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Two effective methods for extract soliton solutions of the reaction-diffusion equations

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Abstract

In this present study, we reduce the fractional reaction–diffusion equation to a traditional differential equation using the fractional complex transformation and consider the Landau Lifshitz (LLG) equation. Moreover, by using the generalized exponential rational function method and Kudryashov's method respectively we extract new exact and solitary wave solutions for these equations. Some plots of some presented new solutions are represented to exhibit wave characteristics. All results in this paper are essential to understand the physical meaning and behavior of the investigated equation that sheds light on the importance of investigating various nonlinear wave phenomena in mathematical physics.

Keywords: fractional reaction–diffusion equation Landau Lifshitz (LLG), Kudryashov's method, solitary wave, rational function method, Partial differential equation
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1 Introduction

Recently, fractional calculus has played an important role in many fields of science, and there are many ways to define the fractional derivative. For example, the Riemann–Liouville derivative, Jumarie's modified Riemann–Liouville derivative [9], conformal fractional derivative [10] and so on. However, Liu has already proved that the formulae proposed by Jumarie about the modified Riemann–Liouville derivative are wrong [14, 15]. On the other hand, since the usual derivative rules hold for conformal fractional derivatives, the corresponding fractional differential equations can be reduced to usual differential equations. In this paper, we consider the (1+1)-dimensional general fractional nonlinear reaction-diffusion equation by using the definition of the conformal fractional derivative, namely

$$\frac{\partial^\alpha u}{\partial t^\alpha} = a\left(u^{n+1}\right)_x + b\left(u^{m+1}\right)_{xx} + \lambda u\left(1-u^k\right), \tag{1.1}$$

where $\frac{\partial^\alpha}{\partial t^\alpha}$ represents the conformal fractional derivative, n, m and $k$are nature numbers, λ is an arbitrary constant and $u$is the function to be determined and Landau-Lifshitz (LLG) equation can be written down as

$$\frac{\partial}{\partial \mathrm{t}}\mathrm{S} = \mathrm{S}\Lambda\Delta\mathrm{S} - \beta\mathrm{S}\Lambda\left(\mathrm{S}\Lambda\Delta\mathrm{S}\right). \tag{1.2}$$

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Here, $S = (\mathrm{S}_1\,(\mathrm{t}, \vec{\mathrm{x}})\,,\ \mathrm{S}_2\,(\mathrm{t}, \vec{\mathrm{x}})\,,\ \mathrm{S}_3\,(\mathrm{t}, \vec{\mathrm{x}})) \in \mathrm{S}^2 \to \mathrm{R}^3$, $\alpha \ \geq 0$, $\alpha\ ^2{+}\beta^2{=}\ 1$, Λ denotes the cross product. The term multiplying with α represents the exchange interaction, while β-term denotes to the Gilbert damping term. The conformal fractional derivative is defined by

$$D_t^\alpha\,(u\,(t)) = \lim_{h\to 0}\frac{u\left(t + ht^{\alpha-1}\right) - u\,(t)}{h}.$$

From Ref. [10], if the limitation (1.2) exists, then we have the following basic properties

$$\begin{array}{ll} 1) & D_t^\alpha\,(u\,(t) \pm v\,(t)) = D_t^\alpha\,(u\,(t)) \pm D_t^\alpha\,(v\,(t))\,, \\ 2) & D_t^\alpha\,(u\,(t)\,v\,(t)) = D_t^\alpha\,(u\,(t))\,v\,(t) \pm D_t^\alpha\,(v\,(t))\,u\,(t)\,, \\ 3) & D_t^\alpha\,(u\,(t)\,/v\,(t)) = \frac{D_t^\alpha(u(t))v(t) - D_t^\alpha(v(t))u(t)}{v^2(t)}, \\ 4) & D_t^\alpha\,(u\,(t)) = t^{1-\alpha}D_t^\alpha\,(u\,(t))\,. \end{array}$$

