

# Comparative comparison of fuzzy logic and classical logic

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## Abstract

When hearing the term fuzzy logic, the first thing that the mind immediately goes to is the field of non-deterministic arguments, and if we want to use a term close to it to make its meaning a little clearer, we can say that fuzzy logic is the same as argument's logic. It is an approximation that, ironically, the literal meaning of the word "argument" also refers to the clarification of something by the same thing (function input or argument). Now the basic question is that while the educated mind of man has considered logic as equivalent to the mechanism of definite perceptions and accurate results for more than a thousand years, how can we talk about the invention of a logical method that is based on uncertainty? This question is doubly important from the aspect that our approximate arguments in the context of our lived life are more than our exact, definite and unambiguous arguments, that is, the same arguments that classical logic, whether in its old or new language, He leaves it aside and does not pay attention to them, and of course, he is forced to do this because he does not have the tools to analyze these arguments. This is where we unknowingly remind ourselves of Bacon's angry regret, who considers Aristotelian logic to be a fantasy that causes waste and diversion of great talents over many centuries, a continuous and constant effort without helping a person to improve the quality of his life; A bunch of delusional nonsense, the result of which, perhaps in the most optimistic state, is Scott's adversary and the victory of futility in verbal disputes, of course, if we do not say only pointless boasting and scrutiny based on rhetoric, the fruit of which is the explanation of rational axioms. If Mr. Bacon had known about fuzzy logic, he would have corrected his view of logic, and to be more precise, he would have separated Aristotle from logic, not logic from science. One of the most important advantages of fuzzy logic is that it can introduce a way to enter the knowledge and experiences of experts into intelligent systems that are based on sensitive and practical decisions and choices and develop the previous absolute correct and incorrect logic. The basis of the analysis in the present research is to examine the components of two types of logic, i.e. classical logic and fuzzy logic, and clarify the understanding of the logical concepts and philosophical foundations of classical logic, as well as compare it with the concepts of fuzzy logic. The research results indicate: that these two types of logic have fundamental differences from an epistemological point of view; Aristotle's epistemological foundation is based on the certainty-oriented duality of "either this/or that", while the fuzzy epistemological foundation is based on a multi-valued system and that both logics are a contradictory understanding of what the mind and the external world are and the connection between They both give hands.

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## 1 Introduction

Perhaps, when the Iranian genius mentioned the name of fuzzy logic for the first time in 1965, few people thought that this discovery would become the basis of the most practical and modern technical sciences and of course even some non-technical sciences in the field of humanities. For this reason, he and his colleagues suffered the special pain of geniuses for 20 years, that is, not being understood and being rejected; It means the first instinctive reaction of a human being against the unknown!

Fuzzy logic was introduced to the world for the first time by this Iranian professor from the University of California, and later it became the focus of researchers' attention due to its many applications in the field of mathematics and computers [6].

Logic is considered a form science because it researches and analyzes the relationships between forms, and its work is to simplify real examples to external examples with components. The Kurds added that's why they call that part systematized by Aristotle himself Aristotelian logic. Aristotelian logic including the margins and additional sections added to it over hundreds of years of thought by scientists, especially by Muslim logicians (such as Ibn Sina), is called classical logic, although Aristotelian logic and classical logic, although the word logic Aristotle is synonymous with the term classical logic [3, 4].

Although mathematics has been one of the most logical sciences since ancient times, until the middle of the 19th century, mathematicians did not pay much attention to the science of logic, and despite its use by them, they still did not consider it the basis of mathematics. In the beginning, Leibniz tried to bring mathematics and logic closer together, and he believed that a common statement based on reasoning should be used in scientific fields, it can be said that he was the inventor of new logic. At the end of the 19th century, with the growth of the science of mathematics and new problems facing this science, scholars of this science researched the foundations of this science. With the development of this research, the need for a more practical tool than Aristotle's logic was suddenly felt, and this was one of the factors in creating a new logic, i.e. inventing the language of signs in logic. In addition to the ease of expressing sentences, the use of symbols had many other benefits. In fact, part of the rules of logic, which is related to mathematical arguments and makes clear the connection between arithmetic and geometry and many similar cases, is known as mathematical logic (logistics).

Because of the value of face in this field of science, this science is also called face logic, and in general, according to the logic before the 19th century AD, which was based on Aristotelian logic, they call the logic that and logics that, like Russell, Frege, Demorgan founded after the 19th century and they use mathematical methods and symbols, they are called mathematical, symbolic and new logics, and if we want to define this type of logic in the exact sense of the word, Because of its scope, we will face a problem unless we give a definition to its examples, but if we limit ourselves to a limited definition, it can be called the science of requirements.

As for fuzzy logic, which should be considered a revolution in the epistemological and functional areas of logic, first and foremost. In its limited sense, it should be said that this science of logic is arguments whose uncertainty is their main characteristic, that is, they are approximate, and in a more complex and scientific sense, it is the same logic related to sets of a fuzzy nature, which by growing through the two walls of zero and 1 which forced mankind to unconscious dogmatism for hundreds of years, opened a horizon for the eyes of the new age that the greatest brains in history did not even dream of and this article tries to explain this wonder.

## 2 History and theoretical foundations of logic

### 2.1 A brief look at history

Logic, as it is known in our minds today, is the same Aristotelian organ that is used in Persian as the knowledge of balance or wisdom. According to history, Aristotle was the first person to establish and organize logic by discovering the laws of logic and organizing them. Before him, the materials and contents of this science were scattered in the culture of the civilized people of that time, but the rich talent of the Greeks was an extremely effective element in creating wisdom and various techniques. Logic also owes its existence to Aristotle, a precious philosopher who knew that formal logic and old logic also refer to the same logic. Ibn Ishaq translated the Greek word into Arabic. Aristotle's logical works were translated into Arabic in the second century of Hijri, and later explanations were added by Farabi, Ibn Sina, Khwaja Nasir, Qutbuddin Razi, etc. Aristotelian logic was dominant in the West until the Renaissance, but it was rejected by Descartes and Francis Bacon in the Renaissance, and Kant accepted and glorified it, and Leibniz considered complete logic until the last two centuries. A new logic prevailed and replaced Aristotelian logic. Linguistic logic and mathematical logic are part of his new logic [2].

## 2.2 Defining logic and stating its root

The word logic comes from the Greek root *logos*, which means reason, so reason and logic are equivalent, and the root of the same meaning is preserved in the Arabic language, and that word is *logic*, which refers to speech. In the definition of logic, they say:

Logic is a scientific tool that prevents the human mind from making mistakes in thinking; Thought and logic can be likened to a building whose formal rules are provided by formal logic and the rules that protect us from mistakes in content are called physical logic [2].

1-2- What are the types of logic?

The basic logical schools that have risen to the task of answering this question are:

Inferential logic originates from the act of thinking, so we must establish conventions so that the act of thinking returns to its base and prevents it from slipping. Inference is the explanation of the relationship between one idea and the validity of another idea, and therefore it is distinguished from induction, and evolutionary thinking is in line with the progress of science and has gone through four stages [4]:

- a) The work that Aristotle began in which he developed a logic that shows how groups attract people.
- b) What Muslim philosophers have done, who developed Aristotle's logic and added chapters to it, and that is a logic that seeks constant connections between facts.
- c) the development of logic towards symbols, which was also done by Frege and others; Symbolic logic is not only a tool for solving problems, but also a tool for clarifying meanings and increasing the power of mental work. The construction of symbolic logic was the research field of these two sciences in relation to each other.
- d) The idea of transforming formal logic into mathematical logic is finally the winner.

