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New approach based on fuzzy hypergraphs in granular computing (An application to the urban vulnerability assessment)

Abdolreza Zarandi Baghini[a], Hojjat Babaei[a], Ramin Tabatabaei Mirhosseini[b,*], Lida Torkzadeh Tabrizi[a]

[a]*Department of Mathematics, Faculty of Science, Kerman Branch, Islamic Azad University, Kerman, Iran*

[b]*Department of Civil Engineering, Faculty of Engineering, Kerman Branch, Islamic Azad University, Kerman, Iran*

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Abstract

Classifying objects based on the simultaneous impact of various parameters has always been challenging due to heterogeneity, impact conflict, and sometimes parameter uncertainty. The purpose of this study is to provide a method for classifying such data. In the proposed method, fuzzy hypergraphs were used to define the granular structures in order to apply the simultaneous effect of heterogeneous and weighted parameters in the classification. This method has been implemented and validated on Fisher's intuitive research in relation to the classification of iris flowers. Evaluation and comparison of the proposed method with Fisher's experimental results showed higher efficiency and accuracy in flower classification. The proposed method has been used to assess the seismic risk of 50,000 buildings based on 10 heterogeneous parameters. Seismic risk classification showed that more than 88% of buildings were classified, and 12% of buildings that could not be classified due to excessive scatter of parameter values were classified using a very small confidence radius. The results indicate the ability of the proposed method to classify objects with the least similarity and number of effective parameters in classification.

Keywords: Hypergraph, Fuzzy membership degree, Fuzzy hypergraph, classification, Granular computing, Urban vulnerability
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1 Introduction

One of the main elements for effective response, before natural disasters, is the assessment of urban vulnerability. Urban vulnerability assessment requires access to information and advanced methods of information analysis. Past experiences have shown that earthquakes, as a complex natural phenomenon, are not predictable. However, their effects can be measured by different mathematical patterns and techniques. Many parameters contribute to the damage caused by an earthquake. Parameters which, in addition to uncertainties, are also conflicting in estimating the damage. The multiplicity of parameters, uncertainty and conflict and the placement of the urban zone faces

*Corresponding author
Email addresses: zarandi@iauk.ac.ir (Abdolreza Zarandi Baghini), babaei@iauk.ac.ir (Hojjat Babaei), tabatabaei@iauk.ac.ir (Ramin Tabatabaei Mirhosseini), ltorkzadeh@yahoo.com (Lida Torkzadeh Tabrizi)

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many challenges in a risk class. Finally, seismic risk assessment is affected by the challenges of classification. So far, different approaches have been proposed to assess the urban vulnerability by focusing on the control of uncertainties and opposite conditions of the parameters.

In spite of the development of a vulnerability structure based on the GIS software, the obtained results are not above an average estimate of physical and life-threatening losses. This limitation implies discarding the sensitivity of infrastructure systems. The lack of attention to infrastructure systems leads to a reduction in reliability and cascading failure [15]. Most of the conventional methods of determining vulnerability risk are merely a specific allocation of risk-taking [12, 14, 18]. Some studies describe seismic risk using the weight scoring method and cumulative measurement (total weight) of individual events in each criterion [5, 8]. A method called Diagram Index (DI) is proposed to determine the interdependence between several properties, based on graph theory. In the DI index, the performance estimation of the fundamental infrastructure of urban systems is investigated, starting with physical damage and then the effects that each node has on the performance of other nodes [15]. Previous research has shown that in principle the resonance descriptors are exacerbating the physical hazards. This increases concerns about social, economic and non-flexibility, or lack of ability to deal with physical hazards [9, 25, 26].

In some methods, by identifying the different structures, the vulnerability of buildings is defined on the basis of being in an urban area. Then the relationship between intensity and damage is obtained by means of the harm matrix [29, 39]. The vulnerability functions and correlation coefficients can also be used according to expected seismic input intensity [2, 11, 22]. Recently, a model of vulnerability information systems is proposed as "Common Relevant Operational Picture" (CROP). In order to minimize the risk of relief in the crop model, the possibility of coordination between regions is enabled by allocating small subset of a common database for each area [34]. Granular Computing (GrC) is a novel computational paradigm for processing complex information entities called "Information Granules", which is used to explore hidden, useful and unknown communication in the dataset [13, 19]. In general, the data granules are a set of entities that are usually placed in a numeric level due to the similarity, functional closeness, consistency or similar cases, as a whole unit [32]. In addition to being able to be an independent seed, granules can be a member of another granules. Then we will have a family of the granules, which are considered as a whole. The granules on one level, though they may be relatively independent, are somehow connected to each other in a certain degree and generally exhibit a particular structure, which is the inner structure of granules. At present, granules are more than a coherent set of methods or principles and an approach for data analysis at different levels in order to detect data, is recommend. As mentioned above, granule computing is not a process or algorithm. There is no specific method which is called "Granule Computing". Granule Computing is used to study the data, which recognizes how different laws in the data can appear at different levels of granularity. With the development of granules, different models of granular computing are proposed and investigated. Among the models are fuzzy sets, rugged sets, and outer space as the main computational models [40]. For example, a granular computing-based model is presented based on data tables, and in this model, different methods for construction, interpretation and representation of the granules were investigated [41]. Also, some other concepts of granular computing are presented based on fuzzy set [31]. Over the past half century, graph theory as a classical mathematical tool, has been applied to many applications in different fields such as economics, computer science, geometry, number theory, optimization, topology and transportation. The graph theory is currently used in granular computing and the representation of seed structures for computer problems research. For example, the adjacency matrix of a simple undirected graph (constrained) is interpreted as a Boolean information table [10].

In graph theory, however, some characteristics of the case study may have some sort of uncertainty. For example, in the earthquake and seismic vulnerability assessment, the effect and distance of buildings from blast centers (gas stations and gas pumps) are not precise. In such cases, fuzzy sets and fuzzy logic are used to deal with uncertainty management. To solve new combinatorial problems, it was necessary to develop the concept of graph theory. In 1960, hypergraph theory was introduced as a generalization of graphs, and one of its primary objectives was to generalize some classical results of graph theory [6]. The hypergraph theory offers a more powerful tool to analyze and solve real-world problems in the modeling of complex systems. Graphs have the ability to communicate with a maximum of two nodes. It is virtually impossible to use graphs for analysis and discussion of more than two node. One of the motivations for introducing hypergraph is for solving this type of graph problems. Hence, the hypergraph can analyze the connection between members of the finite set and even the infinite nodes. Unlike ordinary graphs, each edge in the hypergraph can consist of an arbitrary number of vertices and each edge in this hypergraph is called hyperedge. In the hypergraph, the vertices can be described in a n-ary relation. Some researchers developed a hypergraph segmentation algorithm based on multi-level patterns, in which a sequence of successive hypergraphs are constructed [20]. It shows that a simple granule of the universe can be characterized by a fuzzy equivalence relation or a given partition. For the first time, Zadeh, introduced a special type of subset X and named it a fuzzy subset, by grading the inaccurate degree of belonging of the members of a subset of the reference set X [44]. By combining the benefits of the two fuzzy concepts and hypergraph, fuzzy hypergraph is presented as a modeling structure with fuzzy relation and multiple relationships between objects [38]. Previous studies have shown that fuzzy graphs are expressed on the basis of fuzzy relations [21, 43]. In the studies, some operations have been defined over fuzzy graphs [27, 28]. Fuzzy graph theory has many applications in various fields, including computer networks, data mining, clustering, differentiation, and recording. Analog fuzzy logic has been discussed in many of the theoretical concepts of graph theory and in the case of fuzzy graphs is particularly interesting for researchers [3, 33]. There are also other definitions such as the supplement of fuzzy graph and the characteristics of fuzzy trees [36, 37]. Recently, the concept of M-fuzzy based graphs and some of its properties have been proposed [4].

