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Designing a fuzzy multi-objective mathematical model of four-stage perishable supply chain using an operational, financial and marketing approach under Benders' uncertainty state and decomposition analysis

Fatemeh Fallah[a], Parham Azimi[a,*], Mani Sharifi[b]

[a]*Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

[b]*Department of Information Systems and Analytic, Farmer School of Business, Miami University, Oxford, Ohio, USA*

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Abstract

This article solves the mathematical model of supply, planning, storage, and distribution in a supply chain using an agile supply chain. In modern supply chains, addressing financial risks in purchases and distribution, transportation for supply and distribution of products and lost policies or the storage of products in perishable products supply chains is critical. This article investigates a direct supply chain model for the executive policies of manufacturing companies and a four-stage supply chain problem using the operational financial and marketing approach under an uncertainty state, Lagrangian relaxation and Benders' decomposition evaluation. This study introduced and provided the concepts of supply chains in perishable products. Considering supply chain issues and uncertainty in this environment, a fuzzy mathematical model was provided. In the end, a literature review has shown that most research has used heuristic and meta-heuristic algorithms due to supply chain models being NP-HARD., which are inefficient due to the approximation of optimal solutions in this domain. Meanwhile, the study used a Benders decomposition and Lagrangian relaxation algorithm for mathematical solutions, which would reduce the model's solving time and provide accurate answers.

Keywords: Lagrangian relaxation, Benders relaxation, perishable supply chain, multi-objective optimization
2020 MSC: 90B06

1 Introduction

Intense competition and fast developments across markets, as well as customer priorities and the rapid development of technologies and globalization, have prompted organizations to work as members of a supply chain instead of working individually [11]. Dividing earnings, sharing values and finally modifying risks in a chain of value-creating measures could lead to strategic concepts in a chain supply [1]. Today, environmental factors play a key role in the long-term success of a supply chain, while the purchase process is getting complicated by environmental and

*Corresponding author
Email addresses: fatemeh.fallahnajjar@gmail.com (Fatemeh Fallah), p.azimi@yahoo.com (Parham Azimi), sharifm@miamioh.edu (Mani Sharifi)

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social pressures. Currently, many organizations are concerned about environmental, social and economic issues and measure the performance of suppliers in this regard. In the meantime, there is a wide range of issues with how to select suppliers, with researchers still failing to investigate their internal relations. These issues include the effects of corporate social and humanitarian activities, called "Corporate Social Responsibility" on selecting suppliers. Corporate social responsibility includes a wide variety of activities, including employee welfare plans, and stakeholder groups. The activities of social groups and associations, donations, responsibility against supply chain, moral issues leadership, and environmental issues [8].

Funding and capital required by firms can be provided by various means. Firms' abilities to effectively plan and manage financial flows are considered to be one of the core factors contributing to business progress. If this ability occurs in the form of interactions and communications between businesses across a supply chain, it can provide stable competitive advantages for that chain and thus generate far more values for the entire chain [10]. There are various approaches for financing, as firms can secure their needed funding through equity holders or stakeholders (internal resources-undivided profit) or through foreign resources, which are resources other than stakeholder capital or through money and capital markets [14]. Accordingly, it is required to adopt relevant policies to effectively manage financial flows across a supply chain [3].

Many researchers have recently discussed supply chains. On the other hand, one of the main sections of the supply chain is to design operational supply chain networks. Hence, modelling material flows in the supply chain is very critical due to the importance of material flow and data in the supply chain, among the three sections of financial flows, data and material. It is worth mentioning that in supply chains, certainty is a key aspect that affects the efficiency of a supply chain. Thus, parameters and certain constraints in problems and the way they are encountered in the modelling process are considered major challenges to managers. On the other hand, in the current trading world, cost management is a strategically important tool in this section. Various research has suggested that goods purchased and out-of-the-organization services account for over 60% of the total corporate costs. Saving these costs can be done by effectively applying purchase strategies, although little attention has been paid to the purchase section proportionate to its share in the finished prices of final products, which results from material chain supplies, and no relevant policies have been conducted on securing materials based on financial conditions [12].

As stated, two supply chain generators (financial and operational) constitute the main pattern to realize perspectives in commercial activity development, with most researchers providing models and designs in this domain. Hence, one of the major strategies in capillary distribution networks is to create motivation in product purchasers. Thus, effective advertising is key to an agile and resilient supply chain [9]. Thus, this article aimed to provide a mathematical model for planning supply, production, storage and distribution in the supply chain using an agile supply, approach. Since in modern supply chain issues, dealing with financial risks in purchase and distribution, transportation of the supply and distribution of products and the policies of lost sales or the storage of manufactured products in the supply chain of perishable products is critical, it is required to provide a sour-stage perishable supply chain model to include a supplier, a producer, middle distribution inventories and final customers. This is firstly aimed at minimizing the total supply chain costs, including costs of construction, maintenance and transportation and also advertising costs; secondly minimizing distribution and transfer timing between each level of the supply chain, and thirdly maximizing the social level of supply chains by using maximum manpower. Thus, uncertainty problem components are considered in mathematical model parameters to approach the modelling environment to the real space; for this, a fuzzification approach is used to evaluate the uncertainties. Figure 1 gives a network of supply chains.

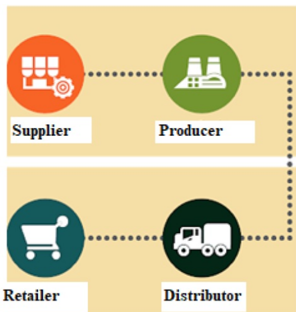

Figure 1: Studied supply chain network

2 Literature review

Elahi and Franchetti [5] presented a non-linear multi-objective mathematical model for the perishable supply chain networks. The chain supply intended includes three various classes of suppliers, wholesale distributors and customers. According to the presented model, a positive correlation is assumed between a product's price and shelf life. Also, wholesale distributors try to sell products of short-term shelf-life earlier and at cheaper prices. The above model has two goals: 1) maximizing total profit for wholesale distributors and 2) minimizing CO_2 emissions in their states and the transportation of products to customers to make the health product supply chain more stable. To solve this mathematical model, a genetic algorithm approach is proposed to determine the optimal solution.