From the point of view of empirical logic, concepts and basic principles are the product of human emotions and experiences, and the main difference with the non-objective method is that empirical logic, instead of organizing observations, seeks knowledge and searches for it. The resulting part of a law or principle is obtained from the observation of phenomena. This school is rooted in the contemporary view of Socrates that "nature arises from nuclei" and the idea of "democracy" is defended by Bacon and other philosophers. But Hume considered the legitimacy of inductive reasoning to be impossible and recognized that it had tarnished his scientific image.

Social logic means that it considers social reality as the primary source of thought and thought and believes that research should be done in the social environment, because the social environment is the basis of thought. Human logic deals with the development of society, and based on that, man has three states: the complete transcendental state, the limited transcendental state, and the Gnostic state, which are the three states in which society also exists, as Durkheim later confirmed.

A field in which philosophical thoughts and research methods are closely related, such as dialectical logic, which was formed by Hegel and his students based on the constant movement and change of existence and the mutual influence of events and existence based on contradictions and the possibility of encountering changes. Pragmatist logic and situational logic were also developed in this context and grew in parallel with philosophical ideas.

Logical mathematics is one of the new disciplines that emerged after the critique of classical logic. In the eyes of some people in the East and the West, Aristotelian logic was regarded as the end of logic and the end of this science, therefore, Haj Molahadi Sabzevari, who wrote the introduction to the description of logical system, called Aristotelian logic as follows: This was the end of logic, and at the same time, everyone's opinion was expressed. that the German Kant cannot add a word to Aristotle's logic [9].

After Kant, mathematical logic was introduced and comprehensively edited by Frege. The importance of Frege's book was then revealed by the publication of the three-volume *Principles of Mathematics*, fully explained by Russell and Whitehead between 1910 and 1913, called the *Concept Mapping*, published and signed in 1879. There are two components or two basic elements, the nominal component and the verb component; The main difference between the new logic and Aristotle's thought is that the verb of the new logic also has a relationship component, that is, it is considered the main relationship, and the important part is the verb and, as a result, all logical objects and predicates. When analyzing a new concept, common words are transferred to the verb part with the difference between the object and the concept of Frege's theory [2].

### 2.3 The history of fuzzy logic

In 1965 at the University of California, Dr. Lotfizadeh published an article called "Fuzzy Sets" and a number of objects were used in this article. He called this collection "loose". The word Los literally means unknown, and as we can see, the choice of this name was not in vain. He did not want to publish any article for two years and finally published this article. He then published another paper with Belman<sup>5</sup> in 2007. This article formed the basis of Zimmerman's important work, which was published in 1987 in the form of an article on the linear vector optimization problem [13]. After this, many people expanded and improved his idea, and since then, this logic has advanced greatly in the field of electrical engineering, so that today dishwashers, cameras, and many control systems with unspecified control functions are on the market. For example, the dishwasher is monitored by a non-specific control system that, depending on the amount of stains and spots that need to be washed, introduces the appropriate detergent into the machine in the same proportion, and that is why it is completely flexible compared to the two-valued logic. And it is more accurate and useful. In previous dishwashers, it didn't matter if a dish was a little dirty or very dirty, and the mechanism of the machine was an unconscious mechanism, but in soft systems, a process similar to artificial intelligence prevails.

Artificial intelligence needs logic, but not Aristotelian logic, but a kind of flexible logic, similar to the way most and perhaps all people think, and the answer to this question is why existing computers cannot perform actions that are easily performed by humans.

Aristotle was one of the first to write about logic, and his work was circulated throughout the world for centuries. He was opposed to Plato's opinions and changed Plato's philosophy over the years, and because of his thought system, Plato's thoughts were quite intelligently transformed into Aristotle's teachings. In the Middle Ages, the conflict with Aristotle's writings increased due to his mystical view. Therefore, during this period, philosophical discussions grew significantly. Aristotle had a broad view of the government and considered the purpose of the government to elevate the elite and people with similar culture to himself, and he believed that people with such ideas should take power and of course divide the people into two categories, good and bad, friends and enemies, or divided rulers and slaves.

If the logic on which the idea is based includes these two and two, it is good, and thus this view of Aristotle's introduction was extended to Aristotelian logic, and according to him, the value of a proposition or sentence is either true or false, and the value of the whole sentence is true when Each of its clauses is true, and the value of the whole sentence is false if even one sentence is false. Aristotle saw values only in absolute darkness and light, as he says in the seventh chapter of the fourth book of Gamma: But in fact, there cannot be a middle ground between two opposites, and for one thing, only one thing must be demanded or denied. Worse, he rejects the thoughts of his predecessors who took the first step into the rational real world, and this is what slowed down the progress of human knowledge and civilization for nearly two thousand years, especially in the West; Such thinking, which sees the world in black and white, is basically unrelated to Eastern thinking and far from their beliefs, because in fact, as in the ancient religions of Iran, except for some religions and religions that arose after the logic of Plato and Aristotle Ahuramazda and Ahriman are two boundaries between good and evil, there were different groups between these two and there was a ratio of good and bad in demons and other creatures, and of course Buddhism also says the same thing with the smallest difference. Therefore, from the perspective of this profession, it is not surprising that an Iranian scientist has created the logical foundations on which the real world operates. But one cannot ignore the important question that Aristotelian logic was the basis of classical mathematics and this logic was the basis of the thinking of thinkers for 2000 years, but they were never sure what was right and wrong and in their opinion everything was either right or wrong, that is Every statement, law or rule is valid or not, and for more than 2 thousand years, these were Aristotle's principles that defined what is right and what is wrong from the point of view of philosophy, and this is, of course, the elegant logic that is necessary for science. sacrificed for ease, and although Aristotelian logic of zero and one is able to make mathematics and computer analysis easy, on the other hand, it removes thousands of other functions from its two-valued equation, which are necessary for various sciences and especially in normal human life. And it is vital [10].

### 2.4 Mind and language in fuzzy logic

As mentioned, logical reasoning is part of the cognitive process associated with unique ontology. In the previous lines, ontology has been briefly discussed in an abstract logic that could be extended to the studied subjects. In abstract, the mind and knowledge also have multiple structures and no mind can be a measure for other minds - as Kant wanted the heart to act as a complete mind - the mind itself does not have a specific and partial destiny and fuzzy logic helps us To understand how the human mind works and its processes, which are often complex, must be developed as a method.

In the case of cognition, before paying attention to the nature of the mind and how it works, the way of facing the outside and the goal is important, and this facing shows itself where it makes an epistemic and practical example. The mind and subconscious mind have a fluid, multi-layered and multi-faceted nature, and it is clear that the rulings that come from this unity in a multi-faceted mind are liquid and floating rulings; We will use the legal system to describe the problem, for example, if a person commits a public crime, the person is often found innocent or guilty, and thus, this term is often seen in law and jurisprudence, but with the application of the principle Rationality without reason in the legal and judicial system cannot be easily placed in the ranks of criminals, depending on the type and level of the person or act, several states and levels can be identified and the results are not certain. Everyone uses these classification rules in everyday language and natural language, and in this context, the deep relationship between mind and language is unclear, because the conceptual processes and their relationship with language and how natural language is used is still a mystery.