Finally, an interesting concept called fuzzy hypergraph based on the original idea was generalized and redefined by Kaufman [7]. In addition, the concept of fuzzy hypergraphs has been considered and the concept of interval values of fuzzy hypergraphs has been introduced [17, 24]. In recent years, the concept of intuitive fuzzy hypergraphs as well as bipolar fuzzy hypergraphs and some of their basic features have been expressed [23, 30]. Classification is easily applicable when faced with a small set of attributes. For example, in a collection of white, black, red and beige cars, we can easily put them in 4 classes. But if the same set includes other features such as year of manufacture, manufacturer, engine size, price, etc., classification becomes a bit complicated. Now suppose you want to classify in a collection of thousands of records and hundreds of unordered attributes with conflicting effects. How can classes and classifying a collection of elements be defined? Increasing the accuracy and focus on some features for classification reduces the accuracy in influencing other features in defining classes and identifying the appropriate class. Considering the hierarchical effect of weighted and heterogeneous parameters in the classification of elements, always reduces the classification accuracy. Highlighting the role of high-level parameters and diminishing the effect of parameters in the lower layers are the main challenges of classifying elements with equal and heterogeneous properties. In this paper, to increase the accuracy in seismic vulnerability assessment and the possibility of better and more accurate decision-making at boundary points, the concepts of fuzzy and granule hypergraphs have been used. To define urban seismic risk classes with flexible and floating boundaries and limits, a granular structure based on fuzzy hypergraph has been used. In this method, much more granules are shown. Previous research has been done to investigate how granules are made in different fields, using hypergraphs [35, 42]. By examining and evaluating each building in the mentioned granular structure, the risk level of the building is calculated and determined. This approach makes it possible to make better decisions at the border points of the classes. In this process, in addition to considering the simultaneous effect of all effective parameters in the classification, the degree of interaction of the parameters with each other is also considered. For verification, the proposed method was implemented in the classification of iris flowers. Fisher has intuitively classified irises [16]. The conformity of the classification with Fisher's experimental and intuitive methods indicates the efficiency of the presented research method. This method is then used to assess seismic risk.

2 General

In this section, the concepts of granules, granule computing, fuzzy, hypergraph and fuzzy hypergraph are defined.

2.1 Granule, granule computing

Granular computing is a well-known method in computation and applied mathematics for data processing. Information is divided into granules through the granulation process [41]. Information granule is a set of entities that are obtained from the numerical analysis of the properties of the entity (data) and then it is classified at a certain level based on similarity, functional proximity, physical proximity, or indistinguishability. Granular data is the lowest level of data that is targeted into a set. This refers to the degree of detail and the size of the class to which the data is divided. For example, dividing people's names into either a single field, consisting of both names and surnames together, or separate fields. The more segmented and specific the data, the more granularity is considered. Putting the data at its proper level depends on the level of knowledge and awareness of the impact of data details on granulation. Classes are described according to whether the grains are large or detailed. The structure of these levels of granulation is determined by the laws of nature or human cognition.

2.2 Fuzzy sets

The key to understanding the difference between a fuzzy set and a classic set is the concept of fuzzy membership degree. In classical logic, the concepts of belonging or non-belonging of a member to a set, are quite definite and precise. So, an object is or is not a member of a set and hence the membership function can only have two values, 0 and 1. In 1965, Lotfi A. Zadeh, introduced a special type of subset X and called it a fuzzy subset by grading the inaccurate degree of belonging members of a subset from a reference set X [45].

Definition 2.1. [45] A "fuzzy subset A" of reference set X or a "fuzzy set A" in reference set X, characterized with membership function $\mu_A : X \to [0,1]$ and the membership of each member is graded. The meaning of $\mu_A(x) \in [0,1]$ degree of membership $x \in X$ in "fuzzy subset A" or "fuzzy set A".

The set $\sup p\mu_A = \{x \in X | \mu_A(x) > 0\}$ is called the support μ_A, and $h(\mu_A)$ is the height display μ such that $h(\mu_A) = \max\{\mu_A(x) | x \in X\}$.

If u and v are fuzzy sets, then $u \wedge v = \min\{u, v\}$ and $u \vee v = \max\{u, v\}$. We write $u \leq v$ if $u(x) \leq v(x)$ for all $x \in X$ also $u < v$ if $u \leq v$ and $u(x) < v(x)$ for some $x \in X$.

2.3 Hypergraph and fuzzy hypergraph

In 1960, the theory of hypergraphs was introduced as a generalization of graphs. Unlike normal graphs, each edge in a hypergraph can contain an arbitrary number of vertices [6].

Definition 2.2. [1] The pair $H = (X, E)$ is called a hypergraph on a finite set X if E is a finite family of non-empty subsets X such that:

1. $\forall\ E_i \in E,\ E_i \neq \emptyset \quad i = 1, ..., m$
2. $\bigcup_{i=1}^{m} E_i = X$

Each $E_i \in E$ is called an edge, and each edge contains a number of elements called vertices. Sometimes a set of vertices is represented by $X(H)$ and the edge set by $E(H)$.

A hypergraph is called a simple hypergraph if E does not contain duplicate edges and $\forall\ A, B \in E, A \subset B$ then $A = B$.

Example 2.3. Suppose $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $E = \{E_1, E_2, E_3, E_4\}$ such that $E_1 = \{1, 3, 8, 10\}, E_2 = \{1, 4, 5\}, E_3 = \{5, 6, 8, 9\}, E_4 = \{2, 7, 8\}$.

By Definition 2.2, $H(X, E)$ is a hypergraph on the set X. Each of the subsets E_1, E_2, E_3, E_4 edges and their members are the vertices of the hypergraph.

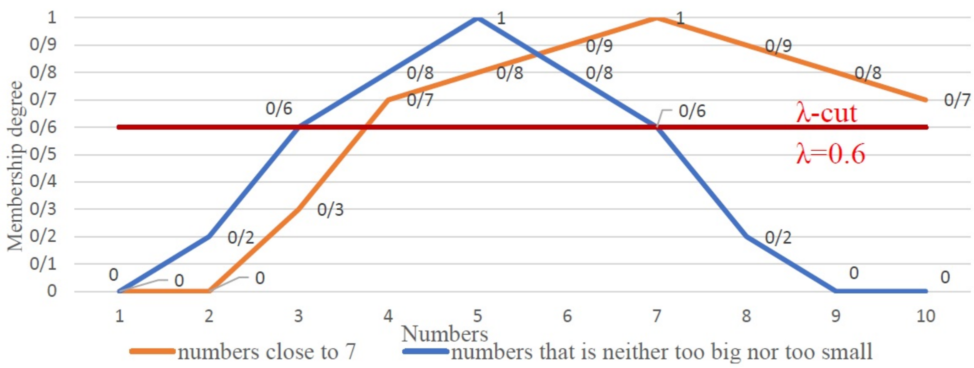

Figure 1: Diagram of two fuzzy relations defined on the X set

Definition 2.4. [17] Let X is a finite set and εa finite family of nontrivial fuzzy subsets on the set$X$ such that $X = \bigcup\{\sup p\mu | \mu \in \varepsilon\}$, then the pair $H = (X, \varepsilon)$ is a fuzzy hypergraph on the set X, and X the set of vertices and ε is called the set of fuzzy edges (the family of edges fuzzy) of H.

Definition 2.5. [17] A fuzzy hypergraph $H = (X, \varepsilon)$ is called simple if ε does not contain duplicate fuzzy edges and whenever $u, v \in \varepsilon, u \leq v$ then $u = v$.

Definition 2.6. [17] Suppose $H = (X, \varepsilon)$ is a fuzzy hypergraph and $\mu \in \varepsilon$ and $0 \leq \lambda \leq 1$. $\lambda - cut$ of μ is defined as follows $\mu_\lambda = \{x \in X | \mu(x) \geq \lambda\}$.

Also, assuming that $E_\lambda = \{\mu_\lambda | \mu \in \varepsilon\}$ and $X_\lambda = \bigcup\{\mu_\lambda | \mu \in \varepsilon\}$. If $E_\lambda \neq \emptyset$ then the hypergraph $H_\lambda = (X_\lambda, E_\lambda)$ is called a λ-level hypergraph of H.

Example 2.7. By defining a fuzzy relation, we define a fuzzy hypergraph on X.

