\quad + \|T(s,t) - T(w,0)\|] + \left(1 - 3\frac{m_0(m_0-1)}{(m_0+1)^2}\right) dist(G, H). \end{aligned}$$

Also, for any $(w,0) \in H$,

$$\begin{aligned} \|T(s,0)-T(w,0)\| &= \|(1,0)-(2,0)\| \\ &= 1 = dist(G,H) \\ &\leq \frac{1}{6}[\|(s,0)-T(s,0)\| + \|(w,0)-T(w,0)\| + \|(s,0)-(w,0)\|] + \left(1-\frac{1}{2}\right) dist(C,D). \end{aligned}$$

Hence, T is a generalized pointwise cyclic contraction but not a relatively nonexpansive mapping. This is because (G,H) does not have T-restriction property.

We present the following two theorems.

Theorem 3.4. Let (L,M) be a pair of nonempty convex weakly compact subsets in a Banach space B. Let T be a generalised pointwise cyclic contraction mapping $L \cup M$ into itself. Suppose (L,M) has T-restriction property. Then there exists $u \in L, v \in M$ so that $\|Tv - v\| = dist(L,M) = \|Tu - u\|$.

Proof . Since T is a generalised pointwise cyclic contraction mapping and (L,M) has the T-restriction property, we have $||Tu - Tv|| \leq ||u - v||$ for each $(u,v) \in L \times M$. Then, by Theorem 2.1 of [4] and Theorem 4 of [11], there exist $u \in L, v \in M$ so that $||Tv - v|| = ||Tu - u|| = dist(L,M)$. □

Theorem 3.5. Let (L,M) be a nonempty, weakly compact convex semi-sharp proximal pair of subsets in a Banach space B, and $T : L \cup M \to L \cup M$ be a generalised pointwise cyclic contraction mapping. Suppose (L,M) has T-restriction property. Then there is $u \in L$ so that $dist(L,M) = \|u - Tu\|$ and T^2 has unique fixed points $u \in L$ and $Tu \in M$.

Moreover, if (L,M) has the projectional property, then for any initial point $u_0 \in L$, the sequences $\{T^{2n}u_0\}$ and $\{T^{2n+1}u_0\}$, respectively converge to u and Tu.

Proof . Since T is a generalised pointwise cyclic contraction mapping and (L,M) has the T-restriction property, then by Theorem 4.5 of [11], there exists $u \in L$ such that $T^2u = u, T^2(Tu) = Tu$ and $||u - Tu|| = dist(L,M)$. Since, (L,M) has projectional property and T-restriction property, by Theorem 4.5 of [11], the sequence $T^{2n}u_0 \to u_0$ and $T^{2n+1}u_0 \to Tu_0$ as $n \to \infty$. □

The following example verifies Theorem 3.5.

Example 3.6. Let $X = (R^2, \|.\|_1)$, $C = \{(1,t) : 0 \leq t \leq 1\}$, and $D = \{(s,0) : 2 \leq s \leq 3\}$. Define $T : D \to C$ by

$$T(s,t) = (1,0) \ \forall \ (s,t) \in D,$$

and $T : C \to D$ by

$$T(s,t) = (2,0) \ \forall \ (s,t) \in C.$$

Clearly, T from $C \cup D$ into itself is cyclic. Also, (C,D) is a nonempty, weakly compact convex semi-sharp proximal pair having T-restriction property and projectional property. Then, there exists $(s,t) = ((1,0),(2,0)) \in C \times D$ so that $\|(1,0) - T(1,0)\| = 1 = \|(2,0) - T(2,0)\|$. Since (C,D) has projectional property, for each $z_0 \in C$, the sequences $\{T^{2n}z_0\}$ and $\{T^{2n+1}z_0\}$ converges to $(1,0)$ and $T(1,0)$, respectively.

Open Problem

Can Theorems 3.4 and 3.5 in **Section 3** hold true without using T-restriction property?

Conclusion

In **Section 2**, we have established some theorems, i.e., Theorems 2.3, 2.4 and 2.7 from [Theorems 4.1, 4.2 and 4.3] of [1] by using generalized cyclic mapping introduced by [9]. In **Section 3**, we introduce T-restriction property and generalized pointwise cyclic contraction mapping. Using this type of mapping, we give some best proximity point theorems which are the generalisation of some results of [1, 3, 4, 5, 9, 11]. Some examples are also given to support our results.

Acknowledgement

The authors wish to thank the editor and the referee for a guidance and a help in the preparation of this paper.

References

[1] A. Abkar and M. Gabeleh, *Best proximity point for cyclic mappings in ordered metric spaces*, J. Optim. Theo. Appl. **151** (2011), 418–424.

[2] M.A. Al-Thafai and N. Shahzad, *Convergence and existence for best proximity points*, Nonlinear Anal. **70** (2009), 3665–3671.

[3] M.J. Anuradha and P. Veeramani, *Proximal pointwise contraction*, Topology Appl. **156** (2009), 2942–2948.

[4] A.A. Eldred, W.A. Kirk and P. Veeramani, *Proximal normal structure and relatively nonexpansive mapping*, Studia Math. **171** (2005), 283—293.

[5] A.A. Eldred and P. Veeramani, *Existence and convergence of best proximity points*, J. Math. Anal. Appl. **323** (2006), 1001–1006.

[6] R. Espinola, *A new approach to relatively nonexpansive mappings*, Proc. Amer. Math. Soc. **136** (2008), 1987–1995.

[7] K. Fan, Extensions of two fixed point theorems of F. E. Browder, Math. Z. **112** (1969), 234–240.

[8] M. Gabeleh and G.S.R. Kosuru, *Some remarks on convergence of best proximity points and semi-cyclic contractions*, Rend. Circ. Mat. Palermo, II. Ser, doi:org/10.1007/s12215-022-00809-9.

[9] E. Karapinar, *Best proximity points of cyclic mappings*, Appl. Math. Lett. **25** (2012), 1761–1766.

[10] E. Karapinar, P. Agarwal, S. S. Yesilkaya and C. Wang, *Fixed point results for Meir-Keeler type contractions in partial metric spaces: A survey*, Mathematics **10** (2022), 1–76.

[11] G.S.R. Kosuru and P. Veeramani, *A note on existence and convergence of best proximity points for pointwise cyclic contractions*, Num. Funct. Anal. Optim. **32** (2013), 821–830.

[12] J.J. Nieto and R. Rodríguez-López, *Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations*, Order **22** (2005), 223–239.

[13] M. Rossafi and Abdelkarim Kari, *Fixed point for weakly contractive mappings in rectangular b-metric space*, Int. J. Nonlinear Anal. Appl. **14** (2022), 763—783.

[14] T. Suzuki, M. Kikkawa and C. Vetro, *The existence of best proximity points in metric spaces with the property UC*, Nonlinear Anal. **71** (2009), 2918–2926.