Dolgui et al. [4] presented a mathematical model to achieve an optimal integrated inventory policy for perishable products within a multi-stage supply chain. This integrated model that includes inventory control and navigation selection can be optimized by an evolutionary technique such as the genetic algorithm. A new genetic algorithm, which avoids a search for solutions and uses a self-adjusting mutation operator without a parameter, is developed to solve the proposed model. Findings were compared using CPLEX software for small-sized problems. In the end, the proposed optimization model provides almost optimal results for various demand scenarios.

Tavakkoli Moghaddam et al. [15] designed a mathematical model for the reverse supply chain of perishable products using a stable product system. According to the presented model, four objective functions of maximizing profitability, satisfaction with using technology, minimizing costs and measuring environmental effects were considered. The results of the proposed model implemented for a manufacturing company indicated that objective functions were sensitive to demands; thus, changes in demands would change objective functions, especially the profitability function.

Patidar and Agrawal [13] proposed a mathematical model to design a traditional Indian Agri-Fresh Food Supply Chain (AFSC) to minimize total function costs and eliminate agricultural products after being harvested. This study developed two mathematical models for the structure and display of product flows in the agri-food supply chain. First, a three-layer, multi-product and multi-period Mixed Integer Linear Programming (MILP) planning model was formulated to minimize the total distribution costs incurred in the chain. Moreover, the developed formula extended to the second model by considering the perishability of the products. To investigate the validity of formulated models, a real case study in the Indian Mandsaur region was used with LING 17.0. The AFSC personable model (the second model) produced better results in terms of costs and minimizing lost products after harvest.

Imran et al. [7] proposed a model for the multi-period and multi-objective inventory routing in the perishable product supply chain under cost uncertainty conditions. Here, they introduced a new objective of maximizing the index of priority in the model, in addition to reducing greenhouse gas emissions. The index of priority quantitatively investigates various qualitative social aspects such as coordination, trust, behaviour and long-term relations between stakeholders. The index of priority measures the relative performance of each of the level factors (supplier/distributor/retailer) in the supply chain. In this state, maximizing the index of priority guarantees access to social stability in the supply chain. In addition, a time-series integrated regression fuzzy meth has been developed to model cost uncertainty. This study consists of three stages: a mixed multi-objective mathematical model involving costs uncertainty is formulated. To determine the parameters of the objective function of the index of priority, the two-stage fuzzy extraction process was used, with other objectives (costs and CHG) modelled mathematically. In the next stage, an interactive multi-objective fuzzy programming model was used to solve the mathematical model, which involves expert preferences for concrete satisfaction based on their experiences. In the end, a case study of the surgery tool supply chain was provided as a sample.

Golinska-Dawson [6] designed a multi-level, multi-product and multi-period Sustainable Close Loop Supply Chain for perishable products. To this aim, an integrated mathematical model was proposed. The main objectives were to minimize the costs of production and distribution and gain customer satisfaction, minimize total CO_2 emissions and maximize social responsibility. This study considers the time of production and delivery of perishable products to design a supply chain network and proposes a new hybrid algorithm based on the Whale Optimization Algorithm (WOA) and Genetic Algorithm (GA). To solve the proposed model and optimize the mathematical model, the proposed hybrid algorithm was run on several experimental problems of different sizes. The results obtained were compared with the strengthened constraint epsilon to examine the proposed algorithm performance. In the end, the findings indicated that the proposed algorithm provides Pareto solutions at acceptable and diverse levels.

3 Modeling

This model presents a mathematical model of supply, production, storage, and distribution planning using an agile approach. Since in modern supply chain issues, dealing with financial risks in purchase and distribution, transportation of the supply and distribution of products and the policies of lost sales or the storage of manufactured products in the supply chain of perishable products is critical, it is required to provide a sour-stage perishable supply chain model to include a supplier, a producer, middle distribution inventories and final customers. This article investigates a direct supply chain model for the executive policies of manufacturing companies and a four-stage supply chain problem. Hence, the flowchart of the study is as follows:

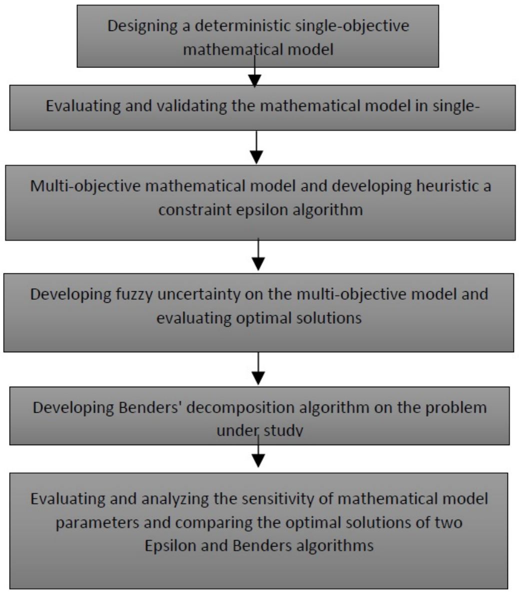

Figure 2: Flowchart of modeling and problem-solving under study

Marking:

Sets:

J Set of suppliers $j = \{1, ..., J\}$

K Set of production centers $k = \{1, ..., K\}$

L Set of distributing centers $l, l' = \{1, ..., L\}$

C Set of fixed customers $m, c = \{1, ..., C\}$

P Set of perishable products $p = \{1, ..., P\}$

T Set of time periods $t = \{1, ..., T\}$

R Sets of production time periods $r = \{1, ..., T\}$

V Set of vehicles $v = \{1, ..., V\}$

Parameters:

Costs of establishing and advertising a production center H_k

Costs of establishing a distribution center l U_l

Fixed costs of using vehicle v F_v

Costs of transportation per product unit between a supplier j and a production center k T_{jk}

Costs of transportation per product unit between a production center k and a distribution center l T_{kl}

Costs of transportation per product unit between wholesale distribution centers l and l' $T_{ll'}$