- Relation μ_A a set of numbers close to 7

Table 1:

x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
μ_A	0	0	0.3	0.7	0.8	0.9	1	0.9	0.8	0.7

Then $\sup p\mu_A = \{3, 4, 5, 6, 7, 9, 10\}$. If $\lambda = 0.6$, then $\mu_{A_{0.6}} = \{4, 5, 6, 7, 8, 9, 10\}$.

- Relation μ_B a set of numbers that is neither too big nor too small.

Table 2:

x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
μ_B	0	0.2	0.6	0.8	1	0.8	0.6	0.2	0	0

Then $\sup p\mu_B = \{2, 3, 4, 5, 6, 7, 8, 9\}$. If $\lambda = 0.6$, then $\mu_{B_{0.6}} = \{4, 5, 6, 7\}$.

If $\varepsilon = \{\mu_A, \mu_B\}$, then $X \neq \sup p\mu_A \bigcup \sup p\mu_B = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$, by Definition 2.4 the pair $H = (X, \varepsilon)$ isn't a fuzzy hypergraph on the X. Figure 1 shows the fuzzy relations μ_A, μ_B with sets $\mu_{A_{0.6}}, \mu_{B_{0.6}}$.

Now we define under Relation μ_C on X set. μ_C a set of numbers close to 0.

Table 3:

x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
μ_C	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.1	0	0

Then $\sup p\mu_B = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

If $E = \{\mu_A, \mu_C\}$, then $X = \sup p\mu_A \bigcup \sup p\mu_C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, by Definition 2.4 the pair $H = (X, E)$ is a fuzzy hypergraph on the X set.

3 Methodology

In hierarchical structures, not all the properties that affect classification are considered at the same time. This problem-solving method actually simplifies the problem by ignoring some features, and then the problem-solving pattern is gradually modified at different levels. Finally, the main problem, which is usually complex, is solved. The proposed method in this paper provides the possibility of classification, taking into account all the effective classification features simultaneously. Defining the structure of classes is one of the main challenges in classifying elements with the simultaneous impact of several properties. In this method, granular structure based on fuzzy hypergraphs is defined to create the class structure. The proposed granular structure is very flexible based on the classes defined in each of the properties. This flexible structure, in addition to increasing the accuracy of decision-making at border points, also has the ability to classify elements with high dispersion of properties and the least similarity. The definition of classes in set X derives from the concept of these classes in each set of properties of the elements of X set. Finally, the proposed methods implemented in classifying 50,000 buildings to determine the seismic risk level by considering the simultaneous impact of 10 properties.

Definition 3.1. Suppose $X = \{(x_1, x_2, ..., x_n) | x_i \in X_i,\ i = 1, 2, ..., n\}$ is a finite set. So that each element of X has n properties. X_i are sets of property values.

let $K_1, K_2, ..., K_m$ are m title classes on set X. Note that at this stage there is no precise definition for the classes on the set X.

Each E_{ij} class is the concept of K_j class on the X_j property for $i = 1, 2, ..., n,\ j = 1, 2, ..., m$. We define the E matrix:

$$E = \begin{pmatrix} E_{11} & \cdots & E_{1m} \\ \vdots & \ddots & \vdots \\ E_{n1} & \cdots & E_{nm} \end{pmatrix}$$

The elements of E matrix in each row are the classes defined on properties value set. For example, elements $E_{11}, E_{12}, ..., E_{1m}$ are m classes defined on X_1 set.

Lemma 3.2. $\bigcup_{j=1}^{m} E_{ij} = X_i$ where E_{ij} "are clases on sets" $X_i,\ i = 1, 2, .., n$ (sets located in a row).

Proof . From Definition 3.1, $E_{ij} \subseteq X_i$ for any $i = 1, 2, ..., n,\ j = 1, 2, ..., m$, thus

$$\bigcup_{j=1}^{m} E_{ij} \subseteq X_i \tag{3.1}$$

Conversely, since for any $i = 1, 2, ..., n,\ j = 1, 2, ..., m$, E_{ij} sets are classes defined on X_i sets so:

$$\forall\ a \in X_i \exists\ t \in \{1, 2, ..., m\} \text{ such that } a \in E_{it},\ \text{ then } X_i \subseteq \bigcup_{j=1}^{m} E_{ij} \tag{3.2}$$

Combining (3.1), (3.2) we have $\bigcup_{j=1}^{m} E_{ij} = X_i$ for $i = 1, 2, ..., n$. □

Lemma 3.3. The pair $H_i = (X_i, E_i)$ is hypergraph on X_i sets, when $E_i = \{E_{ij} | E_{ij} \neq \emptyset, j = 1, 2, .., m\}$ and $i = 1, 2, .., n$.

Proof . From Lemma 3.2, $\bigcup_{j=1}^{m} E_{ij} = X_i$ for $i = 1, 2, .., n$, by Definition 2.2 the pair $H_i = (X_i, E_i)$ is hypergraph on X_i. □

Definition 3.4. Corresponding to matrix E in the Definition 3.1, we define θ_x matrix for all $x \in X$, as follows:

$$\theta_x = \begin{pmatrix} \theta_{11}(x_1) & \cdots & \theta_{1m}(x_1) \\ \vdots & \ddots & \vdots \\ \theta_{n1}(x_n) & \cdots & \theta_{nm}(x_n) \end{pmatrix}$$

such that:

$$\theta_{ij}(x_i) = \begin{cases} \mu_{ij}(x_i), & \text{where } \ x_i \in E_{ij} \\ -\mu^c_{ij}(x_i), & \text{where } \ x_i \notin E_{ij} \end{cases}$$

where:

$$\mu_{ij} : E_{ij} \to [0, 1],\ \mu^c_{ij} : E^c_{ij} \to [0, 1],\ E^c_{ij} = X_i - E_{ij} \text{ for } i = 1, 2, ..., n,\ j = 1, 2, ..., m.$$

For each $e \in E_{ij}$, value of the $\mu_{ij}(e)$ function is called e-membership degree in the E_{ij} set, also the value of the $\mu^c_{ij}(e)$ function is called e-membership degree in the E^c_{ij} set. The elements of the matrix are functions.

Definition 3.5. The jth score of the element $x \in X = \{(x_1, x_2, x_3, ..., x_n) | x_i \in X_i, i = 1, 2, ..., n\}$ is a special number, that can be calculated from the matrix θ_x as follows:

$$score_j(x) = \sum_{i=1}^{n} \theta_{ij}(x) \text{ where } \theta_{ij}(x) \in \theta_x \text{ for } j = 1, 2, ..., m.$$

For example, the sum of the elements of the first column in the matrix θ_x is $score_1(x)$.

Definition 3.6. Let $x \in X = \{(x_1, x_2, x_3, ..., x_n) | x_i \in X_i, i = 1, 2, ..., n\}$, from Definition 3.5 we define:

$$L_j = \{x \in X | score_j(x) \geq 0\} \text{ where } j = 1, 2, ..., m$$

and

$$L_{m+1} = \{x \in X | score_j(x) < 0\} \text{ for all } j = 1, 2, ..., m.$$

Each of sets L_j are granules. By Definition 2.2 $H = (X, L)$ is a hypergraph on X. The proposed granular structure is shown in Figure 2.

The elements of $L_1, L_2, ..., L_m$ granules have at least a $score_j(x) \geq 0$ value (see Figure 2). Zone A contains an element of set X that has no value $score_j(x) \geq 0$ and only has $score_j(x) < 0$ values (set L_{m+1}). In other words, they are not in any granule (class). Zone B is composed of elements that have a $score_j(x) \geq 0$ in all granules (classes).

