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Stochastic learning and control of building dynamics for thermal comfort

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Abstract

In the past few decades, thermal comfort has been considered an aspect of a sustainable building in almost all sustainable building evaluation methods and tools. However, estimating the indoor air temperature of buildings for efficient control is a complicated task due to the nonlinear, complex and uncertain building dynamics characterized by the time-varying environment with disturbances. The primary focus of this paper is designing a predictive and probabilistic room temperature model of buildings using Gaussian Processes (GP) and incorporating it into Model Predictive Control (MPC) to provide thermal comfort. The full probabilistic capabilities of GPs is exploited from two perspectives: the mean prediction is used for the room temperature model, while the uncertainty is involved in the MPC objective not to lose the desired performance and design a robust controller. We illustrated the potentials of the proposed method in a numerical example with simulation results.

Keywords: Gaussian processes, indoor climate, machine learning, model predictive control
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1 Introduction

Due to the disintegrate character of the building dynamics in terms of *optimization* and *control*, achieving an energy-efficient *building climate control* scheme that integrates fully automated heating, ventilation, and air conditioning (HVAC) services is still an open question. In building climate control problems, HVAC systems keep room temperature within a comfortable range. For decades, thermal comfort has been considered an aspect of a sustainable building in almost all sustainable building evaluation methods and tools [12, 14]. However, estimating the indoor air temperature of buildings is a complicated task due to the HVAC system's nonlinear and complex dynamic characterized by the time-varying environment with disturbances. Developing the building model is the most primary and time-consuming task when the modelling technique relies on physics-based and grey-box methods [19] based on energy and mass balance integral–differential equations. On the other hand, the rapid development of Machine Learning (ML) techniques and the increasing data accessibility in buildings have empowered the study of data-driven building models due to their simplicity, high level of automation, and low development engineering effort. In these circumstances, several research works have been investigated considering the time-varying user comfort preference [15]. Optimized energy and comfort

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management scheme for intelligent and sustainable buildings is provided in [21]. A comprehensive review focusing on thermal comfort predictive models and their applicability for energy control purposes is analyzed in [8].

Building climate control must be balance three conflicting demands such as energy efficiency, cost and thermal comfort. Model Predictive Control (MPC) is an optimal control method to design control law by minimizing a performance index while handling these demands. MPC has been implemented successfully in several directions of building control and operation strategies [9, 3]. Better thermal comfort and more energy savings compared to other control techniques can be achieved by combining MPC and ML such as neural networks [20], random forest [10], support vector machines [28]. However, designing accurate building energy/temperature models is the cornerstone to developing MPC for whole building operation and control due to the presence of external disturbances. This issue can be alleviated by modelling the building dynamics using Gaussian Processes (GPs) since it also measures the uncertainty bounds. Unfortunately, most GP based control laws do not take advantage of this information [1, 17]. The main focus of this paper is designing a predictive and probabilistic room temperature model of buildings using GPs. We exploit the GP's full probabilistic capabilities as the mean prediction for the room temperature model and use the model uncertainty in the MPC objective function not to lose the desired performance and to design a robust controller.

The remainder of this paper is organized as follows: Section 2 starts with the preliminary background and framework for data preparation. A methodology for constructing a predictive and probabilistic building model using GPs is discussed in Section 3, while Section 4 deals with theoretical analysis of designing a supervisory control combining GP with MPC scheme to solve the building climate control problem. Afterwards, the potentials of the proposed control scheme are demonstrated in simulation with some numerical results in Section 5. Finally, the conclusions of our work are drawn, and further research challenges are discussed in Section 6.

2 Data Preparation

2.1 Data acquisition

To provide thermal comfort in buildings, heating, ventilation, and air conditioning (HVAC) systems are usually manipulated to keep room temperature within a comfortable range. However, designing a proper controller to minimize cost in the building system while preserving thermal comfort is challenging task due to the HVAC system's complex dynamic characteristics, uncertain and time-varying environment, and disturbances. For this reason, the *data acquisition* system e.g. SCADA(supervisory control and data acquisition), has to be set up in such a way that the gathered data should comprise information both from inside (power consumption, water flow and water temperature, human occupation, CO_2 level, etc.) and outside (air temperature, air humidity, solar radiation, outside wall temperature, wind speed, etc.) the building. One option to correlate these features is to employ NARX(nonlinear autoregressive model with exogenous input)model architecture [17] that incorporates historical information up to a certain lag order. Then a training dataset $\mathbf{D}$ of N samples consisting of input - output pairs $\mathbf{D} = \{\mathbf{X}_{\mathbf{l_y},\mathbf{l_u},\mathbf{l_d}}, \mathbf{Y}\}$ is collected as $\mathbf{X}_{l_y,l_u,l_d} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N]_{l_y,l_u,l_d}$ and $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N]$ with

$$\mathbf{x}_i = [\mathbf{y}_i(l_y) \;\; \mathbf{u}_i(l_u) \;\; \mathbf{d}_i(l_d)] = \begin{cases} \mathbf{y}_i(l_y) = [y^j_{i-1}, \dots, y^j_{i-l_y}], j = 1, 2, \dots N_r. \\ \mathbf{u}_i(l_u) = [u^k_{i-1}, \dots, u^k_{i-l_u}], k = 1, 2, \dots N_u. \\ \mathbf{d}_i(l_d) = [d^k_i, \dots, d^h_{i-l_d}], h = 1, 2, \dots N_d. \end{cases} \tag{2.1}$$

