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Consideration of fuzzy zero based on transmission average

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Abstract

Fuzzy field and arithmetic operators based on transmission average (TA) and neutral member are dealt by Abbasi et al. [1]. Many examples are exist such that based on their definition $\tilde{A}.\tilde{0}_{\tilde{A}}\tilde{0}_{\tilde{A}}.\tilde{A})$ is not equal to $\tilde{0}_{\tilde{A}}$. Therefore, we investigate the conditions that $\tilde{A}.\tilde{0}_{\tilde{A}}(=\tilde{0}_{\tilde{A}}.\tilde{A})=\tilde{0}_{\tilde{A}}$. Numerical examples show the applicability of theorems and mentioned problems.

Keywords: Transmission average, Fuzzy arithmetic, Fuzzy neutral element
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1 Introduction

Field is an algebraic structure, in which the operations of sum, subtraction, multiplication and division are defined and real numbers, complex numbers, and rational numbers are mentioned as the most famous fields. Recently, the concept of fuzzy fields used in different mathematical branches by researchers [2, 7, 9, 10, 11, 12, 14]. The concept of fuzzy ideal is introduced by Wenxiang and Tu [16]. Also, the extension of the defined fuzzy field is presented by them [8]. Recently, fuzzy field and ring are defined by Abbasi et al. [1] which is used to rank fuzzy numbers [3]. Fuzzy equations is solved on the basis of fuzzy field by Allahviranloo et al. [4]. In this paper, the properties of the defined neutral element in [1] is presented. The structure is as follows. In section 2, basic definitions are given. Properties are presented in section 3. In section 4, numerical examples are provided. Finally, conclusion is expressed in section 5.

2 Basic Definitions

Definition 2.1. Let $\tilde{A}$ is a normal, convex and continuous (NCC) fuzzy set on the universal set U. Then, from [10] it is defined:

$$ac(\tilde{A})=\frac{1}{2}(\min core(\tilde{A})+\max core(\tilde{A})).$$

Definition 2.2. [10] A fuzzy number $\tilde{A}$ is called a pseudo-trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x)=\begin{cases} l_{\tilde{A}}(x), & \underline{a}\le x\le a_1,\\ 1, & a_1\le x\le a_2,\\ r_{\tilde{A}}(x), & a_2\le x\le \overline{a},\\ 0, & \text{otherwise}\end{cases}$$

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where $l_{\tilde{A}}(x)$ and $r_{\tilde{A}}(x)$ are non-decreasing and non-increasing functions respectively. The pseudo-triangular fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x))$ and the triangular fuzzy number by $\tilde{A} = (\underline{a}, a_1, a_2, \overline{a}, -, -)$.

Definition 2.3. [10] A fuzzy number $\tilde{A}$ is called a pseudo-triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} l_{\tilde{A}}(x), & \underline{a} \leq x \leq a, \\ r_{\tilde{A}}(x), & a \leq x \leq \overline{a}, \\ 0, & \text{otherwise} \end{cases}$$

where $l_{\tilde{A}}(x)$ and $r_{\tilde{A}}(x)$ are non-decreasing and non-increasing functions respectively. The pseudo-triangular fuzzy number $\tilde{A}$ is denoted by the quintuplet $\tilde{A} = (\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x))$ and the triangular fuzzy number by the quintuplet $\tilde{A} = (\underline{a}, a, \overline{a}, -, -)$.

Definition 2.4. [1] Let $\tilde{A} = (\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x))$ and $\tilde{B} = (\underline{b}, b, \overline{b}, l_{\tilde{B}}(x), r_{\tilde{B}}(x))$ are two pseudo-triangular fuzzy numbers with the following α-cut forms:

$$\tilde{A} = \cup_\alpha A_\alpha, \qquad A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha], \qquad \tilde{B} = \cup_\alpha B_\alpha, \qquad B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha]$$