Logic, which always examines the structure and process of thought, must also analyze the structure of language, and Aristotelian thought only assumes the expression of content - like mathematics and new logic - but, as mentioned, abstract thinking takes the form of everyday and natural language. and both in the abstract sense and in natural language, we deal with different types of predicates. Based on these conditions, it is possible to talk about expressions in an undesirable language. Conducting linguistic and structural research based on formal and informal research on artificial intelligence is a type of natural language for computers. Therefore, this systematic issue should be investigated with a logical issue. It imposes a formal form and finally, technological achievements in terms of mental structure and abstract thinking in terms of language can bring the process of artificial intelligence closer to the human brain and give it creativity and decision-making power. It is also important to consider the relationship between abstract thinking and probability theory and the structure of the human brain here.

## 2.5 The concept of reality and the concept of the world in the dictionary of fuzzy logic

Reality and the world are the basic variables that the examination of the behavior of each approach to the problem shows us the basics of that approach. In fuzzy logic, one can find a unique and clear understanding of the outside world, which in Aristotle's mind has completely dealt with the outside world.

Aristotelian ontology is a kind of essentialism and every object and concept has a basis called meaning, which is realized with the help of signs such as quantity, quality, state and ratio. In this way, by knowing the nature of an object and knowing its related signs, the identity of an object is revealed. But fuzzy logic has a special understanding of the world, which is not only non-essential, but also against essentialism. As if the raw realistic judgment is not enough to fully understand it, but this realistic judgment is made up of twisted textures and layered levels and different contexts, which make us unable to have a correct understanding of existence, so we must Let's accept that the fluidity and current in fuzzy logic is not only caused by different and multiple fields of understanding, but also originates from the layered levels of the outside world itself. This type of identity of the external world is well explained by the results of modern science research such as physics and also the discussions of some advanced philosophers.

## 2.6 Multivalued logics as context

In classical logic, there are only two real values. Many-valued logic systems have more than two real values. The number of actual values can be limited or unlimited. For example, in a three-valued logic system, an expression can be true, false, or unknown. Logics appear with higher values depending on the level between true and false. Multi-valued logic was created by "Lucasy Vech" and Dr. Lotfizadeh laid the foundation of fuzzy logic on this basis, it should be said here that the distinction between these two types of logic is great; Ambiguous functions such as more and less, more and bulkier, etc. are used in fuzzy logic, while multi-valued logic does not accept such indicators. In multi-valued logic, the degree of truth has several parts, for example, a statement in three-valued logic can be true or false or ambiguous, but a statement is neither partially true nor partially false, and this is where fuzzy logic comes in. It is done.

## 2.7 Relationship between fuzzy logic and multi-valued regions

More than 50 years have passed since the discovery of fuzzy logic, and it has been more than three decades that the dominance of this logic has been proven in industry and other fields such as artificial intelligence, but there is still no clear boundary between this multi-valued fuzzy logic. Fuzzy logic and multi-valued domains In the world of science, when some people hear the name fuzzy, they imagine a multi-valued domain, and some keep these two logics completely separate from each other. There are two theories about the formation of fuzzy logic, the first one is that

fuzzy logic was born all at once, turned into a multi-valued field and multi-valued logic had no effect on its creation and growth, and the second theory claims that fuzzy logic It continues the work of scholars such as Lukaszczyk Veitch who believe in multi-valuedness. What we know is that one year before the birth of Dr. Lotfizadeh in 1921, the first published system of ternary logic was presented, and in that first year, the first known system of fields of infinite value was discovered, and Dr. Lotfizadeh in In his paper on fuzzy and in the topic of sets, he used the multi-valued logic of Lukaszczyk Veitch to describe sets and groups of objects and gives vague or multi-valued sets whose components belong to different degrees, or fuzzy names [11].

## 2.8 Differences between fuzzy logic and multi-valued regions

In fuzzy logic, the truth and even the nature of the content can be imprecise; In this logic, we have sentences like “absolutely true or more or less true”, even imprecise probabilities such as “almost” or not so and rarely can be used, and it is clear that the fuzzy logic system is completely different from natural language. It is flexible, whereas in multivalued logic, values other than zero and one are just prepositions. From a spiritual perspective, we are dealing with the human thought process, which is often rational and emotional, and in the form of meaningless words that cannot be clearly explained to humans. Considering what has happened, it seems that it is not without grace to name the factors of the emergence of multi-valued logics in the sense that they have been effective in the emergence of fuzzy logic or at least the motivation for the emergence of this logic [5]:

- The possibility of reception and the problem of algebra
- Reasons
- Uncertainty
- Ambiguity
- The emergence of quantum mechanics
- Semantic paradoxes
- The problem of meaning without precedent
- Undecidable sentences

## 3 Research method

### 3.1 Fuzzy logic

Fuzzy logic is a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false. By contrast, in Boolean logic, the truth values of variables may only be the integer values 0 or 1.

The term fuzzy logic was introduced with the 1965 proposal of fuzzy set theory by mathematician Lotfi Zadeh. studied since the 1920s, as infinite-valued logic-notably by Lukasiewicz and Tarski.

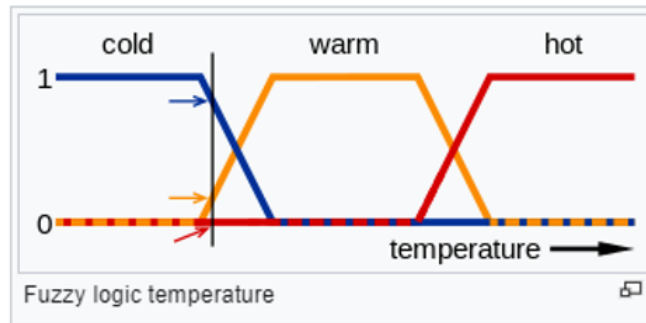
Fuzzy logic is based on the observation that people make decisions based on imprecise and non-numerical information. Fuzzy models or fuzzy sets are mathematical means of representing vagueness and imprecise information (hence the term fuzzy). These models have the capability of recognising, representing, manipulating, interpreting, and using data and information that are vague and lack certainty. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence.

### 3.2 Fuzzification

Fuzzification is the process of assigning the numerical input of a system to fuzzy sets with some degree of membership. This degree of membership may be anywhere within the interval  $[0, 1]$ . If it is 0 then the value does not belong to the given fuzzy set, and if it is 1 then the value completely belongs within the fuzzy set. Any value between 0 and 1 represents the degree of uncertainty that the value belongs in the set. These fuzzy sets are typically described by words, and so by assigning the system input to fuzzy sets, we can reason with it in a linguistically natural manner.



For example, in the image below the meanings of the expressions cold, warm, and hot are represented by functions mapping a temperature scale. A point on that scale has three “truth values”-one for each of the three functions. The vertical line in the image represents a particular temperature that the three arrows (truth values) gauge. Since the red arrow points to zero, this temperature may be interpreted as “not hot”; i.e. this temperature has zero membership in the fuzzy set “hot”. The orange arrow (pointing at 0.2) may describe it as “slightly warm” and the blue arrow (pointing at 0.8) “fairly cold”. Therefore, this temperature has 0.2 membership in the fuzzy set “warm” and 0.8 membership in the fuzzy set “cold”. The degree of membership assigned for each fuzzy set is the result of fuzzification.



