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Estimation of parameters of non-linear regression based on PSOGSA algorithm

Ibtehaj Loqman, Iraq T. Abass*

Department of Mathematics, University of Baghdad, Baghdad 00964, Iraq

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Abstract

Although computational strategies for taking care of Non-linear Regression Based on Hybrid Algorithms (EPNRHA) to Estimation of Parameters have always been available for years, the further application of Evolutionary Algorithms (EAs) to such difficulties provides a framework for addressing a wide range of Multi-Objective Conflicts (MOPs). NRPSOGSA is an Estimation of Parameters of Non-linear Regression Gravitational Search Algorithm with Practical Swarm Optimization that involves the synthesis of hegemony by using the hybrid algorithm (PSOGSA) approach is utilized. Whilst Gravitational Search Algorithm with Practical Swarm Optimization Since the leader hiring process uses the Tchebycheff Strategy as a criterion, simplifying the multi-objective problem (MOP) by rewriting it as a set of Tchebycheff Approach, solving these issues at the same time within the GSA context may lead to rapid resolution. Dominance is important in constructing the leader's library because it allows the chosen leaders to encompass fewer dense places, reducing global optimization problems and producing a more diverse approximated Pareto front. 6 non-linear standard functions yielded this result. PSOGSA appears to be more productive than GSA, PSO, and BAT. All of the outcomes were completed. by MATLAB (R2020b).

Keywords: Estimation Parameter, Practice Swarm Algorithm, Non-Linear Regression
2020 MSC: 90XX, 62J02

1 Introduction

The current study is addressed in this section. It considers the study's context; it states the research aims, research question, significance of the study, scope, and restrictions of the investigation.

Nonlinear Residuals are those in which at least one of the model parameters is expressed nonlinearly in the modelling expression. Nonlinear models are used to model complex interrelationships among variables and are employed in a variety of science and industrial disciplines. Progression, yield intensity, and dose-response equations are types of numerical methods, as are wide variety used to represent physical, biological, economical, and econometric processes [3].

While the statistical theory of parameter estimation in linear models is pretty much complete, many difficulties in regression functions remain unsolved.

*Corresponding author
Email addresses: ibetehaj.loqman@gmail.com (Ibtehaj Loqman), iraq.t@sc.uobaghdad.edu.iq (Iraq T. Abass)

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The underlying principle behind non-linear training is the same as regression models: to link a response y to a vector of predictor variables $x = (x_1., x_2\ ,..\ x_k)$.. The fact that the prediction equation depends nonlinearly on one or more unknown factors characterizes Non-Linear Inference. Non-Linear Regression has been used when there are corporeal reasons to believe that the relationship between the response and the predictors follows a particular functional form. Whereas regression model is frequently used to build a purely econometric technique, Non-Linear Regression has been used when there are physical reasons to believe that the relationship between the response and the factors that influence follows a specific functional form. A Non-Linear Regression model has the form for a pair of $(xi.yi)$ with data instances.

y $= f\,(x.\ \beta) + \varepsilon\ .\ i{=}\ 1 \ldots\ldots. n$ The εi are frequently assumed to be uncorrelated with mean zero and constant variance.

The shape of the Non-Linear Regression function is known in the estimation situation, but the factors $\beta_1 \ldots.\ \beta_p$ are undefined. Ordinary linear model is a popular tool for calculating unknown parameters in a Non-Linear Regression function. [4]. The guesses of $\beta 1 \ldots.\ \beta p$ are derived using this method by decreasing the parameter $s\,(\ \beta) = \sum_{i=1}^{n} \mathrm{e}_{\mathrm{i}}^{2} = \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{yi} - \mathrm{F}\,(\mathrm{Xi}\ .\mathrm{B})\,)^2$.(the sum of squared of errors prediction. An efficiency issue involving nonlinear estimation results is one in which the objective function S(β) is optimized.

The parameter estimation of Non-Linear Regression models has been the subject of various articles. The nonlinearity theory, on the other hand, makes estimation method and statistical analysis of parameter estimates more confusing and hard. Also, the limits of these(traditional)methods for estimating nonlinear parameters. It is also difficult for practitioners to regulate and takes a huge amount of auxiliary data to function successfully. These issues arise as a result of the goal function's huge number of parameters and multimodal nature. Using standard optimization approaches, minimizing the mean squared deviations function S(β) is extremely challenging [6].