The detailed proofs can be found in [10]. For example for proof (1.3), let $h = \varepsilon t^{1-\alpha}$ in definition (3-1) and then $\varepsilon = ht^{\alpha-1}$. Therefore $(D_t^\alpha = T_\alpha)$

$$\begin{aligned} T_\alpha(f)(t) &= \lim_{\varepsilon\to 0}\tfrac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon} \\ &= \lim_{h\to 0}\tfrac{f(t+h)-f(t)}{ht^{1-\alpha}} \\ &= t^{1-\alpha}\lim_{h\to 0}\tfrac{f(t+h)-f(t)}{h} \\ &= t^{1-\alpha}\tfrac{df}{dt}\,(t) \end{aligned}$$

Other results about fractional calculus and conformable derivatives can be found in [1, 3, 4, 5, 6, 7, 8, 13, 19, 21].

Eq. (1.1) contains many famous equations, for example, if $a = \lambda = m = 0$ and $\alpha =\ 1$, then it becomes the well-known linear diffusion equation

$$\frac{\partial u}{\partial t} = u_{xx}. \tag{1.3}$$

Alternatively, if $n = m = 0$ and $a\ = \lambda = 1$, then Eq. (1.1) reduces to a Fisher-type equation. If we also set $k\ =\ 1$then we get the fractional Fisher equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} = u_{xx} + u(1-u). \tag{1.4}$$

Moreover, if we take $\lambda = 0$ and $a\ =\ b\ =\ 1$, Eq. (1.1) just becomes the fractional Fokker–Planck (FP) equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} = {u_x}^{n+1} + {u_{xx}}^{m+1} \tag{1.5}$$

The reaction-diffusion equation has been studied for many years and in a large number of studies. For example, E. V. Krishnan used the hyperbolic function method to obtain exact travelling wave solutions to the reaction-diffusion equation [12]. Deng [11] used the finite-element method to solve the space and time fractional FP equation. He proved that the convergence order is $O(k^{2-\alpha} +\ h^\mu)$, where k is the time step size and h is the space step size. Lie symmetry analysis of the fractional FP equation was conducted by M. S. Hashemi, and he found exact analytical solutions using the reduction method [4]. Mao used the canonical-like transformation method and the trail equation method to investigate the Chaffee-infante equation [2], which is another famous reaction-diffusion equation. Some new solutions in the form of the elliptic functions were shown in that study, which is very difficult to obtain by other methods [16, 18, 20]. In this paper, we focus on constructing exact solutions to Eq. (1.1). Of course, there is no exact solution to the general equation, so we present several situations where an exact solution exists and show how to obtain it by various methods. Discussions about anomalous diffusion are presented and in order to better understand the dynamical properties of the solutions, specific examples are given and plotted.

2 Basic structure for generalized exponential rational function method

Let us consider a typical non-linear PDE for$q\ =\ q\,(x,\ t)$, giving by

$$N\,(q,\ q_x,\ q_t\ ,\ q_{xx},\ \ldots)\ =\ 0. \tag{2.1}$$

Under the wave transformations of $q\,(x,\ t)\ =\ Q(\xi)\text{and}\xi = \sigma x - lt$, equation (1.2) becomes an ordinary differential equation given by:

$$N(Q, \sigma Q', -lQ', \sigma^2 Q'',\ \ldots)\ =\ 0. \tag{2.2}$$

Now, we assume that Equation (2.2) admits the exact solution giving by

$$Q(\xi)\ =\ A_0\ + \sum_{k=1}^{N} A_k \Phi(\xi)^k\ + \sum_{k=1}^{N} B_k \Phi(\xi)^{-k}, \tag{2.3}$$

where

$$\Phi(\xi)\ = \frac{m_1 e^{n_1\xi} +\ m_2 e^{n_2\xi}}{m_3 e^{n_3\xi} +\ m_4 e^{n_4\xi}} \tag{2.4}$$

and m_i, n_i and A_0, A_k, and B_k's are disposal parameters. Finally, N is a constant, which is evaluated by applying the homogeneous balance to equation (2.4).