Fuzzy sets are often defined as triangle or trapezoid-shaped curves, as each value will have a slope where the value is increasing, a peak where the value is equal to 1 (which can have a length of 0 or greater) and a slope where the value is decreasing. They can also be defined using a sigmoid function [14]. One common case is the standard logistic function defined as

$$S(x) = \frac{1}{1 + e^{-x}}$$

which has the following symmetry property

$$S(x) + S(-x) = 1.$$

From this it follows that

$$(S(x) + S(-x)) \cdot (S(y) + S(-y)) \cdot (S(2) + S(-2)) = 1$$

### 3.3 Fuzzy logic operators

Fuzzy logic works with membership values in a way that mimics Boolean logic. To this end, replacements for basic operators AND, OR, NOT must be available. There are several ways to this. A common replacement is called the Zadeh operators:

Boolean	Fuzzy
AND(x,y)	MIN(x,y)
OR(x,y)	MAX(x,y)
NOT(X)	1-X

For TRUE/1 and FALSE/0, the fuzzy expressions produce the same result as the Boolean expressions.

There are also other operators, more linguistic in nature, called hedges that can be applied. These are generally adverbs such as very, or somewhat, which modify the meaning of a set using a mathematical formula.

However, an arbitrary choice table does not always define a fuzzy logic function. In the paper (Zaitsev, et al), a criterion has been formulated to recognize whether a given choice table defines a fuzzy logic function and a simple algorithm of fuzzy logic function synthesis has been proposed based on introduced concepts of constituents of minimum and maximum. A fuzzy logic function represents a disjunction of constituents of minimum, where a constituent of minimum is a conjunction of variables of the current area greater than or equal to the function value in this area (to the right of the function value in the inequality, including the function value).

Another set of AND/OR operators is based on multiplication, where  $x \text{ AND } Y \text{ NOT } x = 1 - X$ . Hence,

$$x \text{ OR } Y = \text{NOT} ( \text{AND} ( \text{NOT} (x), \text{NOT} (y) ) )$$

$$x \text{ OR } y = \text{NOT} ( \text{AND} (1 - x, 1 - y) )$$

$$x \text{ OR } y = \text{NOT} ( (1 - x) * (1 - y) )$$

$$x \text{ OR } y = 1 - (1 - x) * (1 - y)$$

$$x \text{ OR } y = x + y - xy.$$

Given any two of AND/OR/NOT, it is possible to derive the third. The generalization of AND is an instance of a  $t$ -norm.

## 4 Findings: Comparison of fuzzy logic with classical logic

### 4.1 Features of fuzzy logic and its differences with classical logic

Fuzzy logic, as its name suggests, is the logic of approximate arguments, and this issue is important because human arguments are beyond precise arguments, and on the other hand, classical logic abandons this type of arguments because it is the necessary tool for its analysis. The following examples may be useful to clarify the issue:

1. In general, driving from Tehran to Karaj takes about an hour in light traffic.
2. The weather is very hot in the coming days.
3. Most experts believe that the probability of a large earthquake occurring in the near future is low.

Words such as approximately, much, and almost are vague, and this theory covers these vague statements in addition to exact statements, and therefore complements the specific argument of the old logic.

Below we read some of the key features of fuzzy logic that contain its differences with classical logic from Dr. Lotfizadeh's point of view:

- 1) Certain arguments in fuzzy logic are considered as limit examples in approximate arguments.
- 2) Everything can be graded in this logic.
- 3) Logical devices can be fuzzified.
- 4) Knowledge in this logic is considered as changeable sets and also equal to the application of fuzzy process on these sets.
- 5) Being productive is considered as a process of development of variable limits [12].

### 4.2 Fuzzy logic versus Aristotelian logic

Because the basis of Aristotle's logic is not separated from the topics related to knowledge and his way of knowing existence, fuzzy logic analyzes and scrutinizes these areas accordingly. Fuzzy logic generally tries to match the ideological paradigm of its time - coinciding with the emergence of postmodern thought - the general process in the range of fuzzy logic and postmodern is almost coherent. Each element can be used to describe another element, for example: these two spectrums seek to rationalize pluralism in the two fields of thought and action, and both are a natural response to modernity. Of course, before Aristotle's thought, Buddhist thought showed a different approach and it is as if this conflict between East and West existed from the beginning, that is, while Aristotelian thought insists on "either this or not this", the school of Buddhism insists on "both And not this" he says.

The thought issued from these two schools is not exclusive to logic, which existed in the fields of mathematics, physics, etc. and can be traced even among pre-Socratic thinkers.



### 4.3 Fuzzy logic and crossing Aristotle's duality

As we mentioned in the description of Aristotle's thought, as it was mentioned in the analysis of the aspects of Aristotle's thought, Aristotle's thinking is full of this duality, and one of the most significant aspects of this is the concepts of right and wrong in his thought system.

Aristotle pointed out that logic deals with propositions that are true or false, and interrogative sentences and final sentences (interrogative, imperative, etc.) are outside the circle of Aristotelian logic, but true and false news are always limited and in two extremes. There is truth or fiction, and statements are either true or false, and there is no clarity in between. The same principle is expanded in the form of zero and one in the new mathematical logic of Frege and Russell, and the sentences in mathematical logic are either true or false, however, many other judgments can be made between right and false based on fuzzy logic. because there are many numbers and choices between zero and one. In the middle of two political, social, legal, and artistic choices, there are many ideal choices, and these intermediate choices are not only the basis of philosophy but also form the basis of our social and cultural behaviours, even in expression. Elections are the main thing. Both partial science and the man in the street have accepted them as natural things, such as the concept of day and night, which are uncountable; The conventional understanding of day and night, which is based on accurate calendrical and astronomical statistics, tells us that the day has a beginning and an end, and if we ask someone at 9:00 AM and 3:00 PM, is it day or night? He says that it is day and if we ask the same question at 8 pm or 1 am, the answer is: it is night. This view is completely compatible with Aristotelian logic and the principle of resolving contradictions, but when it comes to fuzzy logic, even though day and night are addressed to 5 evening and 8 am, it is true, but if we measure the darkness and light of the night in some parts of the earth, it is certain that sometimes the larger part is light and the smaller part is darkness. If we measure this ratio at noon, we are faced with the maximum brightness of the light, then as the sun goes towards its sunset, the intensity of the light decreases, and it is obvious that if we use fuzzy logic as a criterion, according to the intensity of light and darkness We will have a skeptical judgment on this matter. Of course, in this particular issue, natural language tries to free itself from the absolute rules of the duality of day and night through terms such as sunrise, sunset, and sunset, because we use these ambiguous sets both in the realm of concepts and in the realm of external beings. ; Concepts such as sadness and happiness, height and shortness, cold and heat.

One of the basic criticisms of fuzzy logic is the inaccuracy and confusion in expressing the ruling, but contrary to this claim, it should be said that fuzzy logic became famous precisely because of the elimination and correction of this confusion and inaccuracy in Aristotelian logic. In Aristotelian logic, by simplifying the mental organization in such a way that it is always in the middle of accepting and rejecting a proposition, and the reading of the external facts of the analyzes are far from solidity and accuracy.