In the following, the fuzzy arithmetic operations for pseudo-triangular fuzzy numbers based on TA are defined:

$$\begin{aligned} &\tilde{A} + \tilde{B} = \cup_\alpha (\tilde{A} + \tilde{B})_\alpha, \\ &(\tilde{A} + \tilde{B})_\alpha = \left[\frac{a+b}{2} + \left(\frac{\tilde{A}_\alpha + \tilde{B}_\alpha}{2} \right), \frac{a+b}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2} \right) \right], \end{aligned} \tag{2.1}$$

$$\begin{aligned} &\tilde{A} - \tilde{B} = \cup_\alpha (\tilde{A} - \tilde{B})_\alpha, \\ &(\tilde{A} + \tilde{B})_\alpha = \left[\frac{a-3b}{2} + \left(\frac{\tilde{A}_\alpha + \tilde{B}_\alpha}{2} \right), \frac{a-3b}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2} \right) \right], \end{aligned} \tag{2.2}$$

$$\begin{aligned} &\tilde{A}.\tilde{B} = \cup_\alpha (\tilde{A}.\tilde{B})_\alpha, \\ &(\tilde{A}.\tilde{B})_\alpha \begin{cases} \left[\left(\frac{b}{2}\right) \underline{A}_\alpha + \left(\frac{a}{2}\right) \underline{B}_\alpha, \left(\frac{b}{2}\right) \overline{A}_\alpha + \left(\frac{a}{2}\right) \overline{B}_\alpha \right], & a \geq 0, b \geq 0, \\ \left[\left(\frac{b}{2}\right) \overline{A}_\alpha + \left(\frac{a}{2}\right) \underline{B}_\alpha, \left(\frac{b}{2}\right) \underline{A}_\alpha + \left(\frac{a}{2}\right) \overline{B}_\alpha \right], & a \geq 0, b \leq 0, \\ \left[\left(\frac{b}{2}\right) \overline{A}_\alpha + \left(\frac{a}{2}\right) \overline{B}_\alpha, \left(\frac{b}{2}\right) \underline{A}_\alpha + \left(\frac{a}{2}\right) \underline{B}_\alpha \right], & a \leq 0, b \geq 0, \\ \left[\left(\frac{b}{2}\right) \underline{A}_\alpha + \left(\frac{a}{2}\right) \overline{B}_\alpha, \left(\frac{b}{2}\right) \overline{A}_\alpha + \left(\frac{a}{2}\right) \underline{B}_\alpha \right], & a \leq 0, b \geq 0. \end{cases} \end{aligned} \tag{2.3}$$

Definition 2.5. [1] Let $\tilde{A} = \left(\underline{a}, a_1, a_2, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}(x)}\right)$ and $\tilde{B} = \left(\underline{ba}, b_1, b_2, \overline{b}, l_{\tilde{B}}(x), r_{\tilde{B}(x)}\right)$ are two pseudo-trapezoidal fuzzy numbers with the following α-cut forms:

$$\begin{array}{cc} \tilde{A} = \cup_\alpha A_\alpha, & A_\alpha = \left[\underline{A}_\alpha, \overline{A}_\alpha\right], \\ 0 \leq \alpha\alpha \leq 1, & A_1 = [a_1, a_2], \\ \tilde{B} = \cup_\alpha B_\alpha, & B_\alpha = \left[\underline{B}_\alpha, \overline{B}_\alpha\right], \\ 0 \leq \alpha \leq 1, & B_1 = [b_1, b_2]. \end{array}$$

Let

$$\phi = \frac{a_1 + a_2}{2}, \qquad \varphi = \frac{b_1 + b_2}{2},$$

In the following, the fuzzy arithmetic operations for pseudo-trapezoidal fuzzy numbers based on TA are defined:

$$\begin{aligned} &\tilde{A} + \tilde{B} = \cup_\alpha \left(\tilde{A} + \tilde{B}\right)_\alpha, \\ &\left(\tilde{A} + \tilde{B}\right)_\alpha = \left[\frac{\phi + \varphi}{2} + \left(\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2} \right), \frac{\phi + \varphi}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2} \right) \right], \end{aligned} \tag{2.4}$$

$$\begin{aligned} &\tilde{A} - \tilde{B} = \cup_\alpha \left(\tilde{A} - \tilde{B}\right)_\alpha, \\ &\left(\tilde{A} - \tilde{B}\right)_\alpha = \left[\frac{\phi - +3\varphi}{2} + \left(\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2} \right), \frac{\phi - 3\varphi}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2} \right) \right], \end{aligned} \tag{2.5}$$