Cost of transportation between distribution centers l and customer c, $l, c \in L \cup C$ T_{lc}

Costs of storage per unit of product p in the warehouse of production center k h_{kp}

Costs of storage per unit of product p in distribution center warehouse l h'_{lp}

Costs of distribution per unit of product p by distribution center l C_{lp}

Customer demand c from product p at time interval t D_{cpt}

Product perishability time p u_p

Maximum capacity of supplier j from product supply p ca_{jp}

Maximum production center capacity k from the product production p ca_{kp}

Maximum distribution center capacity l from product distribution p ca_{lp}

Vehicle capacity v ca_v

Large Number M

Decision variables:

An amount of product transferred p between supplier j and production center k at time interval t Y_{jkpt}

An amount of product transferred p between production center k and distribution center l at time interval t W_{klpt}

An amount of product transferred p between the distribution centers l and l' at time interval t $S_{ll'pt}$

Total amount of product transferred p to the distribution centers l at time interval t V'_{lpt}

An amount of product transferred p from distribution center l to customer c at time interval t at time interval r B_{lcptr}

An amount of product transferred p between production center k and distribution center l at time interval t and the produced at time interval r T_{klptr}

An amount of product transferred p between distribution centers l and l' at time interval t and the produced at time interval r $A_{l'lptr}$

Product inventory level p in the production center warehouse k at time interval t and the produced at time interval r Q_{kptr}

Product inventory level p in the distribution center warehouse l at time interval t and the produced at time interval r $Q_{lptr'}$

If production center k is established, it is 1 and otherwise 0. Z_k

If the distribution center l is established, it is 1 and otherwise 0. Z_l

If customer c is assigned to distribution center l at time interval t, it is 1 and otherwise 0. Z_{lct}

If customer c visited the distribution center l of vehicle v at time interval t, it is 1 and otherwise 0. $l, c \in L \cup C$ Z_{lcvt}

If vehicle v goes from supplier j to production center k at time interval t, it is 1, otherwise 0 XY_{jkvt}

If vehicle v goes from the production center k to the distribution center l at time interval t, it is 1, otherwise 0. XW_{klvt}

If vehicle v goes from the production center k to the distribution center l and l' at time interval t, it is 1, otherwise 0. $XS_{ll'pvt}$

Auxiliary variable for the constraint of sub-tour elimination U_{cvt}

3.1 Multi-level network modeling of perishable product supply chains

According to the description of the sets, parameters and decision variables, the problem of designing a multi-level network of perishable products supply chain was modelled as a multi-objective mixed integer non-linear mathematical planning model as follows:

$$\begin{aligned}\min \omega 1 = &\sum_{k=1}^{K} H_k Z_k + \sum_{l=1}^{L} U_l Z_l + \sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{t=1}^{T}\sum_{r=1}^{t} h_{kp} Q_{kptr} + \sum_{l=1}^{L}\sum_{p=1}^{P}\sum_{t=1}^{T}\sum_{r=1}^{t} h'_{lp} Q'_{lptr}\\ &+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{t=1}^{T} T_{jk} Y_{jkpt} + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{p=1}^{P}\sum_{t=1}^{T} T_{kl} W_{klpt} + \sum_{l=1}^{L\cup C}\sum_{c=1}^{L\cup C}\sum_{v=1}^{V}\sum_{t=1}^{T} T_{lc} Z_{lcvt}\\ &+ \sum_{l=1}^{L}\sum_{l'=1}^{L}\sum_{p=1}^{P}\sum_{t=1}^{T} T_{ll'} S_{l'pt} + \sum_{l=1}^{L}\sum_{p=1}^{P}\sum_{t=1}^{T} C_{lp} V'_{lpt} + \sum_{l=1}^{L}\sum_{c=1}^{C}\sum_{v=1}^{V}\sum_{t=1}^{T} F_v Z_{lcvt} \qquad (3.1)\end{aligned}$$

$$\min \omega 2 = \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{t=1}^{T} ti_{jk} XY_{jkvt} + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{v=1}^{V}\sum_{t=1}^{T} tt_{kl} XW_{klvt} + \sum_{l=1}^{L\cup C}\sum_{c=1}^{L\cup C}\sum_{v=1}^{V}\sum_{t=1}^{T} tc_{lc} Z_{lcvt} + \sum_{l=1}^{L}\sum_{l'=1,\ l'\neq 1}^{L}\sum_{v=1}^{V}\sum_{t=1}^{T} tf_{ll'} XS_{ll'vt} \tag{3.2}$$

$$\min \omega 3 = \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{t=1}^{T} h_{jk} XY_{jkvt} + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{v=1}^{V}\sum_{t=1}^{T} ht_{kl} XW_{klvt} + \sum_{l=1}^{L\cup C}\sum_{c=1}^{L\cup C}\sum_{v=1}^{V}\sum_{t=1}^{T} hc_{lc} Z_{lcvt} + \sum_{l=1}^{L}\sum_{l'=1,\ l'\neq 1}^{L}\sum_{v=1}^{V}\sum_{t=1}^{T} hf_{ll'} XS_{ll'vt} \tag{3.3}$$

s.t.:

$$\sum_{r=1}^{t} Q_{kptr} = \sum_{j=1}^{J} Y_{jkpt} - \sum_{l=1}^{L} W_{klpt}, \qquad \forall k, p, t = 1 < u_p \tag{3.4}$$

$$\sum_{r=1}^{t} Q_{kptr} = \sum_{r=1}^{t-1} Q_{kpt-1r} + \sum_{j=1}^{J} Y_{jkpt} - \sum_{l=1}^{L} W_{klpt}, \qquad \forall k, p, 1 < t < u_p \tag{3.5}$$

$$\sum_{r=t+1-u_p}^{t} Q_{kptr} = \sum_{r=t+1-u_p}^{t-1} Q_{kpt-1r} + \sum_{j=1}^{J} Y_{jkpt} - \sum_{l=1}^{L} W_{klpt}, \qquad \forall k, p, t \geq u_p \tag{3.6}$$

