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A note on b-generalized derivations with a quadratic equation in prime rings

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Abstract

Let R be a prime ring of characteristic different from 2, C be its extended centroid and Q_r be its right Martindale quotient ring and $f(t_1,...,t_n)$ be a multilinear polynomial over C, which is not central valued on R. Assume that F is a b-generalized derivation on R and d is a derivation of R such that

$$F(f(s))d(f(s)) + d(f(s))F(f(s)) = 0$$

for all $s = (s_1,...,s_n) \in R^n$. Then either $F = 0$ or $d = 0$, except when d is an inner derivation of R, there exists $\lambda \in C$ such that $F(r) = \lambda r$ for all $r \in R$ and $f(t_1,...,t_n)^2$ is central valued on R.

Keywords: b-generalized derivation, multilinear polynomial, prime ring, generalized polynomial identity
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1 Introduction

A ring R is said to be a prime ring if $r_1Rr_2 = (0)$ implies $r_1 = 0$ or $r_2 = 0$ for any $r_1, r_2 \in R$. Throughout this paper R denotes a prime ring with center $Z(R)$. The Utumi quotient ring of R, the right Martindale quotient ring of R and the extended centroid of R are denoted by U, Q_r and C, respectively. If R is a prime ring, then Q_r is a prime ring and C is a field. We refer the reader to [1] for more properties and details. The commutator of r and s is equal to $rs - sr$ and it is denoted by $[r, s]$ for $r, s \in R$.

An additive map $d : R \to R$ is called a derivation if $d(rs) = d(r)s + rd(s)$ for all $r, s \in R$. An additive map $G : R \to R$ is called a generalized derivation of R if there is a derivation d of R such that $G(rs) = G(r)s + rd(s)$ for all $r, s \in R$. An important example of generalized derivation is a map of the form $g(r) = ar + rb$ for some $a, b \in R$ and such generalized derivation is called inner. In the theory of operator algebras, inner generalized derivations have been primarily studied as an important class of the called elementary operators.

The well-known Posner's result states that if R is a prime ring and d is a nonzero derivation of R such that $[d(r), r] \in Z(R)$ for all $r \in R$, then R is commutative ([17]). This result has led to a lot of work explaining the relationship between the structure of R and some additive map defined on R. As an important example, the additive

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maps $F_1, F_2 : R \to R$ satisfying the relation $H(s) = F_1(s)F_2(s) \mp F_2(s)F_1(s) = 0$ for all $s \in S$, where S is a suitable subset of R. If F_1 and F_2 are derivations of R, then $H(s)$ is called a quadratic differential identity on S.

In [13], Lanski proved that if d and g are nonzero derivations on R such that $[d(r), g(r)] \in Z(R)$ for all $r \in R$, then either there exists $\lambda \in C$ such that $d = \lambda g$ or $char(R) = 2$ and R satisfies s_4, the standard identity of degree 4. After, in [2], Beidar et al showed that if $[d(r), G(r)] = 0$ for all $r \in R$, where d is a derivation of R and G is an additive map on R, then there exist $\gamma \in C$ and $\theta : R \to C$ such that $G(r) = \gamma d(r) + \theta(r)$ for any $r \in R$. In [9], Fosner and Vukman proved that if R is a prime ring of characteristic different from 2, G_1 and G_2 two generalized derivations of R such that $G_1(r)G_2(r) + G_2(r)G_1(r) = 0$ for all $r \in R$, then either $G_1 = 0$ or $G_2 = 0$. Recently, Rania and Scudo extended the above result in [18].

In 2014, Koşan and Lee [12] propose the b-generalized derivation as following: Let $d : R \to Q_r$ be an additive mapping and $b \in Q_r$. An additive map $F : R \to Q_r$ is called a left b-generalized derivation, with associated mapping d, if $F(rs) = F(r)s + brd(s)$ for all $r, s \in R$. In the above paper, they proved that if R is a prime ring then d is a derivation of R. Clearly, a generalized derivation is a 1-generalized derivation. The map $r \longmapsto ar + brc$ is an b-generalized derivation of R, for some $a, b, c \in Q_r$, which is called inner b-generalized derivation of R. The b-generalized derivations on multilinear polynomials were studied in recently ([7], [19]).

This paper motivated by the previous cited results. We investigate in the study of a prime ring R with a quadratic differential identity involving b-generalized derivation and derivation on multilinear polynomial over C, where d is a derivation and F is a b-generalized derivation on R. More precisely, we will prove the following theorem:

Theorem: (Main Theorem) Let R be a prime ring of characteristic different from 2, Q_r be its maximal right ring of quotients and C be its extended centroid and $f(t_1, ..., t_n)$ be a multilinear polynomial over C, not central valued on R. Suppose that F is a b-generalized derivation on R and d is a derivation of R such that

$$F(f(s))d(f(s)) + d(f(s))F(f(s)) = 0$$

for all $s = (s_1, ..., s_n) \in R^n$. Then either $F = 0$ or $d = 0$, except when d is an inner derivation of R, there exists $\lambda \in C$ such that $F(r) = \lambda r$ for all $r \in R$ and $f(t_1, ..., t_n)^2$ is central valued on R.

2 Preliminaries

We will use some important results of generalized polynomial identities and differential identities. We recall the following results that we use in the proof of our results.

Fact 2.1: If I is a two-sided ideal of R, then I, R and Q_r satisfy the same generalized polynomial identities with coefficients in Q_r ([4]).

Fact 2.2: If I is a two-sided ideal of R, then I, R and Q_r satisfy the same differential identities ([14]).

Fact 2.3: We will use the following notation:

$$f(s_1, ..., s_n) = s_1 s_2 ... s_n + \sum_{\sigma \in S_n, \sigma \neq id} \alpha_\sigma s_{\sigma(1)} s_{\sigma(2)} ... s_{\sigma(n)},$$

for some $\alpha_\sigma \in C$ and S_n the symmetric group of degree n.