Fuzzy logic actually plays the role of a facilitator between the mind and the outside world in the field of calculation. Under the nature of fuzzy logic, it is more compatible with the nature of the mind and the world. As an example, it can be said that once upon a time, humans sought to design a robot based on mathematical logic and electronic circuits, but now with the help of fuzzy logic in the field of artificial intelligence, they seek to create a creative and decision-making robot, as in medicine with the help of It can predict the behavior of tumors and there are many examples in this field.

### 4.4 examples of the difference between fuzzy logic and classical logic

#### 4.4.1 Truth

In two-valued logical systems, we have only two values of truth and falsehood, while in multi-valued systems, the truth of a theorem can have one of the following conditions:

- a. A finite set like three-valued logic
- b. An interval like [1 and 0] like fuzzy logic
- c. A boolean algebra.

In fuzzy logic, the value of a theorem can be a fuzzy subset of a partially ordered set, and we say that the relation  $R$  in a set is a partial order in  $S$  if this relation applies to each  $a, b, c \in S$  under the following three conditions:

1. The reflective (reflective) property of  $aRa$
2. The property of antisymmetry  $ab, bra = a = b$

### 3. $ARb, bRcaR$ multiplicity feature

A set  $S$  on which a partial order relation is defined is called a partially ordered set. But we usually assume a fuzzy subset from the interval  $[1 \text{ and } 0]$ , the linguistic values honest, very honest and not completely honest and  $\dots$  are interpreted as the assigned values in this interval [12].

#### 4.4.2 Predicates

In two-valued logic, predictions are precise and definite. Predictions such as “mortal”, “equal” are more like this and unlike these predictions, fuzzy predictions are vague and uncertain; Predicates such as long, soon, fast, much bigger than and heavy are predicates of this type, and the important point is that in natural languages, most predicates are of the type of fuzzy predicates.

#### 4.4.3 Surahs

In classical logical systems, there are only two surs: general surs and existential surs, which respectively express all and some at least one, but in fuzzy logic, the number of surs is more than these, such as: more, most, very, rarely, approximately, usually, Always and in this logic, a fuzzy sur is considered as a fuzzy number or a fuzzy ratio [14].

#### 4.4.4 Predicate Modifiers

In two-valued logic, negation ( $\sim$ ) is the only predicate descriptor, while in fuzzy logic, more examples are used to act like adverbs, such as: more or less, completely infinite and... These predicate descriptors play an essential role in producing different values in linguistic variables. For example, for the predicate young, the following combinations can be presented: very young, quite young, not very young, more or less young [15].

#### 4.4.5 Possibilities

In classical logical systems, the probability is one or, finally, a numerical distance, but in fuzzy logic, other language possibilities or fuzzy probabilities can be used in general... for example: likely, unlikely, very likely, approx. 0.8, with high probability, such probabilities can be converted into fuzzy numbers using fuzzy mathematics. It should be noted that some fuzzy researchers believe that fuzzy probabilities and fuzzy surs can be converted to each other, and with this point of view, any theorem that contains fuzzy probabilities can be considered to be expressed in such a way that those probabilities are transformed into fuzzy surs [12].

#### 4.4.6 Possibility

Unlike classical logic, the possibility in fuzzy logic has more than two degrees. Like fuzzy probability, fuzzy possibility also takes various linguistic variables, words like: possible, completely possible, almost impossible, etc. These values can take fuzzy subsets. Possibility theory and probability distribution are important concepts that have been formed under fuzzy logic theory.

At the end of this section of his article (the section on the differences between two logics), Lotfizadeh notes an important point:

“It is very important to note that in any case, fuzzy logic must comply with the conditions of the classical logic system and provide additional conditions on them. According to this point, fuzzy logic can be considered as an extension of such a system, instead of considering it as a system of inferences contrary to logic and classical systems [14, 15].”

### 4.5 Fuzzy sets

#### 4.5.1 Definitions

A fuzzy set is a pair  $(U, m)$  where  $U$  is a set (often required to be non-empty) and  $m : U \rightarrow [0, 1]$  a membership function. The reference set  $U$  (sometimes denoted by  $\Omega$  or  $X$ ) is called universe of discourse, and for each  $x \in U$ , the value  $m(x)$  is called the grade of membership of  $x$  in  $(U, m)$ . The function  $m = \mu_A$  is called the membership function of the fuzzy set  $A = (U, m)$ . For a finite set  $U = \{x_1, \dots, x_n\}$ , the fuzzy set  $(U, m)$  is often denoted by  $\{m(x_1)/x_1, \dots, m(x_n)/x_n\}$ . Let  $x \in U$ . Then  $x$  is called

- **not included** in the fuzzy set  $(U, m)$  if  $m(x) = 0$  (no member),
- **fully included** if  $m(x) = 1$  (full member),
- **partially included** if  $0 < m(x) < 1$  (fuzzy member) [5].

The (crisp) set of all fuzzy sets on a universe  $U$  is denoted with  $SF(U)$  (or sometimes just  $F(U)$ ) [6].

**Crisp sets related to a fuzzy set**

For any fuzzy set  $A = (U, m)$  and  $\alpha \in [0, 1]$  the following crisp sets are defined:

- $A^{\geq \alpha} = A_{\alpha} = \{x \in U | m(x) \geq \alpha\}$  is called its  $\alpha$ -cut (aka  $\alpha$ -level set)
- $A^{> \alpha} = A'_{\alpha} = \{x \in U | m(x) > \alpha\}$  is called its strong  $\alpha$ -cut (aka strong  $\alpha$ -level set)
- $S(A) = Supp(A) = A^{>0} = \{x \in U | m(x) > 0\}$  is called its support
- $C(A) = Core(A) = A^{-1} = \{x \in U | m(x) = 1\}$  is called its core (or sometimes kernel  $Kern(A)$ ).

Note that some authors understand "kernel" in a different way; see below.

**Other definitions**

- A fuzzy set  $A = (U, m)$  is empty ( $A = \emptyset$ ) iff (if and only if)

$$\forall x \in U : \mu_A(x) = m(x) = 0$$

- Two fuzzy sets  $A$  and  $B$  are equal ( $A = B$ ) iff

$$\forall x \in U : \mu_A(x) = \mu_B(x)$$

- A fuzzy set  $A$  is included in a fuzzy set  $B$  ( $A \subseteq B$ ) iff

$$\forall x \in U : \mu_A(x) \leq \mu_B(x)$$

- For any fuzzy set  $A$ , any element  $x \in U$  that satisfies

$$\mu_A(x) = 0.5$$

is called a crossover point.

- Given a fuzzy set  $A$ , any  $\alpha \in [0, 1]$ , for which  $A^{\alpha} = \{x \in U | \mu_A(x) = \alpha\}$  is not empty, is called a level of  $A$ .
- The level set of  $A$  is the set of all levels  $\alpha \in [0, 1]$  representing distinct cuts. It is the image of  $\mu_A$

$$\Lambda_A = \{\alpha \in [0, 1] : A^{\alpha} \neq \emptyset\} = \{\alpha \in [0, 1] : \exists x \in U (\mu_A(x) = \alpha)\}$$

- For a fuzzy set  $A$ , its height is given by

$$Hgt(A) = \sup\{\mu_A(x) | x \in U\} = \sup(\mu_A(U))$$

where sup denotes the supremum, which exists because  $\mu_A(U)$  is non-empty and bounded above by 1. If  $U$  is finite, we can simply replace the supremum by the maximum.