$$\tilde{A}.\tilde{B} = \cup_\alpha (\tilde{A}.\tilde{B})_\alpha,$$

$$(\tilde{A}.\tilde{B})_\alpha = \begin{cases} \left[\left(\frac{\varphi}{2}\right)\underline{A}_\alpha + \left(\frac{\phi}{2}\right)\underline{B}_\alpha, \frac{\varphi}{2}\overline{A}_\alpha + \left(\frac{\phi}{2}\right)\overline{B}_\alpha\right], & \phi \geq 0, \varphi \geq 0, \\ \left[\left(\frac{\varphi}{2}\right)\overline{A}_\alpha + \left(\frac{\phi}{2}\right)\underline{B}_\alpha, \frac{\varphi}{2}\underline{A}_\alpha + \left(\frac{\phi}{2}\right)\overline{B}_\alpha\right], & \phi \geq 0, \varphi \leq 0, \\ \left[\left(\frac{\varphi}{2}\right)\overline{A}_\alpha + \left(\frac{\phi}{2}\right)\overline{B}_\alpha, \frac{\varphi}{2}\underline{A}_\alpha + \left(\frac{\phi}{2}\right)\underline{B}_\alpha\right], & \phi \leq 0, \varphi \leq 0, \\ \left[\left(\frac{\varphi}{2}\right)\underline{A}_\alpha + \left(\frac{\phi}{2}\right)\overline{B}_\alpha, \frac{\varphi}{2}\overline{A}_\alpha + \left(\frac{\phi}{2}\right)\underline{B}_\alpha\right], & \phi \leq 0, \varphi \geq 0, \end{cases} \tag{2.6}$$

Definition 2.6. [1] Let $F_C(R)$ is a set of pseudo-geometric fuzzy numbers defined on set of real numbers. Then:

$$\forall\ \tilde{A}\ \exists\ 0_{\tilde{A}}\ st;\ \tilde{A} + 0_{\tilde{A}} = 0_{\tilde{A}} + \tilde{A} = \tilde{A},\ \tilde{A} - \tilde{A} = 0_{\tilde{A}},$$

For $\tilde{A} = (\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x))$:

$$0_{\tilde{A}} = (\underline{a} - a, 0, \overline{a} - a, l_{\tilde{A}}(x + a), r_{\tilde{A}}(x + a)), \tag{2.7}$$

For $\tilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x))$:

$$0_{\tilde{A}} = \left(\underline{a} - \phi, \frac{a_1, a_2}{2}, \frac{a_2 - a_1}{2}, \overline{a} - \phi, l_{\tilde{A}}(x + \phi), r_{\tilde{A}}(x + \phi)\right). \tag{2.8}$$

Definition 2.7. [1] Let $\tilde{A}$ and $\tilde{B}$ are two normal, convex and continuous (NCC) fuzzy sets. Then

$$\tilde{A} \cong \tilde{B}\ \ if\ \ and\ \ only\ \ if\ \ ac(\tilde{A}) = ac(\tilde{B})$$

Definition 2.8. [7] Two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are said to be equal and is denoted as $\tilde{A} = \tilde{B}$ if and only if

$$\forall\ x \in X \quad \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x) \tag{2.9}$$

3 The properties of fuzzy zero number

This section is started by using a numerical example.

Example 3.1. Consider that fuzzy number $\tilde{D} = (-2, 0, 100000, 100000, -, -)$. We have:

$$0_{\tilde{D}} = (-2 - 50000, -50000, 50000, 50000, -, -) = \cup_\alpha[2\alpha - 50002, 50000]$$

and

$$\begin{aligned} \tilde{D}.0_{\tilde{D}} &= \cup_\alpha[50000(2\alpha - 50002), 50000(50000)] \\ &= (-2500100000 - 2500000000, 2500000000, 2500000000, -, -) \end{aligned}$$

Hence $ac\left(\tilde{D}.0_{\tilde{D}}\right) = ac\left(0_{\tilde{D}}\right) = 0$. That is $\tilde{D}.0_{\tilde{D}} \cong 0_{\tilde{D}}$. As it is seen in this example two fuzzy numbers $\tilde{D}.0_{\tilde{D}}$ and $0_{\tilde{D}}$ are considered approximately equal. However, the difference spreads are so much. Hence, in the following the conditions are discussed where using fuzzy arithmetic operations based on TA, two fuzzy numbers $\tilde{D}.0_{\tilde{D}}$ and $0_{\tilde{D}}$ be exactly equal.