Let d be a derivation. We denote by $f^d(s_1, ..., s_n)$ the polynomial obtained from $f(s_1, ..., s_n)$ replacing each coefficients α_σ with $d(\alpha_\sigma)$. Then we have

$$d(f(s_1, ..., s_n)) = f^d(s_1, ..., s_n) + \sum_i f(s_1, ..., d(s_i), ..., s_n)$$

for all $s_1, ..., s_n \in R$.

Fact 2.4: ([11]) Let R be a prime ring, d be a nonzero derivation on R and I be a nonzero ideal of R. If I satisfies the differential identity

$$f(a_1, a_2, ..., a_n, d(a_1), d(a_2), ..., d(a_n)) = 0$$

for any $a_1, a_2, ..., a_n \in I$, then either

(i) I satisfies the generalized polynomial identity $f(a_1, a_2, ..., a_n, x_1, x_2, ..., x_n) = 0$ or

(ii) d is Q_r-inner, that is; for some $q \in Q_r$, $d(x) = [q, x]$ and I satisfies the generalized polynomial identity $f(a_1, a_2, ..., a_n, [q, a_1], [q, a_2], ..., [q, a_n]) = 0$.

Fact 2.5: Let $X = \{x_1, x_2, ...\}$ the countable set consisting of the non-commuting indeterminates $x_1, x_2, ...$ Let $C\{X\}$ be the free algebra over C in the set X. We denote $T = Q_r \otimes_C C\{X\}$, the free product of the C-algebra Q_r and $C\{X\}$. The elements of T are called the generalized polynomial with coefficients in Q_r. Let B be a set of C-independent vectors of Q_r. Then any element $f \in T$ can be represented in this form $f = \sum_i \alpha_i m_i$, where $\alpha_i \in C$ and m_i are B-monomials of the form $q_0 y_1 q_1 y_2 q_2 ... y_n q_n$, with $q_0, q_1, ..., q_n \in B$ and $y_1, y_2, ..., y_n \in X$. Any generalized polynomial $f = \sum_i \alpha_i m_i$ is trivial, i.e., zero element in T if and only if $\alpha_i = 0$ for each i. For more detail we refer to [4].

Fact 2.6: ([5]) Let C be an infinite field and $m \geq 2$. Suppose that $A_1, ..., A_k$ are not scalar matrices in $M_m(C)$. Then there exists some invertible matrix $P \in M_m(C)$ such that any matrices $PA_1P^{-1}, ..., PA_kP^{-1}$ have all nonzero entries.

3 The matrix case and inner derivations

In this section we consider the case when both d and F are inner. Let $F(r) = ar + brc$ and $d(r) = [q, r]$ for all $r \in R$. For suitable elements $a, b, c, q \in Q_r$, we consider the generalized polynomial

$$\begin{aligned} P(l_1, ..., l_n) &= (af(l_1, ..., l_n) + bf(l_1, ..., l_n)c)(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) + \\ &\quad (qf(l_1, ..., l_n) - f(l_1, ..., l_n)q)(af(l_1, ..., l_n) + bf(l_1, ..., l_n)c). \end{aligned} \tag{3.1}$$

By our hypothesis, we have $P(r_1, ..., r_n) = 0$ for all $r_1, ..., r_n \in R$, i.e., R satisfies the generalized polynomial identity $P(l_1, ..., l_n)$. We examine the generalized polynomial identity given above. In every case after that, let R be a prime ring of characteristic different from 2 and C be its extended centroid. We assume that $f(l_1, ..., l_n)$ is a multilinear polynomial over C and it is not central valued on R.

Proposition 3.1. Either $P(l_1, ..., l_n)$ is a non-trivial generalized polynomial identity for R or one of the following holds:

(i) $q \in C$;

(ii) $c \in C$ and $a = -bc \in C$;

(iii) $a, b, c \in C$ and $f(l_1, ..., l_n)^2$ is central valued on R.

Proof . Firstly we consider $a \in C, b \in C$ and $c \in C$. Then R satisfies

$$(a + bc)\{f(l_1, ..., l_n)(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) + (qf(l_1, ..., l_n) - f(l_1, ..., l_n)q)f(l_1, ..., l_n)\}.$$

We may assume that $a + bc \neq 0$. Since the characteristic of R different from 2, we have either $q \in C$ or $f(l_1, ..., l_n)^2$ is central valued on R by Theorem 1 in [20].

Now, we suppose that q is not central element of Q_r and $P(l_1, ..., l_n)$ is a trivial generalized polynomial.

Suppose first that b and c are central, $a \notin C$. Then

$$P(l_1, ..., l_n) = (a + bc)f(l_1, ..., l_n)(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) + (qf(l_1, ..., l_n) - f(l_1, ..., l_n)q)(a + bc)f(l_1, ..., l_n).$$

Since $\{1, q\}$ is linearly C-independent and $P(l_1, ..., l_n) = 0 \in T$, we have $(a + bc)f(l_1, ..., l_n)^2 = 0 \in T$, by Fact 2.5. This implies $a + bc = 0$ i.e., $a \in C$, a contradiction. Similarly, suppose that a and c are central, $b \notin C$. Then we arrive that $b \in C$, which is a contradiction. Now, we suppose that $a, b \in C$ and $c \notin C$. Then,

$$P(l_1, ..., l_n) = f(l_1, ..., l_n)(a + bc)(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) + (qf(l_1, ..., l_n) - f(l_1, ..., l_n)q)f(l_1, ..., l_n)(a + bc).$$

Since $\{1, q\}$ is linearly C-independent, using again Fact 2.5, we obtain $f(l_1, ..., l_n)^2(a + bc) = 0 \in T$, it implies $a + bc = 0$. This gives $c \in C$, a contradiction.

Hence, we may suppose that a, b, c and q are non-central element of Q_r and $P(l_1, ..., l_n)$ is a trivial generalized polynomial identity for R. We show that this assumption leads to contradiction. By Fact 2.1, $P(l_1, ..., l_n)$ is a trivial generalized polynomial identity for Q_r.

If $\{1, a, b, q\}$ are linearly C-independent, then we have $af(l_1, ..., l_n)(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) = 0 \in T$ from (3.1). Since $\{1, q\}$ is linearly C-independent, we get $af(l_1, ..., l_n)^2 q = 0 \in T$. Therefore, either $a = 0$ or $q = 0$. Thus we have a contradiction.