Inserting Equation (2.3) into (2.2) with equation (2.4), and then gathering all possible powers of $\xi_i\ =\ e^{n_i\xi}$ for $i\ =\ 1,\ \ldots,\ 4$, forms a polynomial equation as $P\,(\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3,\ \varepsilon_4)\ =\ 0$. Equating coefficients of P to zero, one derives a simultaneous system of equations regarding $m_i,\ n_i(1 \le i \le 4)$, and $\sigma,\ l, A_0, A_k$ and $B_k\ (1 \le k \le N)$.

Finally, solving the non-linear system and substituting the obtained solutions in Equations (1.3) and (1.4), the explicit form of the solutions of (1.2) will be extracted.

3 Application to the fractional reaction–diffusion equation

In this case we consider the fractional transformation is given by:

$$\tau = \left(\frac{1}{\alpha}\right) t^{\alpha}. \tag{3.1}$$

Using this, Eq. (1.1) is reduced to the following traditional partial differential equation:

$$\frac{\partial u}{\partial \tau} = a\left(u^{n+1}\right)_x + b\left(u^{m+1}\right)_{xx} + \lambda u\left(1 - u^k\right), \tag{3.2}$$

Now we will use the traditional wave transformation:

$$u\,(x,\tau) = U\,(\xi)\,, \qquad \xi =\ r\tau + lx$$

then Eq. (3.2)

$$rU' = al\left(U^{n+1}\right)' + bl^2\left(U^{m+1}\right)'' + \lambda U\left(1 - U^k\right). \tag{3.3}$$

In this section, we consider $n = m = 0$ and $\lambda = k = 1$ so we have

$$rU' = alU' + bl^2U'' + U\,(1 - U)\,. \tag{3.4}$$

So

$$U - U^2 + (al - r)\,U' + bl^2U'' = 0. \tag{3.5}$$

The homogeneous balance in equation (3.5) suggests $N = 2$. Setting $N = 2$ along with equation (2.3), one gets

$$U(\xi)\ =\ A_0\ + A_1\Phi(\xi) + A_2\Phi(\xi)^2\ + \frac{B_1}{\Phi(\xi)} + \frac{B_2}{\Phi(\xi)^2}. \tag{3.6}$$

Inserting (3.6) into (3.5) and pursuing the steps outlined for the method, the analytical solutions for the equation (1.1) will be determined consequently.

Family 1: In order we consider $r = [-1, 0, 1, 1]$ and $s = [1, 0, 1, 0]$, then we have equation (3.3) as follows

$$\Psi\,(\xi) = -\frac{1}{1 + e^{\xi}}. \tag{3.7}$$

By substituting (3.7) in (3.5) along with (3.6)

$$A_0 = 0, A_1 = \frac{1}{2}, A_2 = -\frac{1}{2}, B_1 = 1, B_2 = 0, r = \frac{1}{2}, l = \frac{1}{8}.$$

So

$$u_1(x,t) = \frac{1}{2}\left(-\frac{1}{1+e^{\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{1}{8}x}}\right) - \frac{1}{2}\left(-\frac{1}{1+e^{\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{1}{8}x}}\right)^2 - \left(1+e^{\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{1}{8}x}\right)^2.$$

Family 2: in this set we consider $r = [3,1,1,1]$ and $s = [1,1,1,1]$, then we have equation (3.9) as follows

$$\Psi(\xi) = -\frac{2\cosh(\xi) + \sinh(\xi)}{\cosh(\xi)}. \tag{3.8}$$

By substituting (3.8) in (3.5) along with (3.6)

$$A_0 = \frac{1}{2}, A_1 = -\frac{1}{2}, A_2 = 0, B_1 = 0, B_2 = 0, r = -\frac{1}{2}, l = -\frac{1}{8}.$$

So

$$u_2(x,t) = \frac{1}{2} - \frac{1}{2}\left(-\frac{2\cosh\left(-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha} - \frac{1}{8}x\right) + \sinh\left(-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha} - \frac{1}{8}x\right)}{\cosh\left(-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha} - \frac{1}{8}x\right)}\right).$$