Theorem 3.2. Consider the pseudo-triangular fuzzy number $\tilde{A} = (\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x)) = \cup_\alpha\left[\underline{A}_\alpha, \overline{A}_\alpha\right]$ and its corresponding zero $0_{\tilde{A}} = (\underline{a} - a, 0, \overline{a} - a, l_{\tilde{A}}(x + a), r_{\tilde{A}}(x + a)) = \cup_\alpha\left[\underline{A}_\alpha - a, \overline{A}_\alpha - a\right]$. If for $a \geq 0$ we have $a = 2$ or $\underline{A}_\alpha = \overline{A}_\alpha = a$ and for $a \leq 0$ we have $\underline{A}_\alpha = \overline{A}_\alpha = a$ or $a = -2$, then $\tilde{A}.0_{\tilde{A}} = 0_{\tilde{A}}$.

Proof . Two fuzzy numbers are equal if their membership functions are equal or for each $\alpha \in [0, 1]$, their α-cuts be equal. Hence, in order to $\tilde{A}.0_{\tilde{A}} = 0_{\tilde{A}}$, for each $\alpha \in [0, 1]$ we show $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = (0_{\tilde{A}})_\alpha$, we consider two situations:

(i) If $a \geq 0$, then we obtain from (2.3), $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = \cup_\alpha\left[\left(\frac{a}{2}\right)\left(\underline{A}_\alpha - a\right), \left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right)\right]$, we have:

$$\begin{cases} a = 2 \\ or \\ \underline{A}_\alpha = \overline{A}_\alpha = a \end{cases} \Rightarrow \begin{cases} \left(\frac{a}{2}\right)\left(\underline{A}_\alpha - a\right) = \left(\underline{A}_\alpha - a\right) \\ \left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right) = \left(\overline{A}_\alpha - a\right) \end{cases} \Rightarrow \left[\left(\frac{a}{2}\right)\left(\underline{A}_\alpha - a\right), \left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right)\right] = \left[\left(\underline{A}_\alpha - a\right), \left(\overline{A}_\alpha - a\right)\right]$$

Therefore $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = (0_{\tilde{A}})_\alpha$.

(ii) If $a \leq 0$, then we obtain from (2.3), $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = \cup_\alpha \left[\left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right), \left(\frac{a}{2}\right)\left(\underline{A}_\alpha - a\right)\right]$, on other hand:

$$\begin{cases} a = -2 \\ or \\ \underline{A}_\alpha = \overline{A}_\alpha = a \end{cases} \Rightarrow \begin{cases} \left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right) = \left(\underline{A}_\alpha - a\right) \\ \left(\frac{a}{2}\right)\left(\underline{A}_\alpha - a\right) = \left(\overline{A}_\alpha - a\right) \end{cases} \Rightarrow \left[\left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right), \left(\frac{a}{2}\right)\left(\overline{A}_\alpha - a\right)\right] = \left[\left(\underline{A}_\alpha - a\right), \left(\overline{A}_\alpha - a\right)\right]$$

Therefore $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = (0_{\tilde{A}})_\alpha$, so the proof is completed.

□

Theorem 3.3. Consider the pseudo-triangular fuzzy number $\tilde{A} = (\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x)) = \cup_\alpha \left[\underline{A}_\alpha, \overline{A}_\alpha\right]$ and its corresponding zero $0_{\tilde{A}} = \left(\underline{a} - \phi, \frac{a_1 - a_2}{2}, \frac{a_2 - a_1}{2}, \overline{a} - \phi, l_{\tilde{A}}(x + \phi), r_{\tilde{A}}(x + \phi)\right) = \cup_\alpha \left[\underline{A}_\alpha - \phi, \overline{A}_\alpha - \phi\right]$. for $\phi \geq 0$ if $\phi = 2$ or $\underline{A}_\alpha = \overline{A}_\alpha = \phi$ and for $\phi \leq 0$ if $\underline{A}_\alpha = \overline{A}_\alpha = \phi$ or $\phi = -2$, then $\left(\tilde{A}.0_{\tilde{A}}\right)_\alpha = (0_{\tilde{A}})_\alpha$.

Proof . The proof is similar to the previous one. □

4 Numerical Example

In this section the applicability of mentioned topics are presented by solving some numerical examples.