where $\mathbf{y} \in \mathbb{R}^{n_y}$ is power/temperature measurement vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ is heating/cooling set-point vector, $\mathbf{d} \in \mathbb{R}^{n_y}$ is external disturbance vector affecting to the building dynamics, $N_r \in \mathbb{R}$ is the total number of rooms, $N_u \in \mathbb{R}$ is the total number of control inputs, $N_d \in \mathbb{R}$ is the total number of disturbance parameters, and l_y, l_u, $l_d \in \mathbb{R}$ are corresponding minimal autoregressive lags to be determined by feature selection algorithms as we discuss next.

2.2 Feature selection

The *feature selection* process is one of the most critical steps in prediction problems since it finds the smallest subset that significantly affects the prediction accuracy and minimizes the model's complexity. The accuracy of the prediction model dramatically depends on the quality of data and the relevancy of features. A review paper [30] summarizes feature selection applications in building energy management, including filter method [29], wrapper method [16], and embedded method [18]. However, these methods are very general and quite conservative in terms of learning speed. We adopt the algorithm proposed in [23] to select the minimum lag orders to get the most informative features by maximizing the relevancy of the features on the buildings' load consumption and thermal comfort settings.

3 Learning building model with Gaussian Processes

Definition 3.1. Gaussian Processes (GP) is an assembly of stochastic variables that any finite collection of these variables follows a multivariate normal distribution over functions with a continuous domain.

The Bayesian inference of continuous variables leads to Gaussian process regression where the prior GP model is updated with training dataset to obtain a posterior GP distribution [24]. Due to the possibility to include prior knowledge making the method more attractive as compared to other regression algorithms, GP models have been employed in different research fields [22, 5, 25]. This section provides the necessary background about GP and framework to build a probabilistic and predictive model for regression problems mainly, adopted from [2, 26].

3.1 Probabilistic model

Let a triple $(\Omega, \Psi_\theta, \mathcal{P})$ describe a probability space consisting of sample space Ω, corresponding a-algebra Ψ_θ and the probability measure $\mathcal{P}$. Then a stochastic process can be expressed by a measurable function $\Phi_{GP}(\mathbf{x})$ in $\mathcal{X} \subseteq \Omega$, which is fully described by mean function $\mu : \mathcal{X} \to \mathbb{R}$ and covariance function $\kappa : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ such that

$$\Phi_{GP}(\mathbf{x}) \sim \mathcal{GP}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}')) \tag{3.1}$$

$$\mu(\mathbf{x}) = E[\Phi_{GP}(\mathbf{x})] \tag{3.2}$$

$$\kappa(\mathbf{x}, \mathbf{x}') = E[(\Phi_{GP}(\mathbf{x}) - \mu(\mathbf{x}))(\Phi_{GP}(\mathbf{x}') - \mu(\mathbf{x}'))]. \tag{3.3}$$

with pair $(\mathbf{x}, \mathbf{x}') \in \mathcal{X}$. The mean function of the $\mathcal{GP}$ distribution illustrates the point where the samples are more likely located, while the variance of the $\mathcal{GP}$ distribution comes from measuring the correlation of any two samples $(\mathbf{x}, \mathbf{x}')$ that is calculated by the covariance function. We refer to [26] for a variety of mean and covariance functions.

Despite the absence of the existence of the probability density function of the GPs, their finite collection follows multivariate Gaussian distribution. This property allows us to write samples as a joint multivariate Gaussian distribution with a mean μ and variance σ^2 such that

$$\mathbf{y}_i = \Phi_{GP}(\mathbf{x}_i) \sim \mathcal{GP}(\mu(\mathbf{x}_i), \sigma^2(\mathbf{x}_i)|\theta_{\text{ig}}), \quad i = 1, 2, \dots, N. \tag{3.4}$$

where $\theta_{\text{ig}} \in \mathbb{R}^{n_\theta}$ is a set of prior (initial guess) parameters of mean and covariance functions.

3.2 Model learning

Maximum likelihood is commonly used optimisation method in Bayesian framework and its conditional probability function on training input samples $\mathbf{X}$ together with parameters θ is defined as follows

$$\mathcal{P}(\mathbf{Y}|\mathbf{X}, \theta) = \frac{1}{(2\pi)^{\frac{N}{2}} \det(\mathcal{K})^{\frac{1}{2}}} exp\left[-\frac{1}{2}(\mathbf{Y}^T \mathcal{K}^{-1} \mathbf{Y})\right]. \tag{3.5}$$

with the square covariance matrix $\mathcal{K}$

$$\mathcal{K} = \kappa(\mathbf{X}, \mathbf{X}) = \begin{bmatrix} \kappa(\mathbf{x}_1, \mathbf{x}_1) & \kappa(\mathbf{x}_1, \mathbf{x}_2) & \dots & \kappa(\mathbf{x}_1, \mathbf{x}_N) \\ \kappa(\mathbf{x}_2, \mathbf{x}_1) & \kappa(\mathbf{x}_2, \mathbf{x}_2) & \dots & \kappa(\mathbf{x}_2, \mathbf{x}_N) \\ \vdots & \vdots & \dots & \vdots \\ \kappa(\mathbf{x}_N, \mathbf{x}_1) & \kappa(\mathbf{x}_N, \mathbf{x}_2) & \dots & \kappa(\mathbf{x}_N, \mathbf{x}_N) \end{bmatrix} \in \mathbb{R}^{N \times N} \tag{3.6}$$