Let $\{1, a, b, q\}$ is linearly C-dependent. Thus, there exists $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in C$ such that $\alpha_1 a + \alpha_2 b + \alpha_3 q + \alpha_4 = 0$.

Case-1: Let $\alpha_1 = 0$. In this case $\alpha_2 b + \alpha_3 q + \alpha_4 = 0$, where $\alpha_2 \neq 0$ since q is not central. Therefore, $b = \alpha^{'} q + \alpha^{''}$, where $\alpha^{'} = -\alpha_2^{-1}\alpha_3$ and $\alpha^{''} = -\alpha_2^{-1}\alpha_4$. In view of (3.1), we have

$$\begin{aligned} 0 \;=\;& af(l_1, ..., l_n)qf(l_1, ..., l_n) - af(l_1, ..., l_n)^2 q \\ &+(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)cqf(l_1, ..., l_n) - (\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)cf(l_1, ..., l_n)q \\ &+qf(l_1, ..., l_n)af(l_1, ..., l_n) + qf(l_1, ..., l_n)(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)c \\ &-f(l_1, ..., l_n)qaf(l_1, ..., l_n) - f(l_1, ..., l_n)q(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)c. \end{aligned} \tag{3.2}$$

If $\{a, q, 1\}$ is linearly C-independent, then we get a contradiction. Hence, $\beta_1 a + \beta_2 q + \beta_3 = 0$. If $\beta_1 = 0$, then β_2 cannot be zero and so $q \in C$, a contradiction. Thus, suppose that $\beta_1 \neq 0$. Then, there exists $\beta^{'}, \beta^{''} \in C$ such that $a = \beta^{'} q + \beta^{''}$, where $\beta^{'} = -\beta_1^{-1}\beta_2$ and $\beta^{''} = -\beta_1^{-1}\beta_3$. From (3.2), we get

$$\begin{aligned} 0 \;=\;& (\beta^{'} q + \beta^{''})f(l_1, ..., l_n)qf(l_1, ..., l_n) - (\beta^{'} q + \beta^{''})f(l_1, ..., l_n)^2 q \\ &+(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)cqf(l_1, ..., l_n) - (\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)cf(l_1, ..., l_n)q \\ &+qf(l_1, ..., l_n)(\beta^{'} q + \beta^{''})f(l_1, ..., l_n) \\ &+qf(l_1, ..., l_n)(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)c - f(l_1, ..., l_n)q(\beta^{'} q + \beta^{''})f(l_1, ..., l_n) \\ &-f(l_1, ..., l_n)q(\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)c. \end{aligned} \tag{3.3}$$

Since $q \notin C$ and $P(l_1, ..., l_n)$ is a trivial generalized polynomial identity, we obtain

$$qf(l_1, ..., l_n)\left\{ \begin{array}{c} (2\beta^{'} q + \alpha^{'} cq + \beta^{''})f(l_1, ..., l_n) \\ -(\alpha^{'} c + \beta^{'})f(l_1, ..., l_n)q + (\alpha^{'} q + \alpha^{''})f(l_1, ..., l_n)c \end{array} \right\} = 0. \tag{3.4}$$

If $\{c, q, 1\}$ is linearly C-independent, we have $qf(l_1, ..., l_n)(\alpha^{'} c + \beta^{'})f(l_1, ..., l_n)q = 0$. Since $q \notin C$, we have $\alpha^{'} c + \beta^{'} = 0$. Then, we get $\alpha^{'} c = -\beta^{'} \in C$. This implies that $\alpha^{'} = 0$, since $c \notin C$. Hence, we have $b = \alpha^{'} q + \alpha^{''} = \alpha^{''} \in C$, a contradiction. Let $\{c, q, 1\}$ is linearly C-dependent. Then, there exists $\lambda_1, \lambda_2 \in C$ such that $c = \lambda_1 q + \lambda_2$, where $\lambda_1 \neq 0$. By (3.4), we obtain

$$qf(l_1, ..., l_n)\left\{(2\beta^{'} + \alpha^{'}\lambda_1 q + 2\alpha^{'}\lambda_2)q + \beta^{''} + \alpha^{''}\lambda_2)\right\} f(l_1, ..., l_n) = 0.$$

Since $q \notin C$, we get $(2\beta^{'} + \alpha^{'}\lambda_1 q + 2\alpha^{'}\lambda_2)q = -\beta^{''} - \alpha^{''}\lambda_2 \in C$. This implies that $2\beta^{'} + \alpha^{'}\lambda_1 q + 2\alpha^{'}\lambda_2 = 0$, since $q \notin C$. Thus, $\alpha^{'} = 0$ since $\lambda_1 \neq 0$ and $q \notin C$. Hence $b \in C$, a contradiction.

Case-2: Let $\alpha_1 \neq 0$. Then, $\alpha_1 a + \alpha_2 b + \alpha_3 q + \alpha_4 = 0$ yields that $a = \gamma_1 b + \gamma_2 q + \gamma_3$ for some $\gamma_1, \gamma_2, \gamma_3 \in C$. By (3.1), we have

$$\begin{aligned} 0 \;=\;& (\gamma_1 b + \gamma_2 q + \gamma_3)f(l_1, ..., l_n)qf(l_1, ..., l_n) - (\gamma_1 b + \gamma_2 q + \gamma_3)f(l_1, ..., l_n)^2 q \\ &+bf(l_1, ..., l_n)cqf(l_1, ..., l_n) - bf(l_1, ..., l_n)cf(l_1, ..., l_n)q + qf(l_1, ..., l_n)(\gamma_1 b + \gamma_2 q + \gamma_3)f(l_1, ..., l_n) \\ &+qf(l_1, ..., l_n)bf(l_1, ..., l_n)c - f(l_1, ..., l_n)q(\gamma_1 b + \gamma_2 q + \gamma_3)f(l_1, ..., l_n) - f(l_1, ..., l_n)qbf(l_1, ..., l_n)c. \end{aligned} \tag{3.5}$$