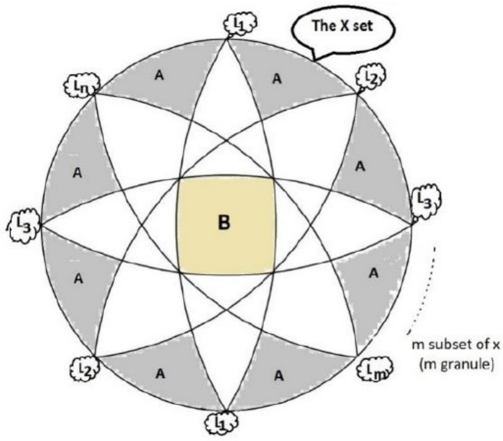

Figure 2: The proposed granular structure

Definition 3.7. Let $x \in X = \{(x_1, x_2, x_3, ..., x_n) | x_i \in X_i, i = 1, 2, ..., n\}$ based on the $score_j(x)$ for $j = 1, 2, ..., m$ defined several DoM_{score_j} values (degree of membership) or $DoNM_{score_j}$ values (non-membership) as follows:

if $score_j(x) \geq 0$, then $DoM_{score_j}(x) = \mu(score_j(x))$ where $\mu : L_j \to [0, 1]$ for $j = 1, 2, ..., m$.

if $score_j(x) < 0$, then $DoNM_{score_j}(x) = \eta(score_j(x))$ where $\eta : L_{m+1} \to [0, 1]$ for $j = 1, 2, ..., m$.

Definition 3.8. Let $x \in X = \{(x_1, x_2, x_3, ..., x_n) | x_i \in X_i, i = 1, 2, ..., n\}$ from Definition 3.7 we define:

$$DoM(x) = \max\{DoM_{score_j}(x) \text{ for } j = 1, 2, ..., m\}$$

$$K_j = \{x | DoM(x) = DoM_{score_j}(x)\} \text{ for } j = 1, 2, ..., m$$

If $DoM(x)$ doesn't exists, define $DoNM(x)$ as follows:

$$DoNM(x) = \min\{DoNM_{score_j}(x) \text{ for } j = 1, 2, ..., m\}$$

$$K_{m+1} = \{x | \exists DoNM(x)\}.$$

By Definition 3.7, a fuzzy hypergraph on the set X is defined. The elements of the set X that have the highest $DoM_{score_j}(x)$ value form the subset, which in turn defines K_j class in Definition 3.1.

This way of constructing subsets defines the granular structure, which is considered as a class. The other values of $DoM_{score_j}(x)$ indicate the belonging of element $x \in X$ to other classes, with different degrees of membership. The absence of a value of $DoM_{score_j}(x)$ for $x \in X$ indicates that the property values are too scattered. Such elements simply have different degrees of $DoNM_{score_j}(x)$ non-belonging to classes. The set of all these elements are considered a subset of X as a class K_{m+1} (the smallest degree of non-belonging $DoNM_{score_j}(x)$).

Lemma 3.9. $X = \bigcup_{j=1}^{m+1} K_j$ where K_j are classes on set X.

Proof . Let $x \in \bigcup_{j=1}^{m+1} K_j \Rightarrow \exists l \leq t \leq m+1$ such that

$$x \in K_t \subset X \Rightarrow \bigcup_{j=1}^{m+1} K_j \subseteq X \tag{3.3}$$

Conversely,

Case 1.

For any $x \in X$ there exists $t \in \{1, 2, ..., m\}$ such that $score_t(x) \geq 0$. From Definition 3.8 $DoM(x)$ value is defined and $x \in K_t$, then

$$x \in \bigcup_{j=1}^{m} K_j \Rightarrow X \subseteq \bigcup_{j=1}^{m} K_j \subseteq \bigcup_{j=1}^{m+1} K_j. \tag{3.4}$$

Case 2.

$score_t(x) < 0$ for any $t \in \{1, 2, ..., m\}$ by Definition 3.6 $x \in L_{m+1}$. From Definition 3.8 $DoNM(x)$ value is defined and $x \in K_{m+1}$, then

$$x \in \bigcup_{j=1}^{m+1} K_j \Rightarrow X \subseteq \bigcup_{j=1}^{m+1} K_j \tag{3.5}$$

By (3.3), (3.4), (3.5) we have $X = \bigcup_{j=1}^{m+1} K_j$. $\square$

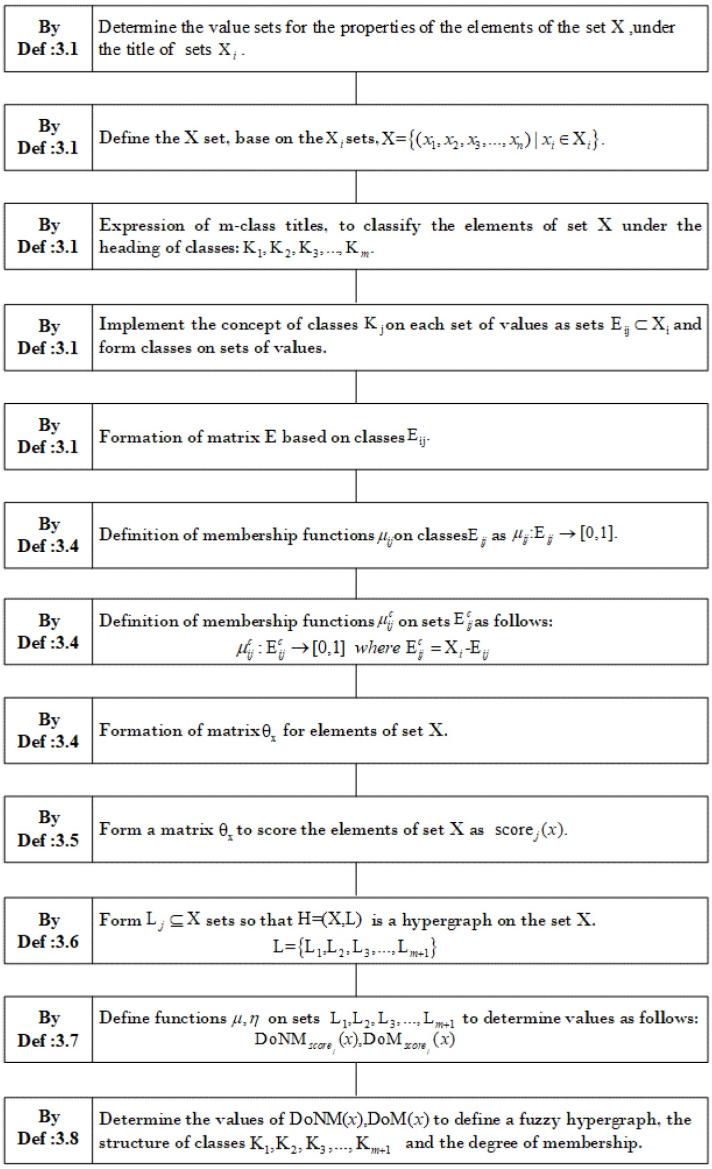

Figure 3: Diagram of the proposed method

Lemma 3.10. $X = (X, E)$ is fuzzy hypergraph on X, where $E = \{K_j | K_j \neq \emptyset,\ j = 1, 2, ..., m+1\}$.

Proof . By Lemma 3.9 $X = \bigcup_{j=1}^{m+1} K_j$, from Definition 2.4 pair $X = (X, E)$ is a fuzzy hypergraph on the set X.

Thus, the flexural granular structure is defined based on fuzzy hypergraphs that have the ability to classify data with the least similarity. An example is given below which uses the proposed method to assess the seismic vulnerability of buildings. The steps of the proposed method are shown in Figure 3. In this diagram $i = 1, 2, ..., n,\ j = 1, 2, ..., m$. □

3.1 Validation

In 1936, Fischer, field-examined 150 irises [16]. As shown in Table 4, he used his intuitive experience to classify these flowers into three categories based on four characteristics (see Table 5). For validation, iris flowers were examined and classified by Fisher (data and results of the study are available) and then classified again using the proposed method.