$$W_{klpt} = \sum_{r=1}^{t} T_{klptr}, \qquad \forall k, l, p, t < u_p \tag{3.7}$$

$$W_{klpt} = \sum_{r=t+1-u_p}^{t} T_{klptr}, \qquad \forall k, l, p, t \geq u_p \tag{3.8}$$

$$Q_{kptr} = \sum_{j=1}^{J} Y_{jkpt} - \sum_{l=1}^{L} T_{klptr}, \qquad \forall k, p, t = r \tag{3.9}$$

$$Q_{kptr} = Q_{kpt-1r} - \sum_{l=1}^{L} T_{klptr}, \qquad \forall k, p, t - r < u_p \tag{3.10}$$

$$\sum_{r=1}^{t} Q'_{lptr} = \sum_{k=1}^{K} W_{klpt} - V'_{lpt} + \sum_{l'=1, l'\neq l}^{L} S_{l'lpt} - \sum_{l'=1, l'\neq l}^{L} S_{ll'pt}, \qquad \forall l, p, t = 1 < u_p \tag{3.11}$$

$$\sum_{r=1}^{t} Q'_{lptr} = \sum_{r=1}^{t-1} Q'_{lpt-1r} + \sum_{k=1}^{K} W_{klpt} - V'_{lpt} + \sum_{l'=1, l'\neq l}^{L} S_{l'lpt} - \sum_{l'=1, l'\neq l}^{L} S_{ll'pt}, \qquad \forall l, p, 1 < t < u_p \tag{3.12}$$

$$\sum_{r=t-u_p+1}^{t} Q'_{lptr} = \sum_{r=t-u_p+1}^{t-1} Q'_{lpt-1r} + \sum_{k=1}^{K} W_{klpt} - V'_{lpt} + \sum_{l'=1, l'\neq l}^{L} S_{l'lpt} - \sum_{l'=1, l'\neq l}^{L} S_{ll'pt}, \qquad \forall l, p, t \geq u_p \tag{3.13}$$

$$V'_{lpt} = \sum_{r=1}^{t}\sum_{c=1}^{C} B_{lcptr}, \qquad \forall l, c, p, t < u_p \tag{3.14}$$

$$V'_{lpt} = \sum_{r=t-u_p+1}^{t}\sum_{c=1}^{C} B_{lcptr}, \qquad \forall l, c, p, t \geq u_p \tag{3.15}$$

$$S_{ll'pt} = \sum_{r=1}^{t} A_{l'lptr}, \qquad \forall l, l', p, t < u_p \tag{3.16}$$

$$S_{ll'pt} = \sum_{r=t-u_p+1}^{t} A_{l'lptr}, \qquad \forall l, l', p, t \geq u_p \tag{3.17}$$

$$Q'_{lptr} = \sum_{k=1}^{K} T_{klptr} - \sum_{c=1}^{C} B_{lcptr} + \sum_{l'=1, l' \neq l}^{L} A_{l'lptr} - \sum_{l'=1, l' \neq l}^{L} A_{ll'ptr}, \qquad \forall l, p, t = r \tag{3.18}$$

$$Q'_{lptr} = Q'_{lpt-1r} - \sum_{c=1}^{C} B_{lcptr} - \sum_{l'=1, l' \neq l}^{L} A_{ll'ptr}, \qquad \forall l, p, t - r < u_p \tag{3.19}$$

$$\sum_{k=1}^{K} Y_{jkpt} \leq ca_{jp}, \qquad \forall j, p, t \tag{3.20}$$

$$\sum_{k=1}^{K} W_{klpt} + \sum_{l'=1, l' \neq l}^{L} S_{l'lpt} \leq ca_{lp} Z_l, \qquad \forall l, p, t \tag{3.21}$$

$$\sum_{j=1}^{J} Y_{jkpt} \leq ca_{kp} Z_k, \qquad \forall k, p, t \tag{3.22}$$

$$V'_{lpt} = \sum_{c=1}^{C} D_{cpt} Z_{lct}, \qquad \forall l, p, t \tag{3.23}$$

$$\sum_{v=1}^{V} \sum_{l=1}^{C \cup L} Z_{lcvt} = 1, \qquad \forall c, t \tag{3.24}$$

$$\sum_{c=1}^{C} \sum_{l=1}^{C \cup L} \sum_{p=1}^{P} D_{cpt} Z_{lcvt} \leq ca_v, \qquad \forall v, t \tag{3.25}$$

$$U_{mvt} - U_{cvt} + C Z_{mcvt} \leq C - 1, \qquad \forall m, c \in C, v, t \tag{3.26}$$

$$\sum_{l=1}^{C \cup L} Z_{lcvt} = \sum_{l=1}^{C \cup L} Z_{clvt}, \qquad \forall v, t, l \in C \cup L \tag{3.27}$$

$$\sum_{l=1}^{L} \sum_{c=1}^{C} Z_{lcvt} \leq 1, \qquad \forall v, t \tag{3.28}$$

$$\sum_{p=1}^{P} V'_{lpt} \leq \sum_{p=1}^{P} ca_{lp} Z_l, \qquad \forall l, t \tag{3.29}$$

$$-Z_{lct} + \sum_{u=1}^{C \cup L} (Z_{luvt} + Z_{ucvt}) \leq 1, \qquad \forall l, c, v, t \tag{3.30}$$

$$\sum_{p} Y_{jkpt} \leq \sum_{v} MXY_{jkVt}, \qquad \forall j, k, t \tag{3.31}$$

$$\sum_{p} W_{klpt} \leq \sum_{v} MXW_{klvt}, \qquad \forall k, l, t \tag{3.32}$$

$$\sum_{p} S_{ll'pt} \leq \sum_{v} MXS_{ll'vt}, \qquad \forall l, l', l \neq l', t \tag{3.33}$$

$$\sum_{j} XY_{jkVt} = \sum_{l} XW_{klvt}, \qquad \forall v, k, t \tag{3.34}$$

$$\sum_{k} XW_{klvt} = \sum_{l' \neq l} XS_{ll'vt} + \sum_{c} Z_{lcvt}, \qquad \forall l, v, t \tag{3.35}$$

$$Q_{kptr} = 0, \qquad \forall k, b, t < r \tag{3.36}$$

$$Q'_{lptr} = 0, \qquad \forall l, b, t < r \tag{3.37}$$

$$Y_{jkpt}, W_{klpt}, S_{l'lpt}, U_{lvt}, \sigma_{cpt} \geq 0, \qquad \forall j, k, c, l, l', p, v, t \tag{3.38}$$

$$B_{lcptr}, A_{ll'ptr}, T_{klptr}, Q'_{lptr}, Q_{kptr} \geq 0, \qquad \forall l, l', c, k, p, t, r \tag{3.39}$$

$$Z_l, Z_k, Z_{lct}, Z_{lcvt} \in \{0, 1\}, \qquad \forall k, l, v, t, c \tag{3.40}$$