Let $\{b, q, 1\}$ is linearly C-independent. Since $P(l_1, ..., l_n)$ is trivial generalized polynomial identity for Q_r, we get

$$bf(l_1, ..., l_n)\left\{(\gamma_1 + c)qf(l_1, ..., l_n) - (\gamma_1 + c)f(l_1, ..., l_n)q\right\} = 0.$$

Since $b, q \notin C$, we find $c = -\gamma_1 \in C$, a contradiction. If $\{b, q, 1\}$ is linearly C-dependent, then there exists $\theta_1, \theta_2 \in C$ such that $b = \theta_1 q + \theta_2$, where $\theta_1 \neq 0$. Hence, by (3.5)

$$\begin{aligned} 0 \quad = \quad & (\gamma_1\theta_1 q + \gamma_1\theta_2 + \gamma_2 q + \gamma_3)f(l_1,...,l_n)qf(l_1,...,l_n) - (\gamma_1\theta_1 q + \gamma_1\theta_2 + \gamma_2 q + \gamma_3)f(l_1,...,l_n)^2 q \\ & +(\theta_1 q + \theta_2)f(l_1,...,l_n)cqf(l_1,...,l_n) - (\theta_1 q + \theta_2)f(l_1,...,l_n)cf(l_1,...,l_n)q \\ & +qf(l_1,...,l_n)(\gamma_1\theta_1 q + \gamma_1\theta_2 + \gamma_2 q + \gamma_3)f(l_1,...,l_n) + qf(l_1,...,l_n)(\theta_1 q + \theta_2)f(l_1,...,l_n)c \\ & -f(l_1,...,l_n)q(\gamma_1\theta_1 q + \gamma_1\theta_2 + \gamma_2 q + \gamma_3)f(l_1,...,l_n) - f(l_1,...,l_n)q(\theta_1 q + \theta_2)f(l_1,...,l_n)c. \end{aligned} \tag{3.6}$$

Since $\{q, 1\}$ is linearly C-independent, we get

$$qf(l_1,...,l_n)\left\{ \begin{array}{c} ((2\gamma_1\theta_1 + 2\gamma_2 + \theta_1 c)q + (\gamma_1\theta_2 + \gamma_3))f(l_1,...,l_n) \\ -(\gamma_1\theta_1 + \gamma_2 + \theta_1 c)f(l_1,...,l_n)q + (\theta_1 q + \theta_2)f(l_1,...,l_n)c \end{array} \right\} = 0.$$

If $\{c, q, 1\}$ is linearly C-independent, we obtain

$$qf(l_1,...,l_n)(\gamma_1\theta_1 + \gamma_2 + \theta_1 c)f(l_1,...,l_n)q = 0.$$

Since $q \notin C$, $\gamma_1\theta_1 + \gamma_2 + \theta_1 c = 0$, that is, $\theta_1 c = -\gamma_1\theta_1 - \gamma_2 \in C$. Since $c \notin C$, we get $\theta_1 = 0$. Thus, $b = \theta_1 q + \theta_2 = \theta_2 \in C$, a contradiction.

Now, let $\{c, q, 1\}$ is linearly C-dependent. Then, there exists $\mu_1, \mu_2 \in C$ such that $c = \mu_1 q + \mu_2$, where $\mu_1 \neq 0$. Since $q \notin C$ and the relation (3.6), we have

$$0 = f(l_1,...,l_n)\left\{ \begin{array}{c} [(-2\theta_2 - \theta_1 q)\mu_1 q - (\gamma_1\theta_2 + \gamma_3 + \theta_2\mu_2)]\, f(l_1,...,l_n)q \\ +(\theta_2\mu_1 - \gamma_1\theta_1 - \gamma_2 - \theta_1\mu_2)q^2 f(l_1,...,l_n) \end{array} \right\}.$$

Hence using again $\{q, 1\}$ is linearly C-independent, we obtain $(-2\theta_2 - \theta_1 q)\mu_1 q = \gamma_1\theta_2 + \gamma_3 + \theta_2\mu_2 \in C$. Since $q \notin C$ and $\mu_1 \neq 0$, we get $\theta_1 q = -2\theta_2 \in C$. This implies that $\theta_1 = 0$ and so $b \in C$, a contradiction.

Therefore, all the cases lead to a contradiction. □

Proposition 3.2. Assume $R = M_m(C)$, $m \geq 2$, the ring of $m \times m$ matrices over the infinite field C. Then one of the following holds:

(*i*) $q \in C$;

(*ii*) $c \in C$ and $a = -bc \in C$;

(*iii*) $a, b, c \in C$ and $f(l_1,...,l_n)^2$ is central valued on R.

Proof . By our assumption R satisfies generalized polynomial identity

$$(af(l_1,...,l_n) + bf(l_1,...,l_n)c)\,(qf(l_1,...,l_n) - f(l_1,...,l_n)q) + (qf(l_1,...,l_n) - f(l_1,...,l_n)q)\,(af(l_1,...,l_n) + bf(l_1,...,l_n)c)\,.$$

Let e_{ij} denotes the matrix whose (i,j)-entry is 1 and rest entries are zero. Since $f(l_1,...,l_n)$ is not central, by [14] (see also [15]), there exist $l_1,...,l_n \in M_m(C)$ and $0 \neq \gamma \in C$ such that $f(l_1,...,l_n) = \gamma e_{mn}$ with $m \neq n$. Moreover, since the set $\{f(r_1,...,r_n) \mid r_1,...,r_n \in M_m(C)\}$is invariant under the action of all C-automorphisms of $M_m(C)$, then for any $i \neq j$ there exist $r_1,...,r_n \in M_m(C)$ such that $f(r_1,...,r_n) = e_{ij}$. Thus, we get

$$0 = ae_{ij}qe_{ij} + be_{ij}cqe_{ij} - be_{ij}ce_{ij}q + qe_{ij}ae_{ij} + qe_{ij}be_{ij}c - e_{ij}qae_{ij} - e_{ij}qbe_{ij}c. \tag{3.7}$$

Left multiplying by e_{ij} in (3.7), we obtain

$$0 = 2a_{ji}q_{ji}e_{ij} + b_{ji}(\sum_{k=1}^{m} c_{jk}q_{ki})e_{ij}.$$

In particular, $2a_{ji}q_{ji} = 0$ and $b_{ji}v_{ji} = 0$. Since $char(R) \neq 2$, it implies that

$$a_{ji}q_{ji} = 0 \text{ and } b_{ji}v_{ji} = 0. \tag{3.8}$$

It is obvious that we may assume q is not scalar, unless we are done. Suppose that a is not scalar. By Fact 2.6, there exists a C-automorphism φ of $M_m(C)$ such that $a^{'} = \varphi(a)$, $q^{'} = \varphi(q)$ have all nonzero entries. Clearly, $a^{'}$, $q^{'}$ and $v^{'} = \varphi(v)$ must satisfy the condition (3.8). This gives a contradiction. Similarly, if we assume b is not scalar, then we get a contradiction.