Maximum likelihood optimising the properties of the GPs prior used to generate new predictive distributions by looking for proper candidate θ that maximises the probability of the training data. The values of the parameters θ depend on the training data quality, and it is not easy to select their prior distribution. For this reason, mostly a uniform prior distribution is selected and the following assumption is used

$$\mathcal{P}(\theta|\mathbf{X}, \mathbf{Y}) \propto \mathcal{P}(\mathbf{Y}|\mathbf{X}, \theta) \tag{3.7}$$

which states that the maximum a posteriori estimate of the hyperparameters θ equals the maximum marginal likelihood estimate. Combining (3.5) with (3.7) and taking advantage of monotic property of the logarithm functions, the objective function to be minimized is written as

$$-\mathcal{L}(\theta) = \frac{N}{2} \ln 2\pi + \frac{1}{2} \ln(det\mathcal{K}) + \frac{1}{2} \mathbf{Y}^T \mathcal{K}^{-1} \mathbf{Y} \tag{3.8}$$

where $\mathcal{L}(\theta) = \mathcal{P}(\mathbf{Y}|\mathbf{X}, \theta)$. Finally, the optimal set of parameters θ_{opt} is provided by solving the following nonlinear and nonconvex optimization problem

$$\theta_{opt} = \min_{\theta} \mathcal{L}(\theta) \tag{3.9}$$

Once the regressors, covariance function, mean function and parameters are selected, the model is validated by measuring the accuracy of the training $\mathbf{D}_{\text{train}}$ and test $\mathbf{D}_{\text{test}}$ datasets using different metrics. Below, we provide standard metrics to validate models in our numerical example and refer [26] to the reader for an overview of the bench of accuracy measuring metrics.

Normalized root mean squared error(nrmse) - the measure that normalizes the root mean squared error between the mean of the model's output and the measured output of the process by the maximum difference of the output values of the training dataset

$$\text{nrmse} = \sqrt{\frac{1}{N} \frac{\sum_{i=1}^{N} (\mathbf{y}_i - \mu(\mathbf{x}_i))^2}{(\mathbf{Y}_{max} - \mathbf{Y}_{min})^2}} \tag{3.10}$$

Mean standardised log loss(msll) - helps us better understand how big model's σ^2 varies and is obtained by subtracting the mean prediction of the model from true measurements and dividing by predicted variance

$$\text{msll} = \frac{1}{2N} \sum_{i=1}^{N} \left[\ln \sigma^2(\mathbf{x}_i) + \frac{(\mathbf{y}_i - \mu(\mathbf{x}_i))^2}{\sigma^2(\mathbf{x}_i)} \right] \tag{3.11}$$

3.3 Predictive model

The GP can be utilized as a prior probability distribution in Bayesian inference [26], allowing function regression to perform. A new given test sample $\mathbf{x}_\star \in \mathcal{X}$ is combined with existing training samples based on the Bayesian framework to obtain a posterior distribution for $\mathbf{y}_\star \in \mathcal{Y}$. Hence, we define the predictive distribution of $\mathbf{y}_\star$ conditioned on $\mathbf{D}, \mathcal{K}, \mathbf{x}_\star, \theta_{opt}$ as follows

$$\mathcal{P}(\mathbf{y}_\star|\mathbf{D}, \mathcal{K}, \mathbf{x}_\star, \theta_{opt}) = \frac{\mathcal{P}([\mathbf{Y}, \mathbf{y}^\star]^T|\mathcal{K}, \mathbf{X}, \mathbf{x}_\star, \theta_{opt})}{\mathcal{P}(\mathbf{Y}^T|\mathcal{K}, \mathbf{X}, \theta_{opt})} \tag{3.12}$$

After adopting the conditional probability functions from (3.5) for $\mathcal{P}([\mathbf{Y}, \mathbf{y}^\star]^T|\mathcal{K}, \mathbf{X}, \mathbf{x}_\star, \theta_{opt})$ and $\mathcal{P}(\mathbf{Y}^T|\mathcal{K}, \mathbf{X}, \theta_{opt})$, the joint multivariate predictions for the batch of random variables $[\mathbf{Y}, \mathbf{y}^\star]^T = [\mathbf{y}_1, \ldots, \mathbf{y}_N, \mathbf{y}^\star]^T \in \mathcal{Y}$ are written as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \\ \mathbf{y}_\star \end{bmatrix} \sim \mathcal{GP} \left(\begin{bmatrix} \mu(\mathbf{x}_1) \\ \mu(\mathbf{x}_2) \\ \vdots \\ \mu(\mathbf{x}_N) \\ \mu_\star(\mathbf{x}_\star) \end{bmatrix}, \begin{bmatrix} \mathcal{K} & \mathcal{K}_\star \\ \mathcal{K}_\star^T & \kappa_\star \end{bmatrix} \right) \tag{3.13}$$

where the covariance matrices $\mathcal{K}_\star = [\kappa(\mathbf{x}_1, \mathbf{x}_\star), \ldots, \kappa(\mathbf{x}_N, \mathbf{x}_\star)]^T \in \mathbb{R}^{N\times 1}$ is the vector of similarity measure between the training samples and the samples and $\kappa_\star = \kappa(\mathbf{x}_\star, \mathbf{x}_\star) \in \mathbb{R}$ is the self-covariance of the test sample.