- A fuzzy set  $A$  is said to be normalized iff

$$Hgt(A) = 1.$$

In the finite case, where the supremum is a maximum, this means that at least one element of the fuzzy set has full membership. A non-empty fuzzy set  $A$  may be normalized with result  $\hat{A}$  by dividing the membership function of the fuzzy set by its height:

$$\forall x \in U : \mu_{\hat{A}}(x) = \mu_A(x) / Hgt(A).$$

Besides similarities this differs from the usual normalization in that the normalizing constant is not a sum.

- For fuzzy sets  $A$  of real numbers ( $U \subseteq R$ ) with bounded support, the width is defined as

$$Width(A) = \sup(Supp(A)) - \inf(Supp(A))$$

In the case when  $Supp(A)$  is a finite set, or more generally a closed set, the width is just

$$Width(A) = \max(Supp(A)) - \min(Supp(A))$$

In the  $n$ -dimensional case ( $U \subseteq R^n$ ) the above can be replaced by the  $n$ -dimensional volume of  $Supp(A)$ .

In general, this can be defined given any measure on  $U$ , for instance by integration (e.g. Lebesgue integration) of  $Supp(A)$ .

- A real fuzzy set  $A(U \subseteq R)$  is said to be convex (in the fuzzy sense, not to be confused with a crisp convex set), iff

$$\forall x, y \in U, \lambda \in [0, 1] : \mu_A(\lambda x + (1 - \lambda)y) > \min(\mu_A(x), \mu_A(y))$$

Without loss of generality, we may take  $x \leq y$ , which gives the equivalent formulation

$$\forall z \in [x, y] : \mu_A(z) \geq \min(\mu_A(x), \mu_A(y)).$$

This definition can be extended to one for a general topological space  $U$ : we say the fuzzy set  $A$  is convex when, for any subset  $Z$  of  $U$ , the condition

$$\forall z \in Z : \mu_A(z) \geq \inf(\mu_A(\partial Z))$$

holds, where  $\partial Z$  denotes the boundary of  $Z$  and  $(fX) = \{f(x)|x \in X\}$  denotes the image of a set  $X$  (here  $\partial Z$ ) under a function  $f$  (here  $\mu_A$ ).

### 4.5.2 Fuzzy set operations

Although the complement of a fuzzy set has a single most common definition, the other main operations, union and intersection, do have some ambiguity.

- For a given fuzzy set  $A$ , its complement  $\neg A$  (sometimes denoted as  $A^c$  or  $cA$ ) is defined by the following membership function:

$$\forall x \in U : \mu_{\neg A}(x) = 1 - \mu_A(x).$$

- Let  $t$  be a  $t$ -norm, and  $s$  the corresponding  $s$ -norm (aka  $t$ -conorm). Given a pair of fuzzy sets  $A, B$ , their intersection  $A \cap B$  is defined by:

$$\forall x \in U : \mu_{A \cap B}(x) = t(\mu_A(x), \mu_B(x)).$$

and their union  $A \cup B$  is defined by:

$$\forall x \in U : \mu_{A \cup B}(x) = S(\mu_A(x), \mu_B(x)).$$

By the definition of the  $t$ -norm, we see that the union and intersection are commutative, monotonic, associative, and have both a null and an identity element. For the intersection, these are  $\emptyset$  and  $U$ , respectively, while for the union, these are reversed. However, the union of a fuzzy set and its complement may not result in the full universe  $U$ , and the intersection of them may not give the empty set  $\emptyset$ . Since the intersection and union are associative, it is natural to define the intersection and union of a finite family of fuzzy sets recursively.

- If the standard negator  $n(\alpha) = 1 - \alpha$ ,  $\alpha \in [0, 1]$  is replaced by another strong negator, the fuzzy set difference may be generalized by

$$\forall x \in U : \mu_{\neg A}(x) = n(\mu_A(x)).$$

- The triple of fuzzy intersection, union and complement form a De Morgan Triplet. That is, De Morgan's laws extend to this triple. Examples for fuzzy intersection/union pairs with standard negator can be derived from samples provided in the article about  $t$ -norms.

The fuzzy intersection is not idempotent in general, because the standard  $t$ -norm  $\min$  is the only one which has this property. Indeed, if the arithmetic multiplication is used as the  $t$ -norm, the resulting fuzzy intersection operation is not idempotent. That is, iteratively taking the intersection of a fuzzy set with itself is not trivial. It instead defines the  $m$ -th power of a fuzzy set, which can be canonically generalized for non-integer exponents in the following way:

- For any fuzzy set  $A$  and  $v \in R^+$  the  $v$ -th power of  $A$  is defined by the membership function:

$$\text{forall } x \in U : \mu_{A^v}(x) = \mu_A(x)^v.$$

the case of exponent two is special enough to be given a name.

- For any fuzzy set  $A$  the concentration  $CON(A) = A^2$  is defined

$$\forall x \in U : \mu_{CON(A)}(x) = \mu_{A^2}(x) = \mu_A(x)^2.$$

Since  $0^0 = 1$ , we have  $A^0 = U$  and  $A^1 = A$ .

- Given fuzzy sets  $A, B$ , the fuzzy set difference  $A \setminus B$ , also denoted  $A - B$ , may be defined straightforwardly via the membership function:

$$\forall x \in U : \mu_{A \setminus B}(x) = t(\mu_A(x), n(\mu_B(x))),$$

which means  $A \setminus B = A \cap \neg B$ , e. g.:

$$\forall x \in U : \mu_{A \setminus B}(x) = \min(\mu_A(x), 1 - \mu_B(x)).$$

Another proposal for a set difference could be:

$$\forall x \in U : \mu_{A-B}(x) = \mu_A(x) - t(\mu_A(x), \mu_B(x)).$$

- Proposals for symmetric fuzzy set differences have been made by Dubois and Prade (1980), either by taking the absolute value, giving

$$\forall x \in U : \mu_{A\Delta B}(x) = |\mu_A(x) - \mu_B(x)|,$$

or by using a combination of just  $\max$ ,  $\min$ , and standard negation, giving

$$\forall x \in U : \mu_{A\Delta B}(x) = \max(\min(\mu_A(x), 1 - \mu_B(x)), \min(\mu_B(x), 1 - \mu_A(x))).$$

Axioms for definition of generalized symmetric differences analogous to those for  $t$ -norms,  $t$ -conorms, and negators have been proposed by Vemur et al. (2014) with predecessors by Alsina et al. (2005) and Bedregal et al. (2009).

### Disjoint fuzzy sets

In contrast to the general ambiguity of intersection and union operations, there is clearness for disjoint fuzzy sets: Two fuzzy sets  $A, B$  are disjoint iff

$$\forall x \in U : \mu_A(x) = 0 \vee \mu_B(x) = 0$$

which is equivalent to

$$\nexists x \in U : \mu_A(x) > 0 \wedge \mu_B(x) > 0$$

and also equivalent to

$$\forall x \in U : \min(\mu_A(x), \mu_B(x)) = 0$$

We keep in mind that  $\min / \max$  is a  $t/s$ -norm pair, and any other will work here as well.

Fuzzy sets are disjoint if and only if their supports are disjoint according to the standard definition for crisp sets.

For disjoint fuzzy sets  $A, B$  any intersection will give  $\emptyset$ , and any union will give the same result, which is denoted as

$$A \dot{\cup} B = A \cup B$$

with its membership function given by

$$\forall x \in U : \mu_{A \dot{\cup} B}(x) = \mu_A(x) + \mu_B(x)$$

Note that only one of both summands is greater than zero.