Now, we assume that a, b and c are central. Thus R satisfies

$$(a+bc)\left\{\begin{array}{c} f(l_1,...,l_n)(qf(l_1,...,l_n)-f(l_1,...,l_n)q) \\ +(qf(l_1,...,l_n)-f(l_1,...,l_n)q)f(l_1,...,l_n) \end{array}\right\}.$$

Hence, if $a+bc=0$ then $a=-bc\in C$, since we can use the similar situation as Proposition 1. By applying Theorem 1 in [20], if $a+bc\neq 0$, either $q\in C$ or $f(l_1,...,l_n)^2$ is central valued on R. □

Proposition 3.3. Assume that $R = M_m(C)$, $m \geq 2$, is the algebra of $m\times m$ matrices over a field C of characteristic different from 2 and $f(l_1,...,l_n)$ is a multilinear polynomial over C, which is not central valued on R. Hence, one of the following holds:

(*i*) $q \in C$;

(*ii*) $c\in C$ and $a=-bc\in C$;

(*iii*) $a,b,c\in C$ and $f(l_1,...,l_n)^2$ is central valued on R.

Proof . In the case of infinite field C, the conclusion follows from Proposition 2. Now we suppose that C is a finite field. Let K be an infinite extension of the field of C and $\overline{R} = M_m(K)\cong R\otimes_C K$. Note that the multilinear polynomial $f(l_1,...,l_n)$ is central valued on R iff it is central valued on $\overline{R}$. Consider the generalized polynomial

$$\begin{aligned} P(l_1,...,l_n) &= (af(l_1,...,l_n)+bf(l_1,...,l_n)c)\,(qf(l_1,...,l_n)-f(l_1,...,l_n)q) \\ &\quad +(qf(l_1,...,l_n)-f(l_1,...,l_n)q)\,(af(l_1,...,l_n)+bf(l_1,...,l_n)c) \end{aligned}$$

which is a generalized polynomial identity for R. Moreover, it is a multihomogeneous of multidegree $(2,2,...,2)$ in the indeterminates $m_1,m_2,...,m_n$. Hence the complete linearization of $P(l_1,...,l_n)$ is a multilinear generalized polynomial $\Theta(l_1,...,l_n,l_1,...,l_n)=2^nP(l_1,...,l_n)$. It is clear that the multilinear polynomial $\Theta(l_1,...,l_n,y_1,...,y_n)$ is a generalized polynomial identity for both R and $\overline{R}$. By $char(C)\neq 2$, we have $P(r_1,...,r_n)=0$ for all $r_1,...,r_n\in\overline{R}$. Therefore, the conclusion follows from Proposition 2. □

Corollary 3.4. Let $R = M_m(C)$, $m\geq 2$, be the ring of $m\times m$ matrices over the field C, which is characteristic different from 2 and $a,b,c,q\in Q_r$. If

$$(ar+brc)(qr-rq)+(qr-rq)(ar+brc)=0 \text{ for all } r\in R,$$

then one of the following holds:

(*i*) $q \in C$;

(*ii*) $c\in C$ and $a=-bc\in C$;

(*iii*) $a,b,c\in C$ and $r^2\in Z(R)$.

Proposition 3.5. Assume that R is a primitive ring which is isomorphic to a dense ring of linear transformations of a vector space V over C such that V is infinite-dimensional over C. Let $a,b,c,q\in R$. If $(ar+brc)(qr-rq)+(qr-rq)(ar+brc)=0$ for all $r\in R$, then either $q\in C$ or $c\in C$ and $a=-bc\in C$.

Proof . We will prove by contradiction. Suppose that q, c and a are noncentral. For any $e=e^2\in Soc(R)$, we have $eRe\cong M_k(C)$ with $k=\dim_C Ve$, since V is infinite dimensional over C. From the fact that $q\notin C$, $c\notin C$ and $a\notin C$, then they not centralize the nonzero ideal $Soc(R)$. Then, $qh_0\neq h_0q$, $ch_1\neq h_1c$ and $ah_2\neq h_2a$ for some $h_0,h_1,h_2\in Soc(R)$. By Litoff's theorem(see Theorem 4.3.11 in [1]), there exists an idempotent $e\in Soc(R)$ such that $h_0,h_1,h_2,h_0q,qh_0,h_1c,ch_1,h_2a,ah_2\in eRe$. Since R satisfies the generalized identity

$$e((aere+berec)(qere-ereq)+(qere-ereq)(aere+berec))e=0,$$

the subring eRe satisfies

$$(eaer + eberece)(eqer - reqe) + (eqer - reqe)(eaer + eberece) = 0.$$

By above finite dimensional case, we have either $eqe \in Z(eRe)$ or $ece \in Z(eRe)$ and $eae \in Z(eRe)$. Thus, we get

$$qh_0 = (eqe)h_0 = h_0(eqe) = h_0q,$$

$$ch_1 = (ece)h_1 = h_1(ece) = h_1c,$$

$$ah_2 = (eae)h_2 = h_2(eae) = h_2a.$$

Clearly, the conclusions contradict with the choices of $h_0, h_1, h_2 \in Soc(R)$, respectively. $\square$

Theorem 3.6. Let R be a noncommutative prime ring of $char(R) \neq 2$, Q_r be its right Martindale quotient ring and C be its extended centroid. Let $f(l_1, ..., l_n)$ be a multilinear polynomial over C, which is not central valued on R and $a, b, c, q \in Q_r$ such that $d(x) = qx - xq$, $F(x) = ax + bxc$ for all $x \in R$. ıf

$$F(f(r))d(f(r)) + d(f(r))F(f(r)) = 0$$

for all $r = (r_1, ..., r_n) \in R^n$, then either $F = 0$ or $d = 0$ unless when there exists $\lambda \in C$ such that $F(x) = \lambda x$ for all $x \in R$ and $f(l_1, ..., l_n)^2$ is central valued on R.