From (3.13), we can deduce that the Gaussian prediction $\mathbf{y}_\star$ for the new input $\mathbf{x}_\star$ with the mean $\mu_\star(x_\star)$ and the variance $\sigma^2_\star(x_\star)$ is given as follows

$$\mathbf{y}_\star = \Phi_{GP}(\mathbf{x}_\star) \sim \mathcal{GP}(\mu_\star(\mathbf{x}_\star), \sigma^2_\star(\mathbf{x}_\star)|\theta_{opt}) \tag{3.14}$$

$$\mu_\star(\mathbf{x}_\star) = \mu(\mathbf{x}_\star) + \mathcal{K}_\star^T \mathcal{K}^{-1}(\mathbf{Y} - \mu(\mathbf{X})) \tag{3.15}$$

$$\sigma^2_\star(\mathbf{x}_\star) = \kappa_\star - \mathcal{K}_\star^T \mathcal{K}^{-1} \mathcal{K}_\star \tag{3.16}$$

The overall GP model learning algorithm and the summary of this section is highlighted in Algorithm 1.

Algorithm 1: GP based MPC at a time step t

Input: training data $\mathbf{D}_{\text{train}}$, test data $\mathbf{D}_{\text{test}}$, validating metric ρ, training data accuracy threshold ε_{train} and test data accuracy threshold ε_{test}.

Output: $\Phi_{GP}(\mathbf{x}) \sim \mathcal{GP}(\mu(\mathbf{x}), \sigma^2(\mathbf{x})|\theta_{opt})$

while $\rho_{train} \geqslant \varepsilon_{train}$ *and* $\rho_{test} \geqslant \varepsilon_{test}$ **do**

- set l_y, l_u, l_d
- assign prior GP model $\Phi_{GP}(\mathbf{x}_i) \sim \mathcal{GP}(\mu(\mathbf{x}_i), \sigma^2(\mathbf{x}_i)|\theta_{\text{ig}}) \quad i = 1, 2, \ldots, N.$
- compute $\mathcal{K}$
- solve minimization problem (3.9) and obtain θ_{opt}
- measure ρ_{train} and ρ_{test} based on $\Phi_{GP}(\mathbf{x}) \sim \mathcal{GP}(\mu(\mathbf{x}), \sigma^2(\mathbf{x})|\theta_{opt})$

4 Building indoor thermal comfort

This section deals with the optimal control problem for building indoor thermal comfort using Model Predictive Control (MPC) methodology applied to stochastic dynamic process. For this purpose, a GP model is used to learn the building model and integrated into the MPC scheme to design a robust control using variance information of the GP model.

4.1 MPC theory

Definition 4.1. Consider the following classic MPC optimization problem with input and output constraints

$$
\begin{aligned}
&\min_{\mathbf{U},E} \sum_{\tau=0}^{N_p-1} \ell_\tau(\mathbf{y}_{\tau+1+t|t}, \mathbf{x}_{\tau+1+t|t}, \mathbf{u}_{\tau+t|t}, \epsilon_{\tau+1+t|t}) && (4.1)\\
&\text{s.t.}\quad \mathbf{x}_{\tau+1+t|t} = f(\mathbf{x}_{\tau+t|t}, \mathbf{u}_{\tau+t|t}, \mathbf{d}_{\tau+t|t}) && \tau \in \mathbb{I}_0^{N_p-1}\\
&\qquad \mathbf{u}^{\min}_{\tau+t|t} \leq u_{\tau+t|t} \leq \mathbf{u}^{\max}_{\tau+t|t} && \tau \in \mathbb{I}_0^{N_p-1}\\
&\qquad \mathbf{y}_{\tau+t|t} = C\mathbf{x}_{\tau+t|t} + \mathbf{v}_{\tau+t|t} && \tau \in \mathbb{I}_1^{N_p}\\
&\qquad -\epsilon_{\tau+t|t} + \mathbf{y}^{\min}_{\tau+t|t} \leq \mathbf{y}_{\tau+t|t} \leq \mathbf{y}^{\max}_{\tau+t|t} + \epsilon_{\tau+t|t} && \tau \in \mathbb{I}_1^{N_p}\\
&\qquad \epsilon_{\tau+t|t} \geq 0 && \tau \in \mathbb{I}_1^{N_p}
\end{aligned}
$$

where t is the current time instant, N_p the prediction horizon, $\mathbb{I}_a^b$ is the set of all integers in the interval $[a, b]$, $\mathbf{U} = [\mathbf{u}_0, \ldots, \mathbf{u}_{N_p-1}]$ is the sequence of manipulated variables $\mathbf{u}_{\tau+t|t} \in \mathbb{R}^{n_u}$ to optimize, $\mathbf{x}_{\tau+t|t} \in \mathbb{R}^{n_x}$ is the state vector at τ-steps ahead, $\mathbf{y}_{\tau+t|t} \in \mathbb{R}^{n_y}$ is the output vector, $\mathbf{d}_{\tau+t|t} \in \mathbb{R}^{n_d}$ is a disturbance vector affecting the prediction model described by $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \to \mathbb{R}^{n_x}$, $\ell_\tau : \mathbb{R}^{n_y} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_y} \to \mathbb{R}_{\geq 0}$ are convex stage cost functions, such as quadratic functions, and $E = (\epsilon_1, \ldots, \epsilon_N)$ is a vector of slack variables $\epsilon_k \in \mathbb{R}^{n_y}_{\geq 0}$ used to soften output constraints.