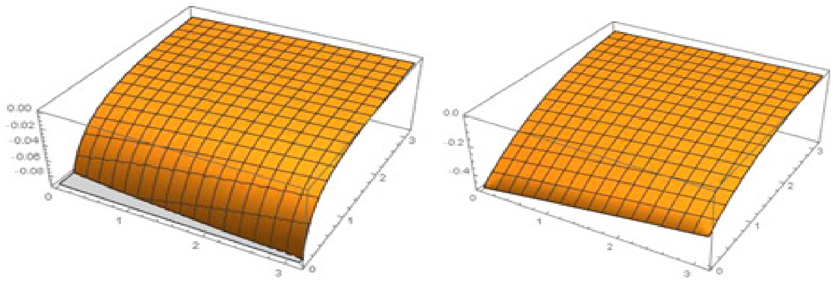


Image 1: Plot3d of $u_2(x,t)$ for $\alpha = 0.3$ and $\alpha = 0.9$ respectively for $x = 0.\pi, t = 0.\pi$.

Family 3: in this set we consider $r = [1;1;1;1]$ and $s = [1,1,1,1]$, then we have equation (3.3) as follows

$$\Psi(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{3.9}$$

By substituting (3.9) in (3.5) along with (3.6)

$$A_0 = \frac{1}{2}, A_1 = \frac{1}{2}, A_2 = 0, B_1 = -1, B_2 = 1, r = 1, l = -1.$$

So

$$u_3(x,t) = \frac{1}{2} + \frac{1}{2}\left(-\frac{\cosh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)}{\sinh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)}\right) + \frac{\sinh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)}{\cosh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)} + \left(\frac{\sinh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)}{\cosh\left(\left(\frac{1}{\alpha}\right)t^{\alpha} - x\right)}\right)^2.$$

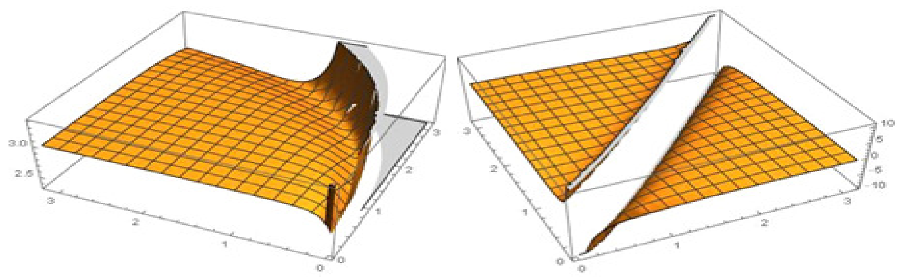


Image 2: Plot3d of $u_2(x,t)$ for $\alpha = 0.3$ and $\alpha = 0.9$ respectively for $x = 0.\pi, t = 0.\pi$.

Family 4: in this set we consider $r = [3;2;1;1]$ and $s = [1;0;1;0]$, then we have equation (3.3) as follows

$$\Psi(\xi) = \frac{3e^{\xi}+2}{1+e^{\xi}}. \tag{3.10}$$

By substituting (3.10) in (3.5) along with (3.6)

$$A_0 = 0,\, A_1 = 0, A_2 = -\frac{1}{2}, B_1 = 0, B_2 = 1, r = -\frac{1}{2}, l = \frac{3}{8}.$$

So

$$u_4(x,t) = -\frac{1}{2}\left(\frac{3e^{-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{3}{8}x}+2}{1+e^{-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{3}{8}x}}\right)^2 + \left(\frac{1+e^{-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{3}{8}x}}{3e^{-\frac{1}{2}\left(\frac{1}{\alpha}\right)t^{\alpha}+\frac{3}{8}x}+2}\right)^2.$$

4 Travelling wave solution of the LLG equation via Kudryashov method

Landau-Lifshitz (LLG) equation (see [12]) can be written down as.