The relation (3.1) shows the first objective function and includes the minimization of total costs of the supply chain network (costs of constriction, storage and transportation of products between centers) and costs of advertising. The relation (3.2) shows the second objective function and includes the minimization of distributing and transfer timing between each level of the supply chain network. The relation (3.3) represents the maximization of the social level of supply chains by the maximum use of manpower. Constraints (3.4) to (3.6) pertain to product storage in production center warehouses at their production times based on the perishability time of each product at each time period. Constraints (3.7) and (3.8) show the transfer of products from production centers to distribution centers based on perishability. Constraints (3.9) and (3.10) show the inventory of each type of product in distribution center warehouses, while constraints (3.11) and (3.12) represent the inventory of each type of product in distribution center warehouses at their total production time. Constraints (3.14) and (3.15) show products being transferred from distribution centers to all areas as demanded by customers at each time period. Constraints (3.16) and (3.17) represent the transfer of products between distribution centers based on customer demands based on perishability time. Constraints (3.18) and (3.19) show an equilibrium relation in distribution centers and guarantee the transfer of the products to customers before they have perished. Constraints (3.20) to (3.22) pertain to the constraints of the capacity of suppliers, production and distribution centers and guarantee that unless a center is established, the maximum capacity of that center cannot be used. The relation (3.23) shows the total demand (products) flow in distribution centers for being transferred to customers. Constraint (3.24) guarantees that each distribution center can only be assigned to a customer. Constraint (3.25) represents the maximum product transportation by an accessible vehicle. Constraint (3.26) pertains to the sub-tour elimination. Constraint (3.27) guarantees that each vehicle can only once enter into or exit from a customer cluster. Constraints (3.28) to (3.30) guarantee that the beginning and ending points of vehicle routing in product distribution to customers are distribution centers. Constraints (3.31), (3.32) and (3.33) pertain to the assignment of paths to transfer perishable materials to target nodes. Constraint (3.34) shows the balance between the vehicle entering and exiting from the producer, and constraint (3.35) pertains to the balance between the middle distribution node for vehicles. Constraints (3.36) and (3.37) represent logic relations within product inventories in production and distribution production centers. Constraints (3.38), (3.39) and (3.40) show the type and genus of the decision variables. To fuzzify the problem, problem parameters, including costs of transportation, storage, customer demands, maximum capacity of production centers and the maximum capacity of distribution centers were considered to have fuzzy trapezoidal uncertainty.

4 Solving method

Model performance validity evaluation is one of the basic concepts to adapt the presented model to the real environment; hence, a mathematical model of small dimensions should be adapted to the real environment or the reference article and the solutions should be compared. Thus, GAMS software was used to evaluate the validity of the model's performance, as the presented model will be investigated in larger dimensions after being validated. On the other hand, considering wide-ranging management and logistic dimensions, it is required to use heuristic and metaheuristic methods to validate the model presented. To test the validity of the proposed model, a problem of small dimensions was solved by using GAMS software. Then, the mathematical model presented was solved by GAMS, which is research in action software, and then investigated by way of a numerical solution method of the proposed method. One of the accurate methods of optimal Pareto solving methods is to use the epsilon constraint method, first developed by Aljdan. On the other hand, the Benders decomposition method depends on decomposing a planning model of mixed integer planning into a core problem and a sub-problem, which are solved repetitively using each other's solutions [2]. The sub-problem includes continuous variables and relevant constraints, while the core problem includes integer variables and a continuous variable, which connects both problems. The optimal solution to the core problem provides a lower bound for the intended objective. Using the solution obtained by the core problem, by assuming the integer variables constant as the input to the sub-problem of a dual for the solved sub-problem, this solution is used an upper limit for the general purpose of the problem; this solution is also used to make a Benders' cut, which includes continuous variables added to the core problem. Thus, the core problem and the sub-problem will be frequently solved until they end a termination condition, which lessens the distance between the upper and lower bounds than a smaller number. Benders' decomposition method in finite repetitions reaches an optimal solution.

5 Numerical results

This section deals with the problem solution of the proposed perishable supply chain using the multi-objective decision method. To this aim, to evaluate the model and observe output variables, a numerical example of small size was designed and solutions obtained from multi-objective decision-making methods (epsilon constraint, Benders' utility objective and Torabi-Hosseini method) were compared.

5.1 Problem solving

This section concerns problems (3.1) to (3.40) of Chapter Three. Table 1 gives a sample state of small sizes.

Table 1: Size of the sample problem designed in small sizes

Sets	Symbols	No.	Sets	Symbols	No.
Drug supply centers	J	3	Type of drugs	P	2
Production centers	K	3	Time intervals	T	2
Distribution centers	L	3	Vehicles	V	2
Fixed customers	C	5			

Deterministic and non-deterministic parameters considered for solving this problem are given in Tables 2 and 3.