MPC is a control technique that selects optimal control action based on the future state predictions of the system model. Optimal control actions are calculated by solving an optimization problem so that an objective function is minimized and constraints are satisfied in every step of a controlled system. Then the only first sample of the commanded inputs is applied to the system as its optimal input. This process is repeated all over again to calculate the control signal in every step [6]. The development of the model to predict the outputs/states in the MPC objective function is the most primary and time-consuming task of MPC design. However, the rapid development of machine learning techniques and the increasing data accessibility in buildings have empowered the study of data-driven models, as we discuss below, due to their simplicity, high level of automation, and low development engineering effort [10, 1].

4.2 GP based MPC solution algorithm

Building climate control must balance three conflicting demands: energy efficiency, cost, and thermal comfort. MPC is an optimal control method to design control law by minimizing a performance index while handling these demands. However, designing accurate building energy/temperature models is the cornerstone to developing MPC for whole building operation and control due to the presence of external disturbances. This issue can be alleviated by including the variance term into the MPC optimization objective enabling the design of a robust controller thanks

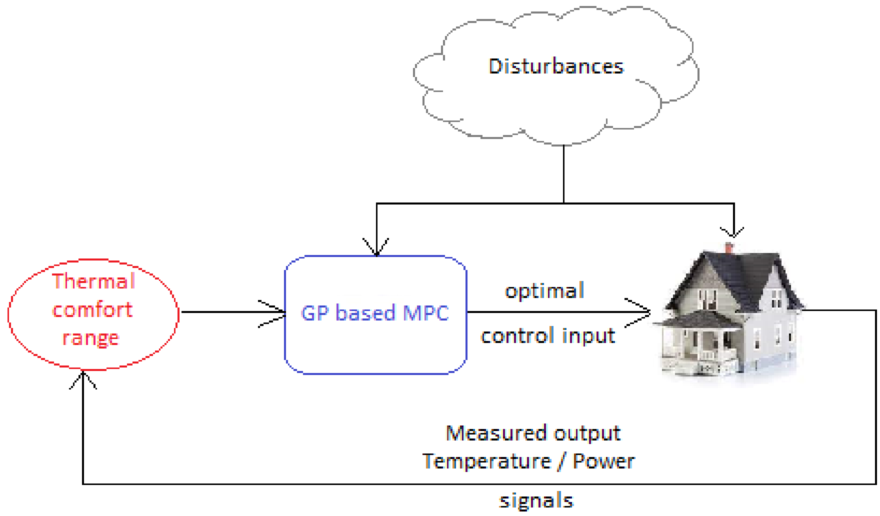


Figure 1. The schematic representation of the building indoor thermal comfort control system using the GP based MPC controller.

to the availability of uncertainty prediction in GP modelling. The MPC scheme based on GP model is illustrated in Figure 1.

One of the most important constraints for the building climate control optimization problem is human thermal comfort. There are two main methods to introduce this constraint to the problem: predicted mean vote [11] and thermal bounds [7]. Treating MPC with the former type as a constraint or objective function increases the computational burden of the optimization problem. For this reason, we consider the latter as a thermal constraint with linear upper and lower bounds in our proposed control problem. We are interested in the use of GPs for predicting the room(s) air temperature $\mathbf{y}$ as a function of the previous temperature measurements, forecasted weather disturbances $\mathbf{d}$ (solar radiation, outside air temperature and internal heat gains) and manipulated variables $\mathbf{u}$. The control task is to keep the room temperature within a predefined comfort range by commanding a set of different actuators $\mathbf{u}$ such as heating, cooling, ventilation and air conditioning. The goal is to select the optimal control inputs automatically using GP based MPC while satisfying the comfort requirements and minimizing energy costs coming from manipulated set-points. GP model-based MPC optimization problem is defined as follows

$$\min_{\mathbf{U},E} \sum_{\tau=0}^{N_p-1} \|\mathbf{y}_{\tau+t+1|t}\|^2_{Q_\mathbf{y}} + \|\sigma^2_{\tau+t+1|t}\|^2_{Q_\sigma} + \|\mathbf{u}_{\tau+t|t}\|^2_{Q_u} + \|\epsilon_{\tau+t+1|t}\|^2_{Q_\epsilon} \tag{4.2}$$