$$\frac{\partial}{\partial \mathrm{t}}\mathrm{S} = \mathrm{S}\Lambda\Delta\mathrm{S} - \beta\mathrm{S}\Lambda\left(\mathrm{S}\Lambda\Delta\mathrm{S}\right). \tag{4.1}$$

Here, $S = (\mathrm{S}_1(\mathrm{t},\vec{\mathrm{x}}), \mathrm{S}_2(\mathrm{t},\vec{\mathrm{x}}), \mathrm{S}_3(\mathrm{t},\vec{\mathrm{x}})) \in \mathrm{S}^2 \to \mathrm{R}^3$, $\alpha \geq 0$, $\alpha^2 + \beta^2 = 1$,Λ denotes the cross product. The term multiplying with α represents the exchange interaction, while β-term denotes to the Gilbert damping term. According to the setting of (4.1), S lies on $\mathbb{S}^2$ which allow us to use the conversion as follows

$$(\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3,) = \left(\frac{2\mathrm{R}(\mathrm{W})}{1+\mathrm{W}\bar{\mathrm{W}}}, \frac{2\mathrm{I}(\mathrm{W})}{1+\mathrm{W}\bar{\mathrm{W}}}, \frac{1-\mathrm{W}\bar{\mathrm{W}}}{1+\mathrm{W}\bar{\mathrm{W}}}\right), \quad \text{and} \quad \mathrm{W} = \frac{\mathrm{S}_1 + i\mathrm{S}_2}{1+\mathrm{S}_3} \tag{4.2}$$

where denotes the conjugate complex numbers of W; the real part and the imaginary part of the complex number W are $\Re(W)$ and $\Im(W)$ respectively.

According to (4.1) and (4.2), we can obtain the complex equation of W as follows

$$-(\alpha+\beta\mathrm{i})\,\mathrm{i}\mathrm{W}_{\mathrm{t}} = \Delta\mathrm{W} - \frac{2\bar{\mathrm{W}}}{1+|\mathrm{W}|^2} < \nabla\mathrm{W} >, \tag{4.3}$$

where <A>= A. A denotes the inner product of the vectors. Under the arbitrary integer n, we set K_i $i = (1,2,\ldots,n)$ are constants satisfying $\Sigma_{i=0}^{n} K_i^2 = 1$; $\vec{K} = (K_1, K_2, K_3, \ldots, K_n)$ and $\bar{r} = \vec{K} \cdot \vec{x}$. In this situation, (4.3) transform into

$$-(\alpha+\beta i)\, iW_t = W_{\bar{r}\bar{r}} - \frac{2\bar{W}\langle W_r\rangle}{1+|W|^2}. \tag{4.4}$$

In this section, we construct a travelling wave solution without the Gilbert term. Assuming $\alpha = 1$ and $\beta = 0$, we suppose that the solution of (4.4) under the condition of Kudryashov method is as follows

$$\mathrm{W}_{\mathrm{c,w}}(\mathrm{t},\bar{\mathrm{r}}) = \mathrm{e}^{-\mathrm{iwt}}\phi(\bar{\mathrm{r}}-\mathrm{ct})\,\mathrm{e}^{\mathrm{i}\psi(\bar{\mathrm{r}}-\mathrm{ct})} \tag{4.5}$$

where c and w are constants undetermined. Here we assume $-c^2+4w > 0$. Substitute (4.5) into (4.4), the separate and the real part and the virtual part respectively as

$$\phi(\xi)\left(w - 2(\xi)^2 + c\psi'(\xi) - \psi'(\xi)^2\right) + \phi(\xi)^3\left(w + c\psi'(\xi) + \psi'(\xi)^2\right) + \phi(\xi)^2\phi''(\xi) = 0 \tag{4.6}$$

and

$$\phi'(\xi)(-c+2\psi'(\xi)) - \phi(\xi)^2\phi'(\xi)(c+2\psi'(\xi)) + \phi(\xi)\psi''(\xi) + \phi(\xi)^3\psi''(\xi) = 0 \tag{4.7}$$