For disjoint fuzzy sets  $A, B$  the following holds true:

$$\sup(A \cup B) = \sup(A) \cup \sup(B).$$

This can be generalized to finite families of fuzzy sets as follows: Given a family  $A = (A_i)_{i \in I}$  of fuzzy sets with index set  $I$  (e.g.  $I = \{1, 2, 3, \dots, n\}$ ). This family is (pairwise) disjoint if and only if

$$\text{for all } x \in U \text{ there exists at most one } i \in I \text{ such that } \mu_{A_i}(x) > 0.$$

A family of fuzzy sets  $A = (A_i)_{i \in I}$  is disjoint, iff the family of underlying supports  $Supp \circ A = (Supp(A_i))_{i \in I}$  is disjoint in the standard sense for families of crisp sets. Independent of the  $t/s$ -norm pair, intersection of a disjoint family of fuzzy sets will give 0 again, while the union has no ambiguity:

$$\dot{\bigcup}_{i \in I} A_i = \bigcup_{i \in I} A_i$$

with its membership function given by

$$\forall x \in U : \mu_{\dot{\bigcup}_{i \in I} A_i}(x) = \sum_{i \in I} \mu_{A_i}(x).$$

Again only one of the summands is greater than zero. For disjoint families of fuzzy sets  $A = (A_i)_{i \in I}$  the following holds true:

$$\sup\left(\dot{\bigcup}_{i \in I} A_i\right) = \bigcup_{i \in I} \sup(A_i)$$

**Scalar cardinality**

For a fuzzy set  $A$  with finite support  $SuppA(A)$  (i.e. a “finite fuzzy set”), its cardinality (aka scalar cardinality or sigma-count) is given by

$$Card(A) = sc(A) = |A| = \sum_{x \in U} \mu_A(x).$$

In the case that  $U$  itself is a finite set, the relative cardinality is given by

$$RelCard(A) = \| A \| = sc(A)/|U| = |A|/|U|.$$

This can be generalized for the divisor to be a non-empty fuzzy set: For fuzzy sets  $A, G$  with  $G \neq \emptyset$ , we can define the relative cardinality by:

$$RelCard(A, G) = sc(AG) = sc(AG)/sc(G).$$

which looks very similar to the expression for conditional probability. Note:

- $sc(G) > 0$  here.
- The result may depend on the specific intersection ( $t$ -norm) chosen.
- For  $G = U$  the result is unambiguous and resembles the prior definition.

**Distance and similarity**

For any fuzzy set  $A$  the membership function  $\mu_A : U \rightarrow [0, 1]$  can be regarded as a family  $\mu_A = (A(x))_{x \in U} \in [0, 1]^U$ . The latter is a metric space with several metrics  $d$  known. A metric can be derived from a norm (vector norm)  $\| \cdot \|$  via

$$d(\alpha, \beta) = \| \alpha - \beta \| .$$

For instance, if  $U$  is finite, i.e.  $U = \{x_1, x_2, \dots, x_n\}$ , such a metric may be defined by:

$$d(\alpha, \beta) := \max\{\mu_A(x_i)|\mu_B(x_i) : i = 1, \dots, n\} \text{ where } \alpha \text{ and } \beta \text{ are sequences of real numbers between 0 and 1.}$$



For infinite  $U$ , the maximum can be replaced by a supremum. Because fuzzy sets are unambiguously defined by their membership function, this metric can be used to measure distances between fuzzy sets on the same universe:

$$d(A, B) := d(\mu_A, \mu_B).$$

which becomes in the above sample:

$$d(A, B) = \max\{|\mu_A(x_i) - \mu_B(x_i)| : i = 1, \dots, n\}.$$

Again for infinite  $U$  the maximum must be replaced by a supremum. Other distances (like the canonical 2-norm) may diverge, if infinite fuzzy sets are too different, e.g.,  $\emptyset$  and  $U$ .

Similarity measures (here denoted by  $S$ ) may then be derived from the distance, e.g. after a proposal by Koczy:

$$S = 1/(1 + d(A, B)) \text{ if } d(A, B) \text{ is finite, } 0 \text{ else,}$$

$$S = \exp(-\alpha d(A, B)) \text{ if } d(A, B) \text{ is finite, } 0 \text{ else,}$$

where  $\alpha > 0$  is a steepness parameter and  $\exp(x) = e^x$ . Another definition for interval valued (rather “fuzzy”) similarity measures  $\xi$  is provided by Beg and Ashraf as well.

**L-fuzzy sets**

Sometimes, more general variants of the notion of fuzzy set are used, with membership functions taking values in a (fixed or variable) algebra or structure  $L$  of a given kind; usually it is required that  $L$  be at least a poset or lattice. These are usually called  $L$ -fuzzy sets, to distinguish them from those valued over the unit interval. The usual membership functions with values in  $[0, 1]$  are then called  $[0, 1]$ -valued membership functions. These kinds of generalizations were first considered in 1967 by Joseph Goguen, who was a student of Zadeh [8]. A classical corollary may be indicating truth and membership values by  $\{f, t\}$  instead of  $\{0, 1\}$ .

An extension of fuzzy sets has been provided by Atanassov. An intuitionistic fuzzy set (IFS)  $A$  is characterized by two functions:

1.  $\mu_A(x)$  - degree of membership of  $x$
2.  $\nu_A(x)$ -degree of non-membership of  $x$

with functions  $\mu_A, \nu_A : U \rightarrow [0, 1]$  with  $\forall x \in U : \mu_A(x) + \nu_A(x) \leq 1$ .

This resembles a situation like some person denoted by  $x$  voting

1. for a proposal  $A : (\mu_A(x) = 1, \nu_A(x) = 0)$ ,
2. against it:  $(\mu_A(x) = 0, \nu_A(x) = 1)$ ,
3. or abstain from voting:  $(\mu_A(x) = \nu_A(x) = 0)$ .

After all, we have a percentage of approvals, a percentage of denials, and a percentage of abstentions.

For this situation, special “intuitive fuzzy” negators,  $t$ - and  $s$ -norms can be defined. With  $D^* = \{(a, b) \in [0, 1]^2 : \alpha + \beta = 1\}$  and by combining both functions to  $(\mu_A, \nu_A) : U \rightarrow D^*$  this situation resembles a special kind of  $L$ -fuzzy sets.

Once more, this has been expanded by defining picture fuzzy sets (PFS) as follows: A PFS  $A$  is characterized by three functions mapping  $U$  to  $[0, 1] : \mu_A, \eta_A, \nu_A$ , “degree of positive membership”, “degree of neutral membership”, and “degree of negative membership” respectively and additional condition  $\forall x \in U : \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$  This expands the voting sample above by an additional possibility of “refusal of voting”.

With  $D^* = \{(\alpha, \beta, \gamma) \in [0, 1]^3 : \alpha + \beta + \gamma = 1\}$  and special “picture fuzzy” negators,  $t$ - and  $s$ -norms this resembles just another type of  $L$ -fuzzy sets.