Table 4: Types of irises

Row	Flower Name
1	Setosa
2	Virginica
3	Versicolor

Table 5: Characteristics of flowers

Row	Characteristics	Size (cm)
1	Sepal length	$\{4.3, 4.4, ..., 7.9\}$
2	Sepal width	$\{2, 2.1, ..., 4.4\}$
3	Petal length	$\{1, 1.1, ..., 6.9\}$
4	Petal Width	$\{0.1, 0.2, ..., 2.5\}$

The collection of flowers is defined and introduced as follows:

$$X = \{(x_1, x_2, x_3, x_4) | x_j \in X_j,\ j = 1, ..., 4\}$$

such that $X_1 = \{4.3, 4.4, ..., 7.9\},\ X_2 = \{2, 2.1, ..., 4.4\},\ X_3 = \{1, 1.1, ..., 6.9\},\ X_4 = \{0.1, 0.2, ..., 2.5\}$. In the next step, we implement the concept of the classes in Table 4 for each of the properties of Table 5 as sets X_i. With this implementation, classes (sets E_{ij}) are defined in each property. Now, according to the definition of 3.1, we form a matrix E:

$$E = \begin{pmatrix} \{4.3, 4.4, ..., 5.8\} & \{4.9, 5.0, ..., 7.0\} & \{4.9, 5.0, ..., 7.9\} \\ \{2.0, 2.1, ..., 3.4\} & \{2.2, 2.3, ..., 3.8\} & \{2.3, 2.4, ..., 4.4\} \\ \{1.0, 1.1, ..., 1.9\} & \{3.0, 3.1, ..., 5.1\} & \{4.5, 4.6, ..., 6.9\} \\ \{0.1, 0.2, ..., 0.6\} & \{1.0, 1.1, ..., 1.8\} & \{1.4, 1.5, ..., 2.5\} \end{pmatrix}$$

3.2 Description

The elements of the first row of the matrix, the classes defined on the set of values of the sepals, are one of the four characteristics of the lily (set X_1). This classification is defined based on Fisher's intuitive observations (the other lines are the same). These classes define $H_1 = (X_1, E_1),\ H_2 = (X_2, E_2),\ H_3 = (X_3, E_3),\ H_4 = (X_4, E_4)$, hypergraphs on four sets of Fisher values (Lemma 3.3), respectively. The sepal length property is classified $E_{11},\ E_{12},\ E_{13}$ such that:

$$E_{11} = \{4.3, 4.4, ..., 5.8\},\ E_{12} = \{4.9, 5.0, ..., 7.0\},\ E_{13} = \{4.9, 5.0, ..., 7.9\}$$

Clearly, $E_{1j} \neq \emptyset$, for $j = 1, 2, 3$ and $E_1 \bigcup_{j=1}^{3} E_{1j}$ (Lemma 3.3).

Each class represents a type of flower.

Each of the classes in Table 5 represents a type of iris. The proposed method for determining the structure and members of each type of iris defines a fuzzy hypergraph on the X set. Based on Definition 3.4, we form a matrix of θ_x for each iris flower. For this purpose, we define fuzzy membership functions $\mu_{ij}(x),\ \mu_{ij}^c(x)$ on sets $E_{ij}^c,\ E_{ij}$.

$$\mu_{ij}^c(x) : E_{ij}^c \to [0, 1] \text{ such that } \mu_{ij}^c(x) = 0\ \mu_{ij}(x) : E_{ij} \to [0, 1]$$

$$\mu_{ij}(x) = \begin{cases} \frac{(x - mode_{E_{ij}})}{mode_{E_{ij}} - \min_{E_{ij}}} + 1, & \text{where } x \leq mode_{E_{ij}}, mode_{E_{ij}} \neq \min_{E_{ij}} \\ \frac{(x - mode_{E_{ij}})}{mode_{E_{ij}} - \max_{E_{ij}}} + 1, & \text{where } x > mode_{E_{ij}}, mode_{E_{ij}} \neq \max_{E_{ij}} \\ 0.00001 \times (x - mode_{E_{ij}}) + 1, & \text{where } mode_{E_{ij}} = \min_{E_{ij}} \text{ or } mode_{E_{ij}} = \max_{E_{ij}} \end{cases} \quad i = 1,2,3,4 \quad j = 1,2,3$$

It is clear that $mode_{E_{ij}}$ is a member of E_{ij} with the most repetition.

Now, using Definition 3.5, the values of $score_j(x) \geq 0$ for the elements of set X are calculated. To better clarify the content, a sample of iris, which is shown with the symbol x, has been placed in the process of the proposed method. Consider the sample iris with specifications $x = (5.1, 3.5, 1.4, 0.2)$. We form matrix θ_x based on membership functions $\mu_{ij}(x)$ and $\mu^c_{ij}(x)$:

$$\theta_x = \begin{pmatrix} 0.875 & 0.95 & 0.967 \\ 0.643 & 0 & 0.375 \\ 0.8 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Then, by Defining 3.5, the values of $score_j(x)$ are calculated:

$$score_1(x) = 3.318$$
$$score_2(x) = 0.95$$
$$score_3(x) = 1.341$$

Now by Defining 3.7 and using the membership function μ, the values of DoM_{score_j} for $j = 1,2,3$ are calculated.

$$\mu : [0,4] \to [0,1] \text{ such that } \mu(x) = x/4$$

$$DoM_{score_1} = 0.829464286$$
$$DoM_{score_2} = 0.2375$$
$$DoM_{score_3} = 0.33512931$$

According to the Definition 3.8, the sample iris belongs to class K_1 (see Table 6) with a membership degree of 0.829464286. In Table 7, the performed classification and Fisher's experimental classification are given and compared for some samples of irises.

In samples 1-2-3-54-56-59-60-61, the experimental results are consistent with the classification of the proposed method. This agreement is more than 90% in 150 samples. In sample 58, the experimental results do not match the classification of the proposed method. But the degree of membership to each class is very low, and this is due to the over-dispersion of the sample property values. The experimental results do not match to the classification of the proposed method for No. 53.

Table 6: Flower Class

Row	**Flower Name**	**Class**
1	Setosa	K_1
2	Virginica	K_2
3	Versicolor	K_3

Table 7: Flower Sample

Sample No	Flower characteristics				Flower type based on experimental method	Flower type based on proposed method	Degree of belonging to the class	Class membership rate (DoM_{score_j})		
	Sepal length	Sepal width	Petal length	Petal Width				Iris-setosa	Iris-virginica	Iris-versicolor
1	5.1	3.5	1.4	0.2	Iris-setosa	Iris-setosa	0.829464286	0.829464286	0.33512931	0.2375
2	4.9	3	1.4	0.2	Iris-setosa	Iris-setosa	0.914285714	0.914285714	0.25	0.25
3	4.7	3.2	1.3	0.2	Iris-setosa	Iris-setosa	0.757142857	0.757142857	0.1875	0.125
53	6.9	3.1	4.9	1.5	Iris-versicolor	Iris-virginica	0.76329023	0.232142857	0.76329023	0.433333333
54	5.5	2.3	4	1.3	Iris-versicolor	Iris-versicolor	0.679166667	0.09375	0.238146552	0.679166667

56	5.7	2.8	4.5	1.3	Iris-versicolor	Iris-versicolor	0.8625	0.209821429	0.627155172	0.8625
58	4.9	2.4	3.3	1	Iris-versicolor	Iris-setosa	0.25	0.25	0.0625	0
59	6.6	2.9	4.6	1.3	Iris-versicolor	Iris-versicolor	0.733333333	0.214285714	0.570402299	0.733333333
60	5.2	2.7	3.9	1.4	Iris-versicolor	Iris-versicolor	0.75	0.330357143	0.389008621	0.75
61	5	2	3.5	1	Iris-versicolor	Iris-versicolor	0.333333333	0.25	0.25	0.333333333

In this method, in addition to the possibility of classifying samples with any number of characteristics and any amount of information scatter, it is also possible to determine the degree of membership to the class. There is also a lot of flexibility in this method by selecting membership functions according to the classification structure. The results of the proposed method are shown in the following diagrams.