Table 2: Deterministic parameters used in the problem based on uniform distribution

Parameters	Range limits	Parameters	Range limits
H_k	$(20000, 30000) \sim U$	ca_{jp}	$(33, 500) \sim U$
U_l	$(50000, 60000) \sim U$	ca_v	$(500, 800) \sim U$
F_v	$(200, 300) \sim U$	$\tilde{T}_{jk}$	$(10, 20) \sim U$
C_{lp}	$(10, 20) \sim U$	$\tilde{T}_{kl}$	$(10, 20) \sim U$
tt_{lc}	$(10, 15) \sim U$	$\tilde{T}_{ll'}$	$(10, 20) \sim U$
u_p	1	$\tilde{T}_{lc}$	$(10, 20) \sim U$

Data were obtained by drug industry expert views and also using published articles in this area based on uniform distribution.

Table 3: Non-deterministic parameters used in the problem based on uniform distribution

Parameter	Range limits
$\tilde{T}_{jk}$	$\sim U((5, 10), (10, 15), (15, 20), (20, 25))$
$\tilde{T}_{kl}$	$\sim U((5, 10), (10, 15), (15, 20), (20, 25)$
$\tilde{T}_{ll'}$	$\sim U((5, 10), (10, 15), (15, 20), (20, 25)$
$\tilde{T}_{lc}$	$\sim U((5, 10), (10, 15), (15, 20), (20, 25)$
$\tilde{h}_{kp}$	$\sim U(1, 2), (2, 3), (3, 4), (4, 5)$
$\tilde{h}'_{kp}$	$\sim U(1, 2), (2, 3), (3, 4), (4, 5)$
$\tilde{D}_{cpt}$	$\sim U((100, 120), (120, 150), (150, 170), (170, 200)$
$\tilde{ca}_{kp}$	$\sim U((200, 300), (300, 400), (400, 500), (500, 600)$
$\tilde{ca}_{lp}$	$\sim U((200, 300), (300, 400), (400, 500), (500, 600)$

Prior to solving models using multi-objective decision-making methods (Epsilon constraint, Benders' utility function and Torabi-Hosseini method), this section solves each and every objective function using individual optimization methods. Output variables include the optimal place for potential facilities and the optimal routing of vehicles. Thus, the relations within solution methods were investigated and comparison indices of efficient solutions were examined. Table 4 gives objective function values, optimal place of facilities and optimal routing of vehicles, as well as the single objective model (first objective function) for the small sample problem.

Table 5 shows objective function values, optimal place of facilities and optimal vehicle routing of the single-objective model (second objective function) for the small sample problem.

Table 6 gives values of the objective function, optimal place of facilities and optimal vehicle routing of the single-objective model (second objective function) for the small sample problem. After determining the best value of the first and second objective functions for the small-size problem, a set of efficient solutions resulting from the problem-solving method using multi-objective decision methods (comprehensive criterion, utility function and Torabi-Hosseini) were examined.

Table 4: Optimal place of potential facilities and transportation routing considering the first objective function using individual optimization methods

Potential centers	**No.**	**Optimal place**
Production centers	2 centers	2 and 3
Distribution centers	2 centers	A, C
Optimal vehicles routing	Vehicle 1 in period 1	$A \to 1 \to 4 \to A$
	Vehicle 2 in period 1	$C \to 3 \to 2 \to 5 \to C$
	Vehicle 1 in period 2	$A \to 4 \to 3 \to A$
	Vehicle 2 in period 2	$C \to 2 \to 5 \to 1 \to C$
First objective function values	541633.336	

Table 5: Optimal place of potential facilities and transportation routing considering the second objective function using individual optimization methods

Potential centers	**No.**	**Optimal place**
Production centers	3 centers	1 and 2 and 3
Distribution centers	2 centers	C, B
Optimal vehicles routing	Vehicle 1 in period 1	$B \to 3 \to 1 \to B$
	Vehicle 2 in period 1	$C \to 2 \to 4 \to 5 \to A$
	Vehicle 1 in period 1	$B \to 3 \to 1 \to B$
	Vehicle 2 in period 2	$C \to 2 \to 4 \to 5 \to A$
Second objective function values	49942.651	

Table 6: Optimal place of potential facilities and transportation routing considering the third objective function using individual optimization methods

Potential centers	**No.**	**Optimal place**
Production centers	3 centers	1 and 2 and 3
Distribution centers	3 centers	C, B, A
	Vehicle 1 in period 1	$A \to 1 \to 4 \to A$
Optimal vehicles routing	Vehicle 2 in period 1	$B \to 3 \to 1 \to B$
	Vehicle 3 in period 1	$C \to 2 \to 4 \to 5 \to A$
	Vehicle 1 in period 2	$A \to 4 \to 3 \to A$
	Vehicle 2 in period 1	$B \to 3 \to 1 \to B$
	Vehicle 3 in period 2	$C \to 2 \to 4 \to 5 \to A$
Third objective function values	420	

Table 7: Set of efficient solutions resulting from solving small sample problem using the comprehensive criteria method

Efficient solutions	**First objective function**	**Second objective function**	**Third objective function**	**Computational time**
1	575779,95	50051,23	402	498.31
2	565412,43	50223,64	398	
3	552152,40	51102,23	390	
4	546081,87	52404,25	418	
5	541713,27	53993,55	408	

Table 8: Set of efficient solutions resulting from solving small sample problem using the utility function method

Efficient solutions	**First objective function**	**Second objective function**	**Third objective function**	**Computational time**
1	572179,09	50136,74	389	490,20
2	562092,72	50199,36	406	
3	552086,77	51007,00	391	
4	541705,29	54006,42	394	

Table 9: Set of efficient solutions resulting from solving small sample problem using the Torabi-Hosseini method

Efficient solutions	**First objective function**	**Second objective function**	**Third objective function**	**Computational time**
1	575772,17	50036,76	404	984,63
2	565427,99	50175,12	410	
3	562317,59	50269,03	395	
4	552155,51	51181,77	412	
5	545106,12	52393,27	388	
6	541704,87	53992,97	396	

Tables 7–9 respectively give a set of efficient solutions for the small sample problem using comprehensive criteria (Table 7), utility functions (Table 8) and Torabi-Hosseini (Table 9).

Concerning the output variables of the problem, Tables 10–12 give the place of facilities, and optimal vehicle routing resulting from the sample problem solution using comprehensive criteria, utility function and Torabi-Hosseini method from the first set of efficient solutions obtained.