### 4.5.3 Applications of fuzzy logic in industries and technical sciences and software sciences

Fuzzy logic has many applications, for example, temperature controllers that operate according to fuzzy principles. Another example is measuring the amount of dirt in dishwashers, this system is available in many household appliances. This logic has been used in automobile-related industries, especially in the field of brake part design, but some countries like Japan have used fuzzy logic to a great extent in the design of subways and underground smart networks. Even in elevators, air conditioners involve fuzzy logic. This logic has been widely used in image and sound analyzers and processors such as digital cameras, facial recognition cameras, and voice recognition devices. This logic has been the cause of the revolution in robotics and artificial intelligence.

Another example in this context can be helpful; Humans are able to see millions of different colours, and now the question is whether a robot will be able to distinguish these colours from each other? The answer is that with the help of fuzzy logic, such an extraordinary thing can be achieved and we can give this diagnostic ability to the robot by coding the colors numerically.

Fuzzy logic is used in various fields such as automotive systems, household goods, environmental control, etc. Some of its common uses are:

Fuzzy logic is used in the aerospace field to control the height of spacecraft and satellites.

Controls speed and traffic in-car navigation systems.

It is used for decision support systems and personal evaluation in the business of large companies.

The phase controller of drying pH controls the chemical distillation process in the chemical industry.

Fuzzy logic is used in natural language processing and various applications in artificial intelligence.

Widely used in modern control systems such as expert systems.

Fuzzy logic mimics how humans make decisions, albeit much faster, so you can use it alongside neural networks [1].

In the automotive industry, the highest energy efficiency is achieved in the design of some microprocessors in the ABS brake system and engine control design. When designing digital cameras, these cameras use the proposed methods of fuzzy logic to detect the movement of the filmed object from the possible vibrations of the cameraman's hand, and as a result, the vibrations in the film are removed. Mineral exploration and drilling in the ground to find metals and minerals, as well as to discover oil and natural gas reservoirs, as well as to control urban transportation systems such as subways and monorails.

When controlling space manoeuvres in medical diagnoses that are otherwise unclear. It is very useful in industrial automation, which we will explain more about below:

All programs called Fuzzy Logic a hardware platforms for creating industrial fuzzy logic technology, and we are trying to reduce costs and improve quality as much as possible through general solutions.

In recent years, the capabilities of fuzzy logic have proven their application in industrial automation. In this practical discussion engineers often use proven concepts for discrete event control and often use ladder logic, and often use continuous Bang-Bang or PID control, although PID control is used when the process under consideration is It is good in fixed mode, it works, but it is not successful in other modes.

This is because the PID control examines the process completely in overdrive, although this simplification is possible in overdrive and this generally works well, but if the variance is large, the operating region of the system can be far from the set point. In this case, the linear hypothesis is often discarded. The same situation occurs when the process parameters change over time, and in such situations, the use of fuzzy control instead of PLD control is shown to perform much better than the current but improved traditional methods, although this is only one case. It is not solved based on it [1].

## 4.6 Application of logic and fuzzy systems in the Holy Quran

In daily life, people use words and concepts that are different, relative and uncountable. The concept of confusion, depending on the origin of the comparison, includes different dimensions in different and changing conditions. Based on this meaning, the Qur'an, which contains transcendental meanings in various fields such as society, politics, culture, ethics, etc., has a lot of credibility and is a personality shaper that can determine the true basis of man.

Most of the Quranic verses are ambiguous and good and bad deeds are very few in the sight of God. Based on this, every human activity, good or bad, has a different scale in the eyes of God, so the Qur'an cannot agree on a

logic that has double value or multiple values. The Qur'an contains more precious and valuable logic and is higher than the logic of people at all levels, and in other words, the logic of the Qur'an is a special logic that should be the basis of human thinking and reasoning and checked with the help of other logics. Looking at the meaning of fuzzy logic and the definability of most of the revelation concepts, it can be claimed that fuzzy logic is a form of logical system governing the Qur'an. In order to understand all the logic of the Qur'an, one should make a lot of effort to find the correct and appropriate way. Also, by applying this thinking, it is possible to develop various systems in the humanities and Islam to meet today's needs with the methods of Islam and the Qur'an, and of course, with the help of obscure methods in the Qur'an and Islamic sciences, the ability of the Qur'an to solve problems and Today's problems are becoming more visible and scientists who know the power of the Qur'anic theory will gain more faith with this work [7].

In terms of Islamic teachings, especially Quranic teachings, we find that the value of a person is measured by analyzing moral principles. It cannot be accurately measured and evaluated in the classical two-dimensional sense. On the other hand, most of the ethical concepts presented in the Qur'an have a systematic and questionable nature of existing perfection, whose examples are not used in the same way.

In the comparison of the two evaluation systems, the systematic evaluation process in behavioural theory has a special feature that categorizes people based on their knowledge and actions. In two-valued ideas, which are in the form of (0-1), a person is a believer or an unbeliever, a god-worshiper or an atheist, on the other hand, in the use of ideas, people are divided into god-worship and so on. Understanding the moral issues of the Holy Quran through the analysis of vague fuzzy concepts is not a way to strengthen the Holy Quran, but it is to achieve a new way to better understand the Word of God. With this explanation, a Muslim tries to understand the Bible better by expanding his knowledge. By referring to the verses of the Qur'an, we see that many of the moral principles of the Bible are constructive and prescriptive in nature, at this time the special system is two-dimensional and two concepts are not useful in understanding. The definition and application of these concepts in relation to the psychological application of their values. In this way, the minimum level of characteristics can be confirmed by maintaining the number of instructions [8].

## 5 Conclusion and summary

As Dr. Lotfizadeh correctly and beautifully stated, the logic that a person uses in doing his daily tasks has caused him to perform actions that the largest smart devices and computers are not able to repeat pattern therefore Based on the idea of simple behaviours in humans, he founded fuzzy logic in the form of a logical system with almost human reasoning. Fuzzy logic, as a necessary generalization of the theory presented by Dr Lotfizadeh, is derived from fuzzy sets, and fuzzy devices use reasoning rules or ambiguous reports and are specified in different situations. It can be said that fuzzy logic is a more powerful and accurate control technology that instead of building a fence around a classification, tries to describe it in a way that is close to the idea, while classical logic, looks for a word A key in the database searches the available data and if it finds a perfect match, it expresses the result, and in this way, the possibility of a good and accurate search by retrieving all relevant information is lost. Although fuzzy systems apparently describe uncertain and uncertain phenomena, fuzzy theory is a very precise theory. Both Aristotelian logic and fuzzy logic have other different concepts, which are necessary to describe as much as possible, and from the above analytical point of view, we conclude that these two logics and their comparison together well showed the development process of thought and science. And the foundation of the recent scientific revolutions can be attributed to the philosophy and logic governing the philosophical flow of the East, because there is no intermediate state in Aristotelian philosophy because the boundaries are clearly defined, but in real thinking, there are specific boundaries. It doesn't exist and the relationship between different elements and different concepts and topics is relative, therefore you can see how much thought is integrated with nature, people and the world; This is because fuzzy thinking actually represents a generalized view of Aristotelian logic, and today the influence of fuzzy thinking cannot be ignored everywhere: from controlling rockets and spaceships to controlling traffic in a city. Large and even in a vacuum cleaner, oven, washing machine, etc. Finally, the biggest advantage of artificial intelligence that has led to its use in industrial growth is its flexibility in data analysis and decision-making. In fact, fuzzy thinking is the right way to think about vague, dark and grey things, and it can be said that many researchers in Iran have done research in this field and more space is needed to produce new research.

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