Proof . Proposition 1 implies that R satisfies the non-trivial generalized polynomial identity

$$\begin{aligned} P(l_1, ..., l_n) \quad = \quad & (af(l_1, ..., l_n) + bf(l_1, ..., l_n)c)\,(qf(l_1, ..., l_n) - f(l_1, ..., l_n)q) + \\ & (qf(l_1, ..., l_n) - f(l_1, ..., l_n)q)\,(af(l_1, ..., l_n) + bf(l_1, ..., l_n)c)\,. \end{aligned}$$

From Fact 2.1, since R and Q_r satisfy same generalized polynomial identities, this generalized polynomial identity is also satisfied by Q_r. If C is infinite, then $P(r_1, ..., r_n) = 0$ for all $r_1, ..., r_n \in Q_r \otimes_C \overline{C}$, where $\overline{C}$ is the algebraic closure of C. By Theorem 3.5 in [8], $Q_r \otimes_C \overline{C}$ is a centrally closed prime $\overline{C}$-algebra and so we may replace R by Q_r or $Q_r \otimes_C \overline{C}$ according as C is finite or infinite. By Theorem 3 in [16], R is a primitive ring having a nonzero socle H and eHe is a simple central algebra finite dimensional over C, for a minimal idempotent $e \in H$. In the light of Jacobson's theorem ([10], p.75), R is isomorphic to a dense ring of linear transformations on any vector space V over C. Firstly, we suppose that V is finite dimensionel over C, i.e., $\dim_C V = k$. By density of R, we get $R \cong M_k(C)$. Then, R must be noncommutative and so $k \geq 2$, since $f(r_1, ..., r_n)$ is not central valued on R. Hence, the conclusion follows from Proposition 3.

Now, we assume that V is infinite dimensional over C. By Lemma 2 in [21], the set $f(R)$ is dense on R and since $P(r_1, ..., r_n) = 0$ for all $r_1, ..., r_n \in R$, we arrive that R satisfies the generalized identity

$$P(x) = (ax + bxc)[q, x] + [q, x](ax + bxc)$$

which gives

$$(ar + brc)(qr - rq) + (qr - rq)(ar + brc) = 0 \text{ for all } r \in R.$$

By Proposition 4, we conclude that $q \in C$ or $c \in C$ and $a = -bc \in C$. The proof is completed. $\square$

4 The Main Result

Theorem 4.1. Suppose that R is a prime ring of characteristic different from 2, Q_r is maximal right ring of quotients and C is extended centroid of R and $f(t_1, ..., t_n)$ is a multilinear polynomial over C which is not central valued on R. If F is a b-generalized derivation on R and d is a derivation of R such that

$$F(f(s))d(f(s)) + d(f(s))F(f(s)) = 0$$

for all $s = (s_1, ..., s_n) \in R^n$, then either $F = 0$ or $d = 0$, except when d is an inner derivation of R, there exists $\lambda \in C$ such that $F(r) = \lambda r$ for all $r \in R$ and $f(t_1, ..., t_n)^2$ is central valued on R.

Proof . Theorem 2.3 in [12] implies that there exist a derivation $\delta : R \to Q_r$ and $a \in Q_r$ such that $F(r) = ar + b\delta(r)$ for all $r \in R$. By hypothesis, R satisfies

$$0 = (af(l_1, ..., l_n) + b\delta(f(l_1, ..., l_n))d(f(l_1, ..., l_n)) + d(f(l_1, ..., l_n))(af(l_1, ..., l_n) + b\delta(f(l_1, ..., l_n)). \tag{4.1}$$

From Lemma 2 in [14], we know that any derivation of R can be uniquely extended to a derivation of Q_r and so Q_r satisfies this differential identity. Substituting the value of $\delta(f(l_1, ..., l_n))$ and $d(f(l_1, ..., l_n))$ in (4.1), we obtain

$$\begin{aligned} 0 = &(af(l_1, ..., l_n) + bf^{\delta}(l_1, ..., l_n) + b\sum_i f(l_1, ..., \delta(l_i), ...l_n)) + (f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., d(l_i), ...l_n)) \\ &+(f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., d(l_i), ...l_n))(af(l_1, ..., l_n) + bf^{\delta}(l_1, ..., l_n) + b\sum_i f(l_1, ..., \delta(l_i), ...l_n)) \end{aligned} \tag{4.2}$$

In case both d and δ are inner derivations, we write $d(x) = [p, x]$ and $F(x) = ax + b[q, x] = (a + bq)x - bxq$ for $p, q \in Q_r$. Thus, the conclusion follows from Theorem 1. Also, if $\delta = 0$, then by (4.2)

$$0 = af(l_1, ..., l_n)(f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., d(l_i), ...l_n)) + (f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., d(l_i), ...l_n))af(l_1, ..., l_n).$$

In light of Kharchenko's theorem (see Fact 2.4), Q_r satisfies

$$0 = af(l_1, ..., l_n)(f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., w_i, ...l_n)) + (f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., w_i, ...l_n))af(l_1, ..., l_n).$$

In particular, Q_r satisfies blended component

$$af(l_1, ..., l_n)\sum_i f(l_1, ..., w_i, ...l_n) + \sum_i f(l_1, ..., w_i, ...l_n)af(l_1, ..., l_n) = 0. \tag{4.3}$$

We replace w_i by $[u, x_i]$, where $u \in Q_r$ and $u \notin C$. Since $f(l_1, ..., l_n)$ is multilinear, then we get

$$af(l_1, ..., l_n)[u, f(l_1, ..., l_n)] + [u, f(l_1, ..., l_n)]af(l_1, ..., l_n) = 0.$$

From Theorem 1, $a \in C$ and $f(l_1, ..., l_n)^2$ is central valued on R. Since $a \in C$ and R is a prime ring, either $a = 0$ or $f(l_1, ..., l_n)^2 = 0$, by equation (4.3). By Main Theorem in [3], $f(l_1, ..., l_n) = 0$ for all $l_i \in R$, in this case we get a contradiction. Thus, we assume in all follows that $d \neq 0$, $\delta \neq 0$ and both are not simultaneously inner derivations.