$$\begin{aligned}
\text{s.t.}\quad & \mathbf{x}_{\tau+t|t} = \begin{bmatrix}\mathbf{y}_{\tau+t|t} & \dots & \mathbf{y}_{\tau-l_\mathbf{y}+t|t} & \mathbf{u}_{\tau+t|t} & \dots & \mathbf{u}_{\tau-l_u+t|t} & \mathbf{d}_{\tau+t+1|t} & \dots & \mathbf{d}_{\tau-l_d+1+t|t}\end{bmatrix} && \tau \in \mathbb{I}_0^{N_p-1}\\
& \mathbf{y}_{\tau+t+1|t} = \mu_t(\mathbf{x}_{\tau+t|t}) + \mathcal{K}^T_{\tau+t|t}\mathcal{K}_t^{-1}(\mathbf{Y}_t - \mu_t(\mathbf{X}_t)) && \tau \in \mathbb{I}_0^{N_p-1}\\
& \sigma^2_{\tau+t+1|t} = \kappa_{\tau+t|t} - \mathcal{K}^T_{\tau+t|t}\mathcal{K}_t^{-1}\mathcal{K}_{\tau+t|t} && \tau \in \mathbb{I}_0^{N_p-1}\\
& \mathbf{u}^{\min}_{\tau+t|t} \le \mathbf{u}_{\tau+t|t} \le \mathbf{u}^{\max}_{\tau+t|t} && \tau \in \mathbb{I}_0^{N_p-1}\\
& -\epsilon_{\tau+t|t} + \mathbf{y}^{\min}_{\tau+t|t} \le \mathbf{y}_{\tau+t|t} \le \mathbf{y}^{\max}_{\tau+t|t} + \epsilon_{\tau+t|t} && \tau \in \mathbb{I}_1^{N_p}\\
& \epsilon_{\tau+t|t} \ge 0 && \tau \in \mathbb{I}_1^{N_p}
\end{aligned}$$

where $\|s\|^2_Q = s^TQs$ is a weighted quadratic norm and $Q_y, Q_\sigma, Q_u, Q_\epsilon$ are corresponding positive definite matrices. The summary of GP based MPC scheme is given in the Algorithm 2.

5 Numerical Example

In this section, we demonstrate the potentials of the proposed strategy on a simulation example using a simplified version of the building given in [13]. We consider the following discrete nonlinear system

$$\begin{cases}\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + E\mathbf{d}_t\\ \mathbf{y}_t = C\mathbf{x}_t + \mathbf{v}_t\end{cases} \tag{5.1}$$

Algorithm 2: GP based MPC at a time step t

Input: Training data: $\mathbf{D}_t = \{\mathbf{X}_t, \mathbf{Y}_t\}$, autoregressive lags: l_y, l_u, l_d, GP model $\Phi_{GP}(\mathbf{x}_t) \sim \mathcal{GP}(\mu_t(\mathbf{x}_t), \sigma_t^2(\mathbf{x}_t)|\theta_{opt})$.

Output: $\mathbf{u}_t$

1 calculate the matrices $\mathcal{K}_t^{-1}$ and $\mu_t(\mathbf{X}_t)$
2 solve MPC problem (4.2) online for $\mathbf{u}_t, \ldots, \mathbf{u}_{t+N_h-1}$
3 apply only $\mathbf{u}_t$ to the building

with $A = \begin{bmatrix} 0.8511 & 0.0541 & 0.0707 \\ 0.1293 & 0.8635 & 0.0055 \\ 0.0989 & 0.0032 & 0.7541 \end{bmatrix}$, $B = \begin{bmatrix} 0.070 \\ 0.006 \\ 0.004 \end{bmatrix}$, $E = \begin{bmatrix} 0.02221 & 0.00018 & 0.0035 \\ 0.00153 & 0.00007 & 0.0003 \\ 0.10318 & 0.00001 & 0.0002 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T$.

The primary purpose of the control task is to achieve temperature $\mathbf{y}$ comfort while minimizing energy consumption by manipulating the control signal $\mathbf{u}$. In order to solve both Classic MPC (4.1) and GP based MPC (4.2) problems, we choose the weights as $Q_y = 1$, $Q_\sigma = 1$, $Q_u = 1$ and $Q_\epsilon = 10$ and use the values of variables frequently used throughout this paper and summarized in Table 2 for this particular problem. We solve nonlinear optimization problems associated with both MPCs using the IPOPT algorithm in the CasADi framework [4] and execute all simulations in MATLAB 2018b on a machine equipped with an Intel Core i5-5200U (2.7GHz) processor.

Table 1. Meaning and values of the variables used in control optimization problems.

Variable	Units	Description	Control setup
$\mathbf{x}$	$[^0C]$	Indoor wall/room/outside wall temperatures	States
$\mathbf{u}$	$[W/m^2]$	Heating set-point	Control input
$\mathbf{d}$	$[^0C]$, $[W/m^2]$	Outside temperature, solar radiation, internal heat gain	State disturbances
$\mathbf{u}^{\min} = 0$	$[W/m^2]$	Minimum heating capacity	Input constraint
$\mathbf{u}^{\min} = 30$	$[W/m^2]$	Maximum heating capacity	Input constraint
$\mathbf{y}$	$[^0C]$	Room temperature	Output
$\mathbf{y}^{\min} = 21$	$[^0C]$	Lower comfort boundary	Output constraint
$\mathbf{y}^{\max} = 23.5$	$[^0C]$	Upper comfort boundary	Output constraint
$\mathbf{v} \sim \mathcal{N}(0, 0.02)$	$[^0C]$	Measurement Gaussian noise	Output disturbance
ϵ	$[^0C]$	Comfort band violation	Slack