where $\xi = \bar{r} - ct$. (4.6)-(4.7) are the nonlinear constant coefficients ordinary differential equation system with the variable ξ. According to (4.7), we can obtain a relationship between ψ and ϕ

$$\psi'(\xi) = \frac{\left(1+\phi(\xi)^2\right)\left(-c+2C_1+2C_1(\xi)^2\right)}{2\phi(\xi)^2}, \tag{4.8}$$

where C_1 is arbitrary constant. If we set C_1=0, we have

$$\psi'(\xi) = -\frac{c\left(1+\phi(\xi)^2\right)}{2\phi(\xi)^2}. \tag{4.9}$$

Substituting (4.9) into (4.6) to get

$$\begin{aligned} &c^2 + 3c^2\,\phi(\xi)^2 + \left(c^2 - 4w\right)\phi(\xi)^6 + \phi(\xi)^4\left(3c^2 - 4w + 8\phi'(\xi)^2\,\phi(\xi)^6\right) - \\ &\phi(\xi)^3\left(1 + \phi(\xi)^2\right)\phi''(\xi) = 0, \end{aligned} \tag{4.10}$$

to solve (4.10), we assume that the solution $\phi(\xi)$ of the nonlinear Eq. (4.10) can be presented as

$$\phi(\xi) = \sum_{i=0}^{M} A_i \Upsilon^i(\xi), \tag{4.11}$$

and Υ satisfied in following Riccati equation

$$\Upsilon'(\xi) = \Upsilon^2(\xi) - \Upsilon(\xi). \tag{4.12}$$

Eq. (1.5) gives the solution, as follows:

$$\Upsilon(\xi) = \frac{1}{1+e^{\xi}}. \tag{4.13}$$

Substituting Eqs (4.12)-(4.11) into Eq. (4.10) and collecting all terms with the same order of Υ^j together, we convert the left-hand side of Eq. (4.10) into a polynomial in Υ^j. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations forA_0, A_1, A_2 and h. By solving these algebraic equations, we obtain several case of variables solutions [11, 12].

Remark: This Riccati equation (4.12) also admits the following exact solutions:

$$\phi_1(\xi) = \frac{1}{2}\left(1 - \tanh\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 > 0, \tag{4.14}$$

$$\phi_2(\xi) = \frac{1}{2}\left(1 - \coth\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 < 0. \tag{4.15}$$

Stage 3: By substituting the obtained solutions in stage ***2*** into Eq. (4.10) along with general solutions of Eq. (4.12), finally generates new exact solutions for the nonlinear PDE (1.2).

5 Results

By Kudryashov's method, the solution of (4.10) is assumed as

$$\phi(\xi) = A_1\Upsilon(\xi) + A_0,$$

where A_1 and $A_0$are constants. Substituting (3.5) into (3.3) and comparing the coefficients of alike powers of $\Upsilon(\xi)$ provides algebraic system of equations. After solving the system, the A_i, $i = 0, 1$ are obtained and produces following new sets of solution for (4.10).

Case-1

$$A_1 = \frac{12}{5}, A_0 = \frac{3}{4}, c = -1, w = \frac{1}{2}.$$

From (2.3) we have

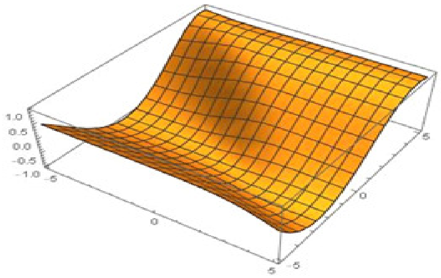
$$\psi(\xi) = -\frac{1}{2}c\left(\phi^{-1}(\xi) + \phi(\xi) + 2\right).$$

So

$$\psi(\xi) = -\frac{1}{2}c\left(\left(A_1\frac{1}{1+e^{\bar{r}-ct}} + A_0\right)^{-1} + \left(A_1\frac{1}{1+e^{\bar{r}-ct}} + A_0\right) + 2\right).$$