Table 10: Place of potential facilities and transportation routing from solving small size problem using comprehensive criterion

Potential centers	No.	**Optimal place**
Production centers	2 centers	2 and 3
Distribution centers	2 centers	B , C
Optimal vehicles routing	Vehicle 1 in period 1	$B \to 3 \to 1 \to B$
	Vehicle 2 in period 1	$C \to 2 \to 4 \to 5 \to C$
	Vehicle 1 in period 2	$B \to 3 \to 1 \to B$
	Vehicle 2 in period 2	$C \to 2 \to 4 \to 5 \to C$

Table 11: Place of potential facilities and transportation routing from solving small size problem using utility function

Potential centers	**No.**	**Optimal place**
Production centers	2 centers	2 and 3
Distribution centers	2 centers	B , C
Optimal vehicles routing	Vehicle 1 in period 1	$B \to 1 \to 5 \to \mathrm{B}$
	Vehicle 2 in period 1	$C \to 3 \to 2 \to 4 \to \mathrm{C}$
	Vehicle 1 in period 2	$B \to 3 \to 1 \to \mathrm{B}$
	Vehicle 2 in period 2	$C \to 2 \to 4 \to 5 \to \mathrm{C}$

Table 12: Place of potential facilities and transportation routing from solving small size problem using Torabi-Hosseini method

Potential centers	**No.**	**Optimal place**
Production centers	2 centers	2 and 3
Distribution centers	2 centers	A , C
Optimal vehicles routing	Vehicle 1 in period 1	$A \to 2 \to 3 \to 4 \to A$
	Vehicle 2 in period 1	$C \to 5 \to 1 \to A$
	Vehicle 1 in period 2	$C \to 2 \to 4 \to 1 \to A$
	Vehicle 2 in period 2	$A \to 5 \to 3 \to A$

As inferred from the results of the dual-objective problem solving using multi-objective decision methods, changing optimal numbers and places of facilities as well as optimal transportation routing helps create a set of new and different efficiency solutions. This suggests that the effects of the two objective functions and the simultaneous consideration of these two functions change their output variables. To better compare the efficiency solutions of solving methods, comparison indices of efficient solutions (e.g., mean objective functions, the number of efficient solutions, the highest-expansion index, the distancing index and computational time) were used. Table 13 gives indices studied by the efficient solutions of multi-objective decision-making methods.

Table 13: Indicators for comparing efficient solutions from solving a small sample problem

Index	**Comprehensive criterion**	**Utility function**	**Torabi-Hosseini**
Mean objective function 1	556227,985	557015,96	557080,71
Mean objective function 2	51554,98	51337,37	51341,48
Mean objective function 3	410	412	412
Number of effective answers	5	4	6
Highest expansion index	34294,03	30718,51	34296,24
Distancing index	0,239	0,067	0,450
Computational time	498,31	490,20	984,63

As noted by Table 13, the comprehensive criterion decision-making method outperformed another method in gaining the mean first objective function indices, while the utility function was found to be most efficient in gaining mean second objective function indices, distancing indices and computational times. In the end, the Torabi-Hosseini method was more efficient than others in gaining a mean number of efficient solutions and the highest expansion indices.

5.2 Sentiment analysis

This section analyzes the sentiment of objective functions by changing the two parameters of the time of perishability and capacity of the supplier. Below are the analysis results of each of the parameters changed.

5.2.1 Perishability time

The time of perishability considered for the above problem is 1 time period, with a problem with a perishability period of 1-6 solved to determine the changes of the first and second objective functions. Table 14 gives changes in the first and second objective functions at different time periods.

Table 14: First and second objective function changes with changing period of perishability

Perishability period	First objective function value	Second objective function value	Third objective function value
1	541633,336	49942,651	420
2	531254,263	49647,126	422
3	52476,145	49034,842	428
4	521486,328	48625,661	430
5	519784,625	48369,824	432
6	515777,934	48060,074	436

Diagrams 3–5 illustrate changes in the first, second, and third objective functions based on perishability time period changes.

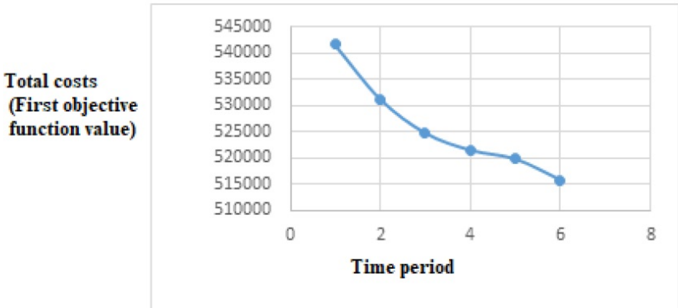

Figure 3: First objective function value changes at different perishability time periods

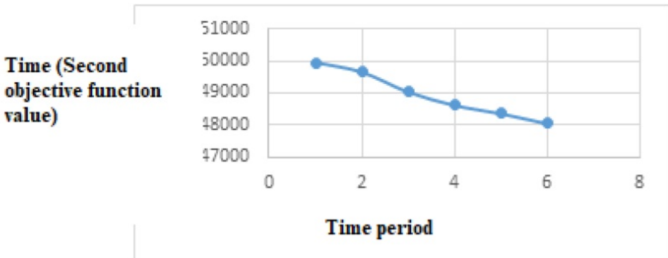

Figure 4: Second objective function value changes at different perishability time periods

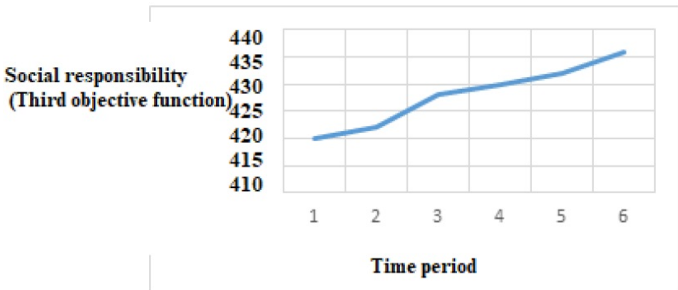

Figure 5: Third objective function value changes at different perishability time periods

According to the sentiment analysis, total amounts costs and transportation reach their highest level if the time period of perishability is 1; optimally speaking, if perishability decreases, the third objective function requires more transfer personnel due to the need for faster transportation, with the social objective function reaching its highest value at the perishability value of 1. This is because in this state, distributors and retailers do not have inventory in their stockpiles and for this, they should meet customer needs at every time period. It is also noted that with increasing the time period of perishability, total network costs will decrease, which is due to relevant planning in inventory storage.