First we consider the case when d and δ are linearly C-independent modulo inner derivations. Applying Kharchenko's theorem by (4.2), Q_r satisfies the differential identity

$$\begin{aligned} &(af(l_1, ..., l_n) + bf^{\delta}(l_1, ..., l_n) + b\sum_i f(l_1, ..., w_i, ...l_n)) + (f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., z_i, ...l_n)) \\ &+(f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., z_i, ...l_n)) + (af(l_1, ..., l_n) + bf^{\delta}(l_1, ..., l_n) + b\sum_i f(l_1, ..., w_i, ...l_n)) \end{aligned}$$

and in particular

$$b\sum_i f(l_1, ..., w_i, ...l_n)\sum_i f(l_1, ..., z_i, ...l_n) + \sum_i f(l_1, ..., z_i, ...l_n)b\sum_i f(l_1, ..., w_i, ...l_n). \tag{4.4}$$

is a generalized identity for Q_r. For some non-central element $q \in Q_r$, replacing w_i and z_i by $[q, x_i]$ in (4.4), for any $i = 1, ..., n$, we get R satisfies

$$b[q, f(l_1, ..., l_n)]^2 + [q, f(l_1, ..., l_n)]b[q, f(l_1, ..., l_n)].$$

This relation is a particular case of Theorem 1 and so result follows from Theorem 1.

Now, let d and δ be C-dependent modulo inner derivations. In this case, there exist $\alpha, \beta \in C$ such that $\alpha d + \beta\delta$ is inner derivation, i.e., there exists $q \in Q_r$ such that $\alpha d(x) + \beta\delta(x) = [q, x]$ for all $x \in Q_r$.

Suppose first $\alpha = 0$ and $\beta \neq 0$. Then $\delta(x) = [p, x]$ for all $x \in Q_r$, where $p = \beta^{-1}q \notin C$. As mentioned above, d is not an inner derivation of R. By (4.1), R satisfies

$$(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)])d(f(l_1, ..., l_n)) + d(f(l_1, ..., l_n))(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)]). \tag{4.5}$$

Since d is not inner and by using Kharchenko's theorem, R satisfies the generalized identity

$$(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)])(f^d(l_1, ..., l_n) + \sum_i f(l_1, ..., w_i, ...l_n)) + (f^d(l_1, ..., l_n)$$
$$+ \sum_i f(l_1, ..., w_i, ...l_n))(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)]).$$

Hence, R satisfies the blended component

$$(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)]) \sum_i f(l_1, ..., w_i, ...l_n) + \sum_i f(l_1, ..., w_i, ...l_n)(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)]).$$

Replace w_i by $[q, x_i]$ for some $q \notin C$, then R satisfies

$$(af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)])[q, f(l_1, ..., l_n)] + [q, f(l_1, ..., l_n)](af(l_1, ..., l_n) + b[p, f(l_1, ..., l_n)]).$$

The desired result is obtained with Theorem 1. Now, we consider the case when $\beta = 0$ and $\alpha \neq 0$. Then $d(x) = [u, x]$, where $u = \alpha^{-1}q \notin C$. As above remarked, δ cannot be an inner derivation of R. From (4.1), R satisfies the differential identity

$$(af(l_1, ..., l_n) + bf^\delta(l_1, ..., l_n) + b\sum_i f(l_1, ..., w_i, ...l_n))[u, f(l_1, ..., l_n)]$$

$$[u, f(l_1, ..., l_n)](af(l_1, ..., l_n) + bf^\delta(l_1, ..., l_n) + b\sum_i f(l_1, ..., w_i, ...l_n)).$$

Thus, R satisfies the blended component

$$b\sum_i f(l_1, ..., w_i, ...l_n)[u, f(l_1, ..., l_n)] + [u, f(l_1, ..., l_n)]b\sum_i f(l_1, ..., w_i, ...l_n).$$

Similar to above, replace w_i by $[q, x_i]$ for some $q \notin C$, R satisfies the generalized identity

$$b[q, f(l_1, ..., l_n)][u, f(l_1, ..., l_n)] + [u, f(l_1, ..., l_n)]b[q, f(l_1, ..., l_n)].$$

From Theorem 1, either $q \in C$ or $u \in C$. In this case, we get a contradiction.

Finally, we consider the case both $\alpha \neq 0$ and $\beta \neq 0$. Thus, $d(x) = \gamma\delta(x) + [v, x]$, where $\gamma = -\alpha^{-1}\beta$, $v = \alpha^{-1}q$. If δ is an inner derivation, then d must also be inner. Thus, we suppose that δ is not inner. From our hypothesis,

$$(af(l_1, ..., l_n) + bf^\delta(l_1, ..., l_n) + b\sum_i f(l_1, ..., w_i, ...l_n))$$
$$(\gamma f^\delta(l_1, ..., l_n) + \gamma\sum_i f(l_1, ..., w_i, ...l_n) + \sum_i f(l_1, ..., [v, l_i], ...l_n))$$
$$+(\gamma f^\delta(l_1, ..., l_n) + \gamma\sum_i f(l_1, ..., \delta(l_i), ...l_n) + \sum_i f(l_1, ..., [v, l_i], ...l_n))$$
$$(af(l_1, ..., l_n) + bf^\delta(l_1, ..., l_n) + b\sum_i f(l_1, ..., \delta(l_i), ...l_n))$$

is a differential identity for Q_r. Since δ is not inner, using again Kharchenko's theorem, Q_r satisfies

$$(af(l_1,...,l_n)+bf^{\delta}(l_1,...,l_n)+b\sum_i f(l_1,...,w_i,...l_n))$$
$$(\gamma f^{\delta}(l_1,...,l_n)+\gamma\sum_i f(l_1,...,w_i,...l_n)+\sum_i f(l_1,...,[v,l_i],...l_n))$$
$$+(\gamma f^{\delta}(l_1,...,l_n)+\gamma\sum_i f(l_1,...,w_i,...l_n)+\sum_i f(l_1,...,[v,l_i],...l_n))$$
$$(af(l_1,...,l_n)+bf^{\delta}(l_1,...,l_n)+b\sum_i f(l_1,...,w_i,...l_n)).$$