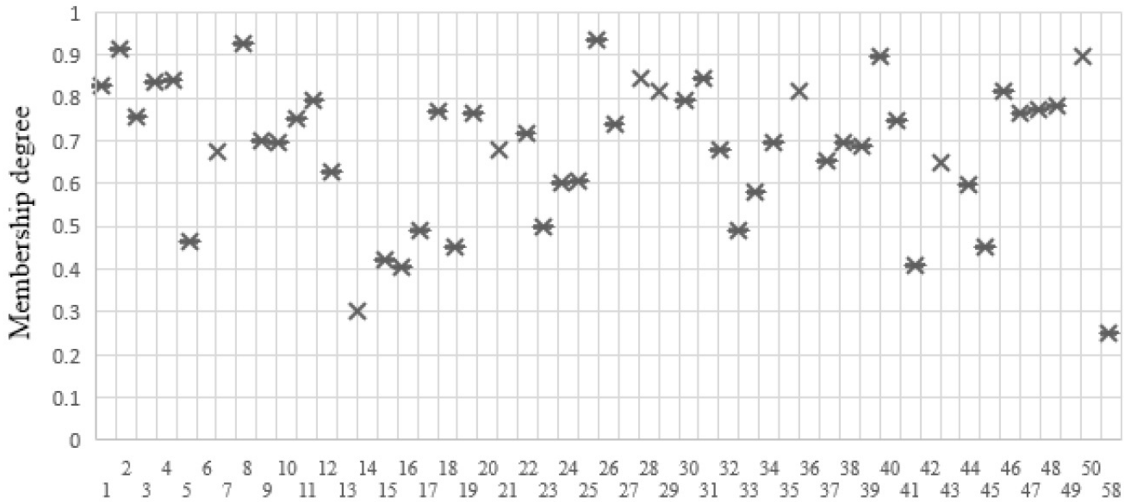

Figure 4: Setosa membership degree chart

Figure 4 shows that 95% of the elements with a membership degree of more than 0.3 are in the Iris-setosa flower class, and this shows the accuracy of the method.

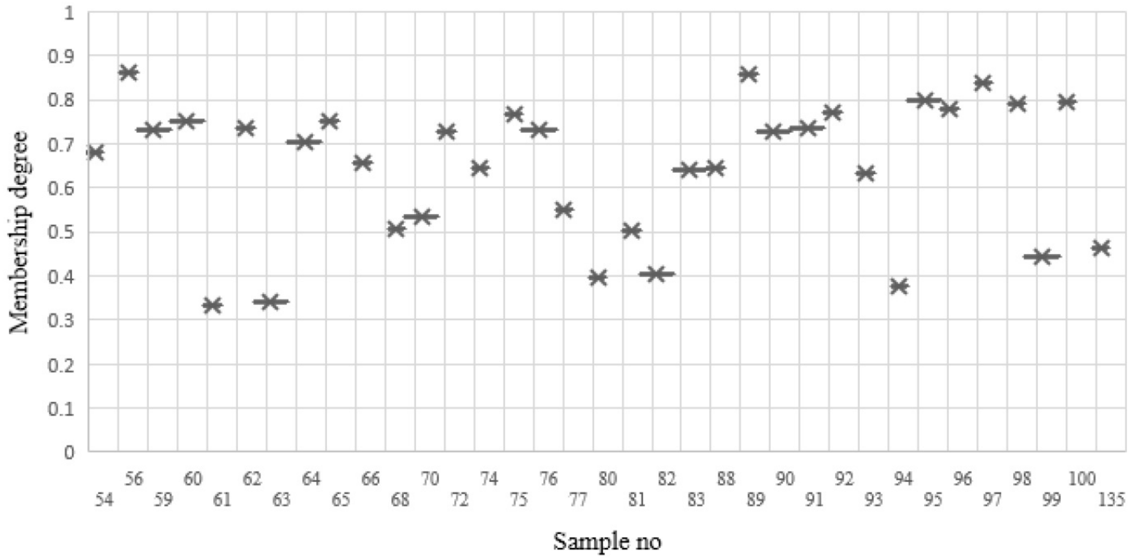

Figure 5: Versicolor membership degree chart

Figure 5 also shows that the membership of elements in the Iris-versicolor flower class is more than 0.3.

Figure 6 shows that a number of elements with a membership of less than 0.2 are in the Iris-virginica class. This is due to the high dispersion of data, but the proposed method has the ability to classify, which is one of the strengths

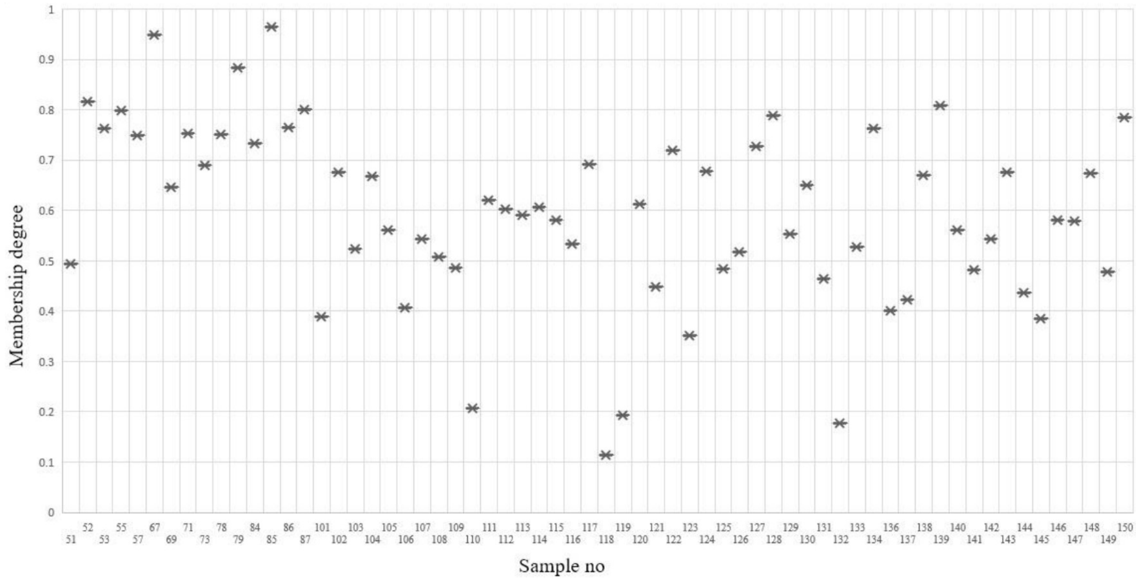

Figure 6: Virginica membership degree chart

of this method. The comparison of the classes of Table 8 shows the differences between the proposed method and the results of the field study and Fisher's intuitive experiences.

Table 8: Comparison of the classes

Sample No	Flower characteristics				Flower type based on experimental method	Flower type based on proposed method	Degree of belonging to the class	Class membership rate		
	Sepal length	Sepal width	Petal length	Petal Width				Iris-setosa	Iris-virginica	Iris-versicolor
51	7	3.2	4.7	1.4	Iris-versicolor	Iris-virginica	0.494252874	0.214285714	0.494252874	0.491666667
52	6.4	3.2	4.5	1.5	Iris-versicolor	Iris-virginica	0.816810345	0.214285714	0.816810345	0.6
53	6.9	3.1	4.9	1.5	Iris-versicolor	Iris-virginica	0.76329023	0.232142857	0.76329023	0.433333333
55	6.5	2.8	4.6	1.5	Iris-versicolor	Iris-virginica	0.797772989	0.178571429	0.797772989	0.620833333
57	6.3	3.3	4.7	1.6	Iris-versicolor	Iris-virginica	0.748347701	0.196428571	0.748347701	0.416666667
58	4.9	2.4	3.3	1	Iris-versicolor	Iris-setosa	0.25	0.25	0.0625	0
67	5.6	3	4.5	1.5	Iris-versicolor	Iris-virginica	0.948275862	0.3125	0.948275862	0.825
69	6.2	2.2	4.5	1.5	Iris-versicolor	Iris-virginica	0.646551724	0	0.646551724	0.55
71	5.9	3.2	4.8	1.8	Iris-versicolor	Iris-virginica	0.753663793	0.214285714	0.753663793	0.3875
73	6.3	2.5	4.9	1.5	Iris-versicolor	Iris-virginica	0.690014368	0.071428571	0.690014368	0.445833333
78	6.7	3	5	1.7	Iris-versicolor	Iris-virginica	0.751364943	0.25	0.751364943	0.379166667
79	6	2.9	4.5	1.5	Iris-versicolor	Iris-virginica	0.882543103	0.214285714	0.882543103	0.75
84	6	2.7	5.1	1.6	Iris-versicolor	Iris-virginica	0.732543103	0.142857143	0.732543103	0.4
85	5.4	3	4.5	1.5	Iris-versicolor	Iris-virginica	0.965517241	0.375	0.965517241	0.85
86	6	3.4	4.5	1.6	Iris-versicolor	Iris-virginica	0.763793103	0.178571429	0.763793103	0.475
87	6.7	3.1	4.7	1.5	Iris-versicolor	Iris-virginica	0.801364943	0.232142857	0.801364943	0.541666667
135	6.1	2.6	5.6	1.4	Iris-versicolor	Iris-versicolor	0.4625	0.107142857	0.41558908	0.4625