5.2.2 Supplier capacity

To perform a greater sentiment analysis on the model, the capacity of supplier parameters was considered whose changes changed the first and second objective's function values. Hence, the capacity of supplier values was respectively 10, 20, 30,40 and 50% lower or higher than the basic capacity. Table 15 gives values of objectives functions.

Table 15: Objective function changes with changing the capacity of producers

Supplier capacity	First objective function values	Second objective function values	Third objective function values
-50%	546971.249	51766.533	282
-40%	545661.023	51247.250	301
-30%	543945.345	51024.556	323
-20%	542964.365	50534.681	344
-10%	542364.125	50018.614	383
basis	541633.336	49942.651	420
+10%	540148.964	49641.220	447
+20%	535476.152	49368.472	459
+30%	532647.332	49026.587	485
+40%	532148.002	48514.556	503
+50%	530486.772	48365.299	523

Figures 6–8 show changes in the objective functions relative to changes in producer capacity.

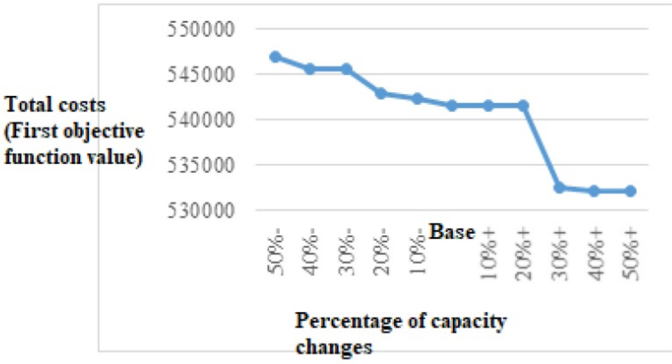

Figure 6: First objective function changes at various producer capacities

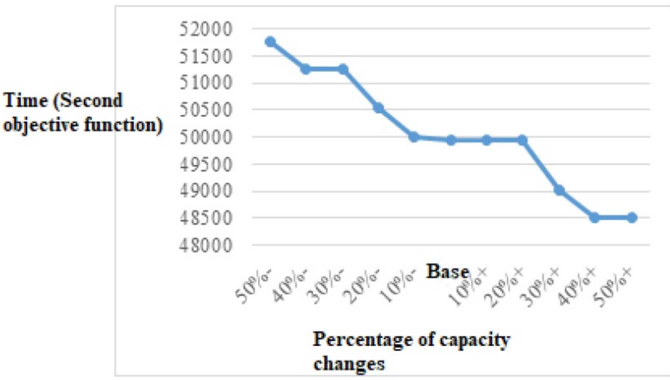

Figure 7: Changes in the value of the second objective function at different producer capacity

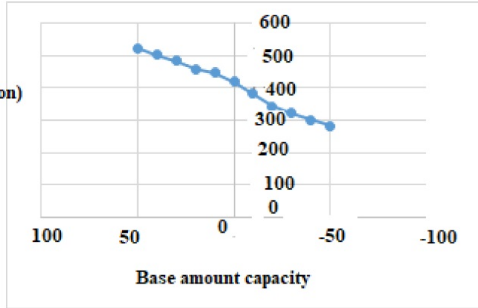

Figure 8: Changes in the value of the third objective function at different producer capacities

As noted by Table 15, as the capacity of the supplier increases. Total costs decrease due to the transfer of drugs being closer to other levels of the supply chain network, with the time required for drug transfer decreased, while social responsibility has increased.

6 Conclusion

This article solved a four-level perishable product supply chain problem using operational and marketing approaches under uncertainty states and Benders' Decomposition and Lagrangian Relaxation methods. From an innovation point of view, perishable product supply chain concepts were introduced and a final four-level model was designed. Another innovative aspect was the introduction of a fuzzy mathematical model in line with the existing real space in supply chain robes and uncertainty states. The last innovation was the inefficiency of the literature that had used heuristic and meta-heuristic algorithms due to the supply chain models being NP-HARD. On the other hand, this study used a Benders' Decomposition and Lagrangian Relaxation algorithm to mathematically solve the problem. This algorithm not only provided accurate solutions but also improved the solving time. The following is a list of results:

1. The comprehensive criterion decision-making method outperformed another method in gaining the mean first objective function indices, while the utility function was found to be most efficient in gaining mean second objective function indices, distancing indices and computational times. In the end, the Torabi-Hosseini method was more efficient than others in gaining the mean number of efficient solutions and highest expansion indices.
2. After minimizing the first and second objective function values and maximizing the third objective functions, the comprehensive criterion, utility function and Torabi-Hosseini method respectively worked effectively in gaining mean indices of the first, second and third objective functions.
3. The less the distancing indices and computational time, the more efficient the methods used to solve the problem. Hence, the utility function was more efficient in gaining indices of computational time and distance from the ideal point.
4. According to the sentiment analysis, total amounts costs and transportation reached their highest level if the time period of perishability is 1. This is because in this state, distributors and retailers do not have inventory in their stockpiles and for this, they should meet customer needs at every time period. It is also noted that with increasing the time period of perishability, total network costs will decrease, which is due to relevant planning in inventory storage.
5. Total costs decrease due to the transfer of drugs being closer to other levels of the supply chain network, with the time required for drug transfer decreased, while social responsibility has increased.
6. The computational costs obtained from problem-solving using the MOPSO algorithm were lower than those by the NSGA II, while the NSGA II algorithm outperformed the former algorithm when it came to finding efficient solutions.

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