So, the exact solution for (1.2) is constructed as

$$w_{c,m}\left(t,\bar{r}\right)=e^{-\frac{1}{2}it}\left(\frac{12}{5}\frac{1}{1+e^{\bar{r}+t}}+\frac{3}{4}\right)e^{\frac{1}{2}i\left(\left(\frac{12}{5}\frac{1}{1+e^{\bar{r}+t}}+\frac{3}{4}\right)^{-1}+\left(\frac{12}{5}\frac{1}{1+e^{\bar{r}+t}}+\frac{3}{4}\right)+2\right)}.$$



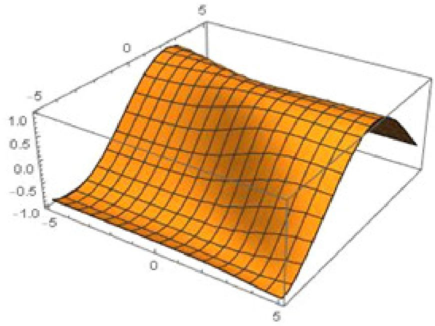


Image 3: Real part of $w_{c,m}\left(t,\bar{r}\right)$ Image 4: Imaginarily part of $w_{c,m}\left(t,\bar{r}\right)$

Case-2:

$$A_1=-\frac{12}{5},A_0=-\frac{3}{4},c=-\frac{1}{2},w=1.$$

The exact solution for (1.2) is obtained as

$$w_{c,m}\left(t,\bar{r}\right)=e^{-it}\left(-\frac{12}{5}\frac{1}{1+e^{\bar{r}+\frac{1}{2}t}}-\frac{3}{4}\right)e^{-\frac{1}{4}i\left(\left(-\frac{12}{5}\frac{1}{1+e^{\bar{r}+\frac{1}{2}t}}-\frac{3}{4}\right)^{-1}+\left(-\frac{12}{5}\frac{1}{1+e^{\bar{r}+\frac{1}{2}t}}-\frac{3}{4}\right)+2\right)}.$$

Case-3 $A_1=-\frac{1}{2},A_0=-1,c=\frac{3}{2},w=-1$ $A_1=-\frac{1}{2},A_0=-1,c=\frac{3}{2},w=-1.$

The exact solution of (1.2) is attained as

$$w_{c,m}\left(t,\bar{r}\right)=e^{it}\left(-\frac{1}{2}\frac{1}{1+e^{\bar{r}-\frac{3}{2}t}}-1\right)e^{-\frac{3}{4}i\left(\left(-\frac{1}{2}\frac{1}{1+e^{\bar{r}-\frac{3}{2}t}}-1\right)^{-1}+\left(-\frac{1}{2}\frac{1}{1+e^{\bar{r}-\frac{3}{2}t}}-1\right)+2\right)}.$$

Case-4

$$A_1=\frac{1}{2},A_0=1,c=-\frac{3}{2},w=1.$$

The exact solution for (1.2) is given by

$$w_{c,m}\left(t,\bar{r}\right)=e^{-it}\left(\frac{1}{2}\frac{1}{1+e^{\bar{r}+\frac{3}{2}t}}+1\right)e^{\frac{3}{4}i\left(\left(\frac{1}{2}\frac{1}{1+e^{\bar{r}+\frac{3}{2}t}}+1\right)^{-1}+\left(\frac{1}{2}\frac{1}{1+e^{\bar{r}+\frac{3}{2}t}}+1\right)+2\right)}.$$

6 Concluding remarks

In this paper, some new solitary soliton solutions of the fractional reaction-diffusion equation and Landau-Lifshitz (LLG) equation are obtained with the aid of efficient analytic methods. The structure considered for the equation consists of a series of arbitrary parameters that lead to many well-known models by considering certain options for them. One of the main advantages of this method is the determination of different categories of solutions for the equation in a single framework.

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