Table 9: Title of risk

Row	Risk level	Class
1	Low-risk	K_1
2	Moderate-risk	K_2
3	High-risk	K_3
4	Very High-risk	K_4

4 Case study

In this example, the proposed method is used to assess the seismic risk of 50,000 buildings. The classification of buildings is done in four classes in Table 9, taking into account the simultaneous impact of ten building Characteristics of Table 10.

Table 10: Characteristics of building

Index	Rate	Risk level in Characteristics			
		Low	Moderate	High	Very High
Danger of earth liquefaction (Slope of the earth in percentage)	$X_1 = [0 - 45]$	$E_{11} = [0 - 3.86]$	$E_{12} = [3.86 - 12.11]$	$E_{13} = [12.11 - 23.35]$	$E_{14} = [23.35 - 45]$
Earth movement (Magnitude of Mercalli earthquake)	$X_2 = [1 - 10]$	$E_{21} = [1 - 3]$	$E_{22} = [3 - 5]$	$E_{23} = [5 - 6]$	$E_{24} = [6 - 10]$
Distance of buildings from explosive stations (gas and gas stations) in meters	$X_3 = [0 - 2000]$	$E_{31} = [100 - 2000]$	$E_{32} = [50 - 100]$	$E_{33} = [100 - 2000]$	$E_{34} = [0 - 20]$
Type of materials and structures used in buildings	$X_4 = \{1, 2, ..., 8\}$	$E_{41} = \{1, 2\}$	$E_{42} = \{3, 4\}$	$E_{43} = \{5\}$	$E_{44} = \{7, 8\}$
Type of work of buildings (per person)	$X_5 = [0 - 1000]$	$E_{51} = [1 - 25]$	$E_{52} = [25 - 75]$	$E_{53} = [70 - 100]$	$E_{54} = [100 - 1000]$
Age of buildings (by year)	$X_6 = [0 - 100]$	$E_{61} = [0 - 5]$	$E_{62} = [5 - 15]$	$E_{63} = [15 - 25]$	$E_{64} = [25 - 100]$
Height of buildings (in meters)	$X_7 = [1 - 200]$	$E_{71} = [1 - 3]$	$E_{72} = [3 - 5]$	$E_{73} = [5 - 15]$	$E_{74} = [15 - 200]$
Building quality	$X_8 = \{1, 2, 3, 4\}$	$E_{81} = 1$	$E_{82} = 2$	$E_{83} = 3$	$E_{84} = 4$
Density of buildings (in percentage)	$X_9 = [0 - 100]$	$E_{91} = [0 - 25]$	$E_{92} = [25 - 50]$	$E_{93} = [50 - 75]$	$E_{94} = [75 - 100]$
Passage width (in meters)	$X_{10} = [0 - 100]$	$E_{10\ 1} = [20 - 100]$	$E_{10\ 2} = [15 - 20]$	$E_{10\ 3} = [12 - 15]$	$E_{10\ 4} = [1 - 12]$

We display the set of buildings as set X as follows:

$X = \{(x_1, x_2, x_3, ..., x_{10}) | x_i \in X_i,\ i = 1, ..., 10\}$, X_i sets are defined in Table10.

Table 11: Characteristics frequency table

Characteristics	Min	Max	Average	Standard deviation
x_1	1	44.998	3.7758	12.6818
x_2	1	10	2.3182	2.8091
x_3	0	2000	19.6833	514.5335
x_4	1	8	2.1224	2.2875
x_5	1	1000	16.2825	406.8242
x_6	0	100	5.9288	31.8835
x_7	2	200	7.2686	53.8127
x_8	1	4	1.5818	1.1188
x_9	0	100	5.9418	31.8808
x_{10}	1	100	5.9477	31.7565

The boundaries of the classes for each of the ten properties are specified in Table 10 (risk level in characteristics columns). As mentioned earlier, defining the class structure in Table 9 for buildings (set X) is one of the main challenges. For this purpose, based on the proposed method, we define a granular structure based on fuzzy hypergraphs. In the first step, based on the Definition 3.1 elements of the matrix E, which are the classes of each property, X_i is defined based on the concept of the classes in Table 9. As shown in Table 10, these classes are listed in four columns: Low, Moderate, High, Very High. To provide the best data, the frequency tables of properties (see Table 11) and the percentage of class properties (Table 12) and the composition chart (see Figure 7) of the class properties are given.

Table 12: Percentage of characteristics belonging to classes table

Characteristics Symbol	Low-risk	Moderate-risk	High-risk	Very high-risk
x_1	18.996%	43.714%	12.704%	24.586%
x_2	20.434%	20.798%	10.2%	48.568%
x_3	46.37%	14.552%	8.486%	30.592%

x_4	24.846%	25.108%	25.116%	24.93%
x_5	31.882%	15.174%	7.558%	45.386%
x_6	18.034%	24.956%	6.452%	50.558%
x_7	4.31%	8.978%	26.526%	60.186%
x_8	25.08%	24.628%	25.288%	25.004%
x_9	49.276%	16.82%	16.534%	17.37%
x_{10}	53.928%	3.328%	2.112%	40.632%

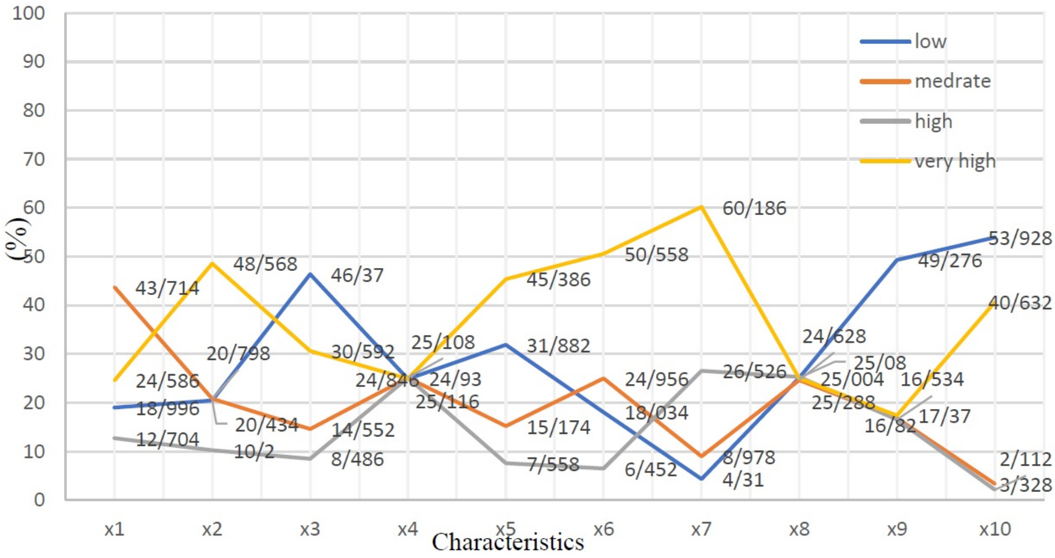

Figure 7: Percentage of characteristics belonging to classes chart

Now we define the fuzzy membership functions on the E_{ij} interval. In this example, Gaussian functions are used to determine the degree of membership of properties to each of the four classes defined on the characteristics set. Functions are defined as follows:

It is clear $E_{ij} \subset X_i,\ E_{ij}^c = X_i - E_{ij}$ for $i = 1, 2, ..., 10,\ j = 1, 2, 3, 4$.

$$\mu_{i1} : E_{i1} \to [0,1] \text{ such that } \mu_{i1}(x) = e^{\frac{\ln(0.001)\times(x-a_{i1})^2}{c_{i1}^2}} \text{ where } E_{i1} = [a_{i1}, c_{i1}],\ i = 1, 2, 3, ..., 10$$

$$\mu_{i1}^c : E_{i1}^c \to [0,1] \text{ such that } \mu_{i1}^c(x) = e^{\frac{\ln(0.001)\times(x-b_{i1})^2}{b_{i1}^2}} \text{ where } E_{i1}^c = (c_{i1}, b_{i1}],\ i = 1, 2, 3, ..., 10$$

$$\mu_{i2} : E_{i2} \to [0,1] \text{ such that } \mu_{i2}(x) = e^{\frac{\ln(0.001)\times(x-\frac{c_{i2}+d_{i2}}{2})^2}{(\frac{c_{i2}+d_{i2}}{2})^2}} \text{ where } E_{i2} = [c_{i2}, d_{i2}],\ i = 1, 2, 3, ..., 10$$

$$\mu_{i2}^c : E_{i2}^c \to [0,1] \text{ such that } \mu_{i2}^c(x) = \begin{cases} e^{\frac{\ln(0.001)\times(x+(b_{i2}-\frac{(c_{i2}+d_{i2})}{2}))^2}{b_{i2}^2}}, & \text{where } x \in [a_{i2}, c_{i2}) \\ e^{\frac{\ln(0.001)\times(x-(b_{i2}-\frac{(c_{i2}+d_{i2})}{2}))^2}{b_{i2}^2}}, & \text{where } x \in (d_{i2}, b_{i2}] \end{cases} \quad i = 1, 2, 3, ..., 10$$

$$\mu_{i3} : E_{i3} \to [0,1] \text{ such that } \mu_{i3}(x) = e^{\frac{\ln(0.001)\times(x-\frac{d_{i3}+e_{i3}}{2})^2}{(\frac{d_{i3}+e_{i3}}{2})^2}} \text{ where } E_{i3} = [d_{i3}, e_{i3}],\ i = 1, 2, 3, ..., 10$$

$$\mu_{ic}^c : E_{i3}^c \to [0,1] \text{ such that } \mu_{i3}^c(x) = \begin{cases} e^{\frac{\ln(0.001)\times(x+(b_{i3}-\frac{(e_{i3}+d_{i3})}{2}))^2}{b_{i3}^2}}, & \text{where } x \in [a_{i3}, d_{i3}) \\ e^{\frac{\ln(0.001)\times(x-(b_{i3}-\frac{(e_{i3}+d_{i3})}{2}))^2}{b_{i3}^2}}, & \text{where } x \in (e_{i3}, b_{i3}] \end{cases} \quad i = 1, 2, 3, ..., 10$$

$$\mu_{i4} : E_{i4} \to [0,1] \text{ such that } \mu_{i4}(x) = e^{\frac{\ln(0.001)\times(x-b_{i4})^2}{(b_{i4}-e_{i4})^2}} \text{ where } E_{i4} = [e_{i4}, b_{i4}],\ i = 1, 2, 3, ..., 10$$

$$\mu_{i4}^c : E_{i4}^c \to [0,1] \text{ such that } \mu_{i4}^c(x) = e^{\frac{\ln(0.001)\times(x-a_{i4})^2}{e_{i4}^2}} \text{ where } E_{i4}^c = [a_{i4}, e_{i4}),\ i = 1, 2, 3, ..., 10$$

In the following, the membership of $x_{10} = (5, 4, 669, 2, 60, 43, 103, 3, 11, 49)$ element in the four risk categories is examined. We form matrix θ_x (Definition 3.4) based on the above membership functions. By forming matrix θ_x, the score values are calculated based on the Definition 3.5.

$$\theta_x = \begin{pmatrix} -0.0043 & 0.3809 & -0.02870 & -0.9183 \\ -0.0832 & 1 & -0.0068 & -0.537 \\ 0.0469 & -0.0329 & -0.0399 & -0.0441 \\ 1 & -0.1667 & -0.5 & -0.8333 \\ -0.0022 & 0.7586 & -0.0015 & -0.9762 \\ -0.106 & -0.045 & -0.0166 & 0.0185 \\ -0.1969 & -0.1718 & -0.1359 & 0.1497 \\ -0.6667 & -0.3333 & 1 & -0.3333 \\ 0.2625 & -0.024 & -0.1969 & -0.9198 \\ 0.1658 & -0.0391 & -0.0565 & -0.0983 \end{pmatrix}$$

From Definition 3.5:

$$score_1(x) = 0.4159$$
$$score_2(x) = 1.3267$$
$$score_3(x) = 0.0172$$
$$score_4(x) = -4.5179$$

By Definition 3.7 and functions μ, η the values $DoNM_{score_j}$, DoM_{score_j} are calculated for $i = 1, 2, 3, 4$.

$$\mu : [0, n] \to [0, 1] \text{ such that } \mu(x) = e^{\frac{-6.90\times(x-10)^2}{100}}$$
$$\eta : [-n, 0] \to [0, 1] \text{ such that } \eta(x) = e^{\frac{-6.90\times(x+10)^2}{100}}$$

From Definition 3.7

$$DoM_{score_1}(x) = \mu(score_1(x)) = \mu(0.4159) = 0.0018$$
$$DoM_{score_2}(x) = \mu(score_2(x)) = \mu(1.3267) = 0.0056$$
$$DoM_{score_3}(x) = \mu(score_3(x)) = \mu(0.0172) = 0.001$$
$$DoM_{score_4}(x) = \eta(score_4(x)) = \eta(-4.5179) = 0.1253.$$

Because there exists the largest membership degree therefore, by Definition 3.8 we consider element x_{10}= (5, 4, 669, 2, 60, 43, 103, 3, 11, 49) to belong to the moderate-risk class with a membership grade of 0.0056. Due to the high dispersion of the properties of this element, the membership grade is not strong in any of the grade classes, but this method has the ability to classify with the least similarity. This is how other buildings are classified. Table 13 shows the percentage of 50,000 buildings belonging to the four classes.

Table 13: Percentage of affiliation

Low-risk	Moderate-risk	High-risk	Very-high	No classification
15.154%	10.882%	26.936%	35.468%	11.56%

5 Conclusions

In this paper, a method for assessing the vulnerability of urban areas was presented using granular calculations and the concepts of fuzzy hypergraphs. In the proposed model, it was shown that fuzzy hypergraphs, compared to other methods, have the ability to define a flexible granular structure for different urban hazard classes. As an innovation of the proposed method, we can mention its flexibility in classifying elements, considering the role of all effective parameters, both equal weight and heterogeneity in classification. In verifying the proposed method, it has been shown that the classification of iris flowers with the proposed method is more than 90% consistent with the intuitive classification of Fisher.The low degree of membership of some flowers in all three classes indicates an excessive dispersion of characteristics. Using the proposed method in assessing the seismic vulnerability of 50,000 buildings, it is possible to simultaneously play the role of 10 heterogeneous and sometimes equal weight properties in the classification. Examination of the obtained results indicates the flexibility and accuracy of the proposed method in classifying elements with any number of heterogeneous and equal parameters. This study shows that fuzzy hypergraphs have the ability to define a flexible granular structure, taking into account the simultaneous role of all effective parameters in classification.

Declarations

- Availability of data and materials

  The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

- Competing interests

  The authors declare that they have no competing interests.

- Funding

  There are no sources of funding for the research reported.

- Authors' contributions

  Zarandi Baghini performed a new method of granular computing and developed the fuzzy hypergraphs and also analyzed and interpreted the computer proceeding data regarding the urban vulnerability and the verification of the previous experimental study. Tabatabaei Mirhosseini performed the background to the study, and was a major contributor in writing the manuscript. Babaei read and approved the final manuscript.

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