Example 4.1. Consider the fuzzy number $\tilde{A}_1 = \left(1, 2, 4, \left(1 - (x-2)^2\right)^{\frac{1}{2}}, \left(1 - \frac{1}{4}(x-2)^2\right)^{\frac{1}{2}}\right)$ in example 3.1 of [1]. From (2.7) it is obtained that $0_{\tilde{A}_1} = \left(-1, 0, 2, \left(1 - x^2\right)^{\frac{1}{2}}, \left(1 - \frac{1}{4}x^2\right)^{\frac{1}{2}}\right)$. We have:

$$\tilde{A}_1 = \cup_\alpha \left[2 - \sqrt{1-\alpha^2}, 2 + 2\sqrt{1-\alpha^2}\right], 0_{\tilde{A}_1} = \cup_\alpha \left[-\sqrt{1-\alpha^2}, 2\sqrt{1-\alpha^2}\right].$$

From (2.3), it is obtained:

$$\tilde{A}_1, 0_{\tilde{A}_1} = \cup_\alpha \left[-\sqrt{1-\alpha^2}, 2\sqrt{1-\alpha^2}\right] = 0_{\tilde{A}_1}.$$

As $a = 2$, it is obtained that $\tilde{A}_1.0_{\tilde{A}_1} = 0_{\tilde{A}_1}$. (theorem 3.1 is holed)

Example 4.2. Consider the fuzzy number $\tilde{A}_7 = \left(\frac{3}{4}, 1, \frac{5}{4}, -, -\right)$ in example 3.1 of [1]. From (2.7), $0_{\tilde{A}_7} = \left(-\frac{1}{4}, 0, \frac{1}{4}, 4(x + \frac{1}{4}), -4(x - \frac{1}{4})\right)$, we have:

$$\tilde{A}_7 = \cup_\alpha \left[\frac{\alpha}{4} + \frac{3}{4}, -\frac{\alpha}{4} + \frac{5}{4}\right], \qquad 0_{\tilde{A}_7} = \cup_\alpha \left[\frac{\alpha}{4} - \frac{1}{4}, -\frac{\alpha}{4} + \frac{1}{4}\right].$$

From (2.3), it is obtained:

$$\tilde{A}_7.0_{\tilde{A}_7} = \cup_\alpha \left[\frac{1}{2}\left(\frac{\alpha}{4} - \frac{1}{4}\right), \frac{1}{2}\left(-\frac{\alpha}{4} + \frac{1}{4}\right)\right].$$

It is observed that as the conditions of the theorem 3.2 are not held the equality is not held and we have $\tilde{A}_7.0_{\tilde{A}_7} \neq 0_{\tilde{A}_7}$.

Example 4.3. Consider the fuzzy number $\tilde{D} = (-2, 0, 1, 1, -, -)$. From (2.8), $0_{\tilde{D}} = \left(-\frac{5}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\left(x + \frac{5}{2}\right), \frac{1}{2}\right)$, we have:

$$\tilde{D} = \cup_\alpha [2\alpha - 2, 1], \qquad 0_{\tilde{D}} = \cup_\alpha \left[2\alpha - \frac{5}{2}, \frac{1}{2}\right].$$

From (6), it is obtained:

$$\tilde{D}.0_{\tilde{D}} = \cup_\alpha \left[\frac{1}{4}\left(2\alpha - \frac{5}{2}\right), \frac{1}{4}\left(\frac{1}{2}\right)\right] \neq 0_{\tilde{D}}.$$

It is observed that as the conditions of the theorem 3.3 are not held the equality is not held, we have $\tilde{D}.0_{\tilde{D}} \neq 0_{\tilde{D}}$.

5 Conclusion

In this paper, first; the defined arithmetical operations and neutral element on the basis of TA by Abbasi et al. [1] are indicated. Following some examples and theorems are presented to show that using the defined arithmetical operations and neutral element on the basis of TA it is concluded that $\tilde{A}.0_{\tilde{A}} \neq 0_{\tilde{A}}$. At the end the condition where $\tilde{A}.0_{\tilde{A}} = 0_{\tilde{A}}.\tilde{A} = 0_{\tilde{A}}$ is expressed. In future work, we are hoped to define neutral member and arithmetic operators based on TA such that the field has a unique neutral member.

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