Then, Q_r satisfies the blended component

$$b(\sum_i f(l_1,...,w_i,...l_n))^2+\sum_i f(l_1,...,w_i,...l_n)b\sum_i f(l_1,...,w_i,...l_n) \tag{4.6}$$

since $0\neq\gamma\in C$. In particular, $w_1=l_1$ and $w_i=0$ for all $i\geq 2$, by (4.6), Q_r satisfies $bf(l_1,...,l_n)^2+f(l_1,...,l_n)bf(l_1,...,l_n)$. By [6], this yields $b\in C$. Since $b\neq 0$ and $char(R)\neq 2$, $f(l_1,...,l_n)^2=0$ for all $l_1,...,l_n\in R$. By Main Theorem in [3], it implies a contradiction. □

Example 4.2. Assume that K be a field with characteristic 2 and

$$R=\left\{\left(\begin{array}{cc} k & l\\ 0 & m\end{array}\right)\middle|\, k,l,m\in K\right\}.$$

We define mappings $F,d:R\to R$ by $F(r)=e_{12}r+e_{22}re_{12}$ and $d(r)=e_{11}re_{22}$ for all $r\in R$, where $\{e_{ij}\,|\,i,j=1,2\}$ denotes a set of matrix units in R. It is easy to see that F is a nonzero b-generalized derivation and d is a nonzero derivation of R. Moreover, F and d satisfy the hypotheses in Theorem 4.1, but F is not in the form $F(r)=\lambda r$ for all $r\in R$. Therefore, the condition of primeness and the characteristic hypotheses are crucial.

5 Conclusions

Many articles in the literature, involving different types of derivations on multilinear polynomials of a prime ring, show that there is a relationship between the structure of a prime ring R and the behavior of additive maps defined on R. We investigate in the study of a prime ring R of characteristic different from 2 with a quadratic differential identity involving a b-generalized derivation and a derivation on a non-central multilinear polynomial $f(t_1,...,t_n)$ over C. We show that if $F(f(s))d(f(s))+d(f(s))F(f(s))=0$ for all $s=(s_1,...,s_n)\in R^n$, then there exists $\lambda\in C$ such that $F(r)=\lambda r$ for all $r\in R$, d is an inner derivation of R and $f(t_1,...,t_n)^2$ is central valued on R, where $F\neq 0$ is a b-generalized derivation and $d\neq 0$ is a derivation on R.

References

[1] K.I. Beidar, W.S. III Martindale and A.V. Mikhalev, *Rings with Generalized Identities*, Pure and Applied Mathematics, Dekker, New York, 1996.

[2] K.I. Beidar, M. Bresar and M.A. Chebotar, *Functional identities with r-independent coefficients*, Commun. Algebra **30** (2002), no.12, 5725–5755.

[3] C.L. Chuang and T.K. Lee, *Rings with annihilator conditions on multilinear polynomials*, Chin J. Math. **24** (1996), no. 2, 177–185.

[4] C.L. Chuang, *GPI's having coefficients in Utumi quotient rings*, Proc. Am. Math. Soc. **103** (1998), no. 3, 723–728.

[5] V. De Filippis, *Product of two generalized derivations on polynomials in prime rings*, Collectanea Mathematica **61** (2010), no. 3, 303–322.

[6] Ç. Demir and N. Argaç, *Prime rings with generalized derivations on right ideals*, Algebra Colloq. **18** (2011) no. 1, 987–998.

[7] B. Dhara, *b-generalized derivations on multilinear polynomials in prime rings*, Bull. Korean Math. Soc. **55** (2018), no. 2, 573–586.

[8] T.S. Erickson, W.S. III, Martindale and J.M. Osborn, *Prime non-associative algebras*, Pac. J. Math. **60** (1975), 49–63.

[9] M. Fosner and J. Vukman, *Identities with generalized derivations in prime rings*, Mediterr J. Math. **9** (2012), no. 4, 847–863.

[10] N. Jacobson, *Structure of rings*, American Mathematical Society, USA, 1964.

[11] V.K. Kharchenko, *Differential identities of prime rings*, Algebra Log. **17** (1978), 155–168.

[12] M.T. Koşan and T.K. Lee, *b-Generalized derivations having nilpotent values*, J. Aust. Math. Soc. **96** (2014), no. 3, 326–337.

[13] C. Lanski, *Differential identities of prime rings, Kharchenko's theorem and applications*, Contemp. Math. **124** (1992), 111–128.

[14] T.K. Lee, *Derivations with invertible values on a multilinear polynomial*, Proc. Am. Math. Soc. **119** (1993), no. 4, 1077–1083.

[15] U. Leron, *Nil and power central polynomials in rings*, Trans. Amer. Math. Soc. **202** (1975), 97–103.

[16] W.S. III Martindale, *Prime rings satisfying a generalized polynomial identity*, J. Algebra Colloq. **3** (1996), no. 4, 369–478.

[17] E.C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093–1100.

[18] F. Rania and G. Scudo, *A quadratic differential identity with generalized derivations on multilinear polynomials in prime rings*, Mediterr. J. Math. **11** (2014), 273–285.

[19] S.K. Tiwari and B. Prajapati *Centralizing b-generalized derivations on multilinear polynomials*, Filomat **33** (2019), no. 19, 6251–6266.

[20] T.L. Wong, *Derivations with power central values on multilinear polynomials*, Algebra Colloq. **3** (1996), no. 4, 369–378.

[21] T.L. Wong, *Derivations cocentralizing multilinear polynomials*, Taiwan J. Math. **1** (1997), no. 1, 31–37.