To learn the GP model in (3.14), we generate the data of $M = 2000$ samples as follows: (i) the control signal $\mathbf{u}$ is frozen for three consecutive time steps with uniform distribution in the magnitude between $\mathbf{u}^{\min}$ and $\mathbf{u}^{\max}$ as specified in Table 1, (ii) obtained signals are applied to the building model described by (5.1), and the corresponding measurements are collected. We use $M_{\text{train}} = 0.6M$ samples for learning the parameters of the GP model and $M_{\text{test}} = 0.4M$ test samples used to assess the performance of the specified model. We validate the GP model by measuring the prediction accuracy using the commonly used Normalized Root Mean Square Error (nrmse) and Mean Standard Log Loss (MSLL) provided in [26]. GP models with zero mean are common in practice, so we set $\mu = 0$ and look for a proper covariance function candidate by considering squared exponential se in (5.2a) and rational quadratic rq in (5.2b) covariance functions with several combinations of corresponding autoregressive lags. The Ipopt [27] optimization algoritmh is implemented to solve problem (3.9). We choose the composite covariance function in (5.2c) with l_y=2, l_u=2, and l_d=0 as it performs better accuracy compared to other candidates, see Table 2.

$$\kappa_1(\mathbf{x}, \mathbf{x}') = \theta_{f_1} exp\left(-\frac{1}{2}\sum_{s=1}^{n_\theta}\frac{(\mathbf{x}-\mathbf{x}')^2}{\theta_{1,s}^2}\right) \tag{5.2a}$$

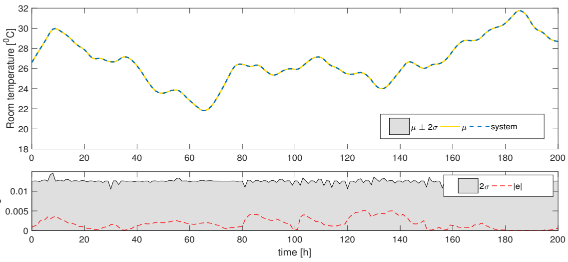
$$\kappa_2(\mathbf{x}, \mathbf{x}') = \theta_{f_2} exp\left(-\frac{1}{2\alpha}\sum_{s=1}^{n_\theta}\frac{(\mathbf{x}-\mathbf{x}')^2}{\theta_{2,s}^2}\right)^{-\alpha} \tag{5.2b}$$

$$\kappa(\mathbf{x}, \mathbf{x}') = \kappa_1(\mathbf{x}, \mathbf{x}') + \kappa_1(\mathbf{x}, \mathbf{x}') \tag{5.2c}$$

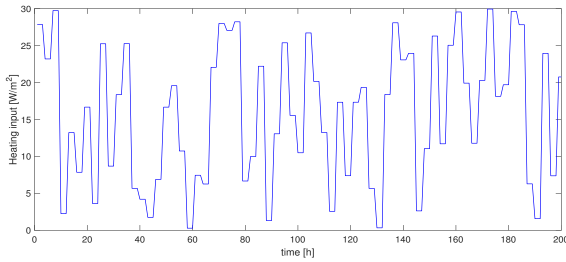
Table 2. GP modeling accuracy results (nrmse/msll) on the training data for different autoregressive lags and covariance functions (se - squared exponential, rq - rational quadratic).

Covariance function	Autoregressive lags				
	l_y=3, l_u=2, l_d=1	l_y=2, l_u=2, l_d=0	l_y=2, l_u=1, l_d=1	l_y=1, l_u=1, l_d=1	l_y=1, l_u=1, l_d=0
se	0.061/-1.770	0.002/-4.124	0.017/-2.946	0.045/-1.122	0.108/-1.360
se+rq	0.045/-1.910	0.001/-4.829	0.024/-3.846	0.035/-1.208	0.096/-1.642
rq	0.061/-1.770	0.115/-1.284	0.097/-2.556	0.067/-1.520	0.137/-1.595

Figure 2 illustrates trajectories and corresponding uncertainty regions predicted by the GP model for applied control signals, where the mean values are indistinguishable from the true ones. The prediction for the test data is depicted in the left top corner of Figure 3. Moreover, the robustness of the chosen GP model is tested with different Gaussian noises $\mathbf{v}$ and the corresponding trajectory forecasts are demonstrated in Figure 3, where one can see that the uncertainty region enlarges as the noise variance increases. For the sake of better visualization, we cut the first 200 samples off in all Figures.

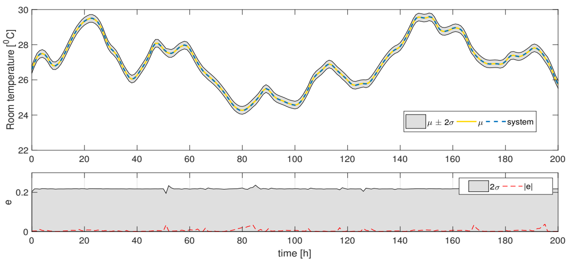


(a)

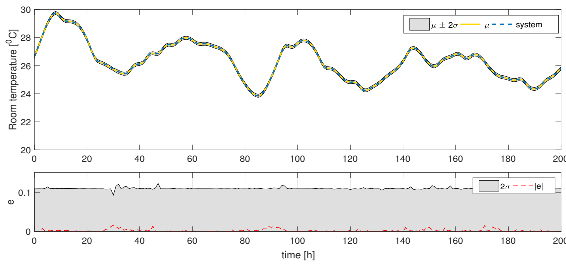


(b)

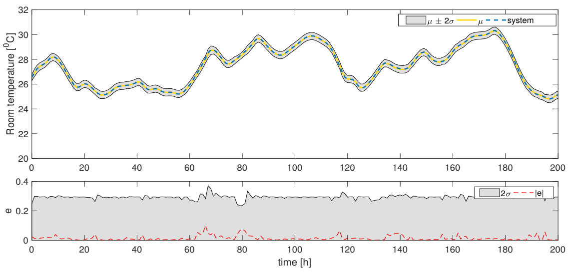
Figure 2. The prediction accuracy of the GP model for the training data.
(a)Top plot draws the true (blue), the predicted mean μ (yellow) and 95 % confidence intervals $\mu + 2\sigma$ (gray) values, while bottom plot shows the absolute error e between true and predicted values, (b) Control signal



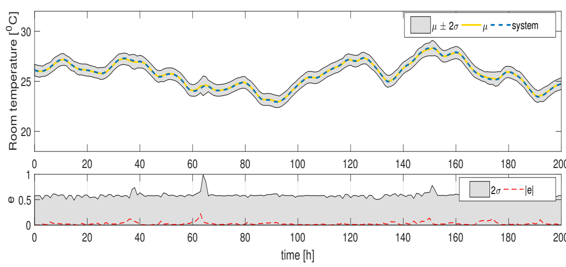
(a)



(b)



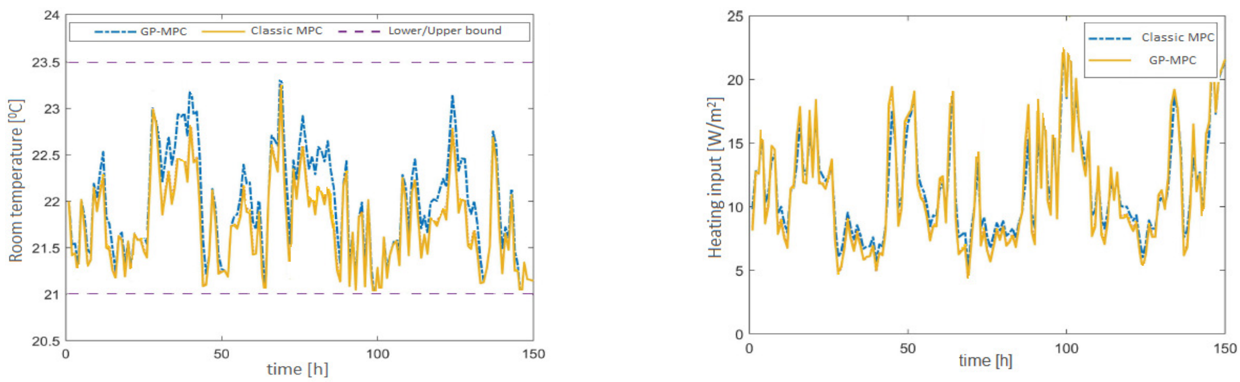
(c)



(d)

Figure 3. Effects of introducing different Gaussian noises to the system output,
(a) $\mathbf{v} \sim \mathcal{N}(0, 0.02)$, (b) $\mathbf{v} \sim \mathcal{N}(0, 0.01)$, (c) $\mathbf{v} \sim \mathcal{N}(0, 0.03)$, (d) $\mathbf{v} \sim \mathcal{N}(0, 0.05)$

The Classic MPC with $N_p = 10$ and the GP-MPC controllers are tested in simulation by running a temperature from a feasible initial state $\mathbf{y}_0 = 22$ $[{}^0C]$ and simulation results are obtained for 150 hours. Figure 4 show that the GP-MPC scheme is able to keep the temperature within the thermal comfort margins and recovers a good closed-loop performance by using the variance prediction preview information to compute the objective function.



(a) (b)

Figure 4. The closed-loop performances of Classic MPC and GP based MPC laws.
(a) Room temperature, (b) Heating input.

6 Conclusion

This paper discussed the use of Gaussian Processes (GPs) for predictive and probabilistic modelling of a building's complex dynamics for thermal comfort. We learned a GP model that predicts a room air temperature as output for a given input vector which is the combination of the previous temperature measurements, forecasted weather disturbances such as solar radiation, outside air temperature and internal heat gains, and manipulated heating set-point. Model Predictive Control (MPC) strategy based on GP model was implemented to obtain optimal heating set-points providing user predefined min-max thermal comfort. We exploited the GP model's mean prediction for the room temperature and used the corresponding provided uncertainty bounds in the MPC objective function not to lose the desired performance as compared with classic MPC law in simulation results. Our future work will be focused on designing a robust decision making of the GP-based MPC scheme if an uncertain weather forecast is provided and one of the measuring sensors is broken.

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