As a result, the adoption of sophisticated meta-heuristic processes, which have several advantages including ease of implementation, is trustworthy, robust, and helpful in overcoming these challenges. To solve Non-Linear Multiple regression, three meta heuristic methods termed Gravitational Search Algorithm with Practical Swarm Optimization (PSOGSA) were utilized in this paper.

2 Non-Linear Regression

Non-Linear Regression is a type of regression that is not logarithmic. Among the most extensively used measurement methods for analyzing the relationship between two or more variables is statistical technique [1]. It is a crucial instrument for the accurate evaluation and thorough utilization of data gained in investigations. It illustrates the link between two types of measures: autonomous or pr+edictor observations, designated by the letter I, and dependent or predictor measurements, denoted by the letter P. $x{=}\,(x_1,\ x_2,\ \ldots\ ..,x_k\ )$, and the dependent or response measurement, denoted Y.

The goal is to create a regressive model that allows us to accurately characterize, predict, and manipulate the dependent variable using just the independent variables [3, 4], $y = f(x,\, \beta) + \varepsilon$ where y is the dependent variable, x is a vector of independent variable(s), β is a vector of characteristic(s), β *and s* is an array of specifications with zero means and standard deviations. Linear and nonlinear regression are the two main types of regression analysis.

When the regression product f is linear regression, coefficient of determination [10] becomes a very commonly used statistical deductive reasoning approach. Sir Francis Galton first introduce the theory of linear regression in 1894 [10]. However, linear models are not always applicable; consequently, a Non-Linear Statistical method, where f is nonlinear in β [j], is frequently used. Non-linear regression analysis might have non-linear characteristics, non-linear indicators, or non-linear parameters and variables. Even when the variables in a forecast are linear, the model is characterized as a Non-Linear Regression model if the characteristics are non-linear [11].

The Non-Linear Resistance is a useful technique for assessing scientific data, especially when data must be transformed to suit a linear interpolation. As a result, linear regression can be thought of as a subset of the more broad non-linear regression. [12]. Nonlinear models are used in a variety of fields, including physical, biological, and inferential statistics, as well as business, construction, mathematics, and managing. Nonlinear model construction is a fresh and exciting subject of research in applied mathematics currently. A researcher in mathematics and any other scientific area is almost definitely confronted with the difficulty of formulating a multiple linear regression model.

In the literature, a great number of nonlinear models have been specified and effectively applied to a variety of real-world scenarios relating to a variety of research topics in various domains of mathematical modeling. However, a huge number of scenarios have yet to be nonlinearly represented due to the complexity of the circumstances or their statistical and mathematical intractability [13]. Numerical methods have a wide range of applications in real-world situations, and we'll look at two instances of nonlinear regressors next.

Table 1: Models of Non-Linear Dependent variable:-

N	Name	Models
1	Meryer1	$\frac{\beta1\beta2x1}{1+\beta1x1+\beta2x2}$
2	Meryer4	$\beta3(e^{-\beta1x1}+e^{\beta2x2})$
3	Meryer7	$\beta1+\beta2e^{\beta3x}$
4	Militky4	$\beta1e^{\beta3x}+\beta2e^{\beta2x}$
5	Militky5	$\beta1x^{\beta2}+\beta3^{\beta2/x}$
6	Gompertz	$\beta1e^{-e(\beta2-\beta3x)}$
7	Logistic	$\frac{\beta1}{1+e^{(\beta2-\beta3x)}}$
8	Richards	$\frac{\beta1}{\left(1+e^{(\beta2-\beta3x)}\right)^{1/\beta4}}$
9	Jennrich	$e^{\beta1x1}+e^{\beta2x2}$
10	Militky2	$e^{\beta1x}+e^{\beta2x}$
11	Ratkowsky2	$\frac{\beta1}{1+e^{(\beta2-\beta3x)}}$
12	Eckerle4	$\frac{\beta1}{\beta2}e^{\left(-\frac{(x-\beta3)^2}{2\beta_2^2}\right)}$
13	Ratkowsky3	$\frac{\beta1}{(1+e^{(\beta2-\beta3x)})^{1/\beta4}}$
14	BoxBOD	$\beta1(1-e^{-\beta2X})$
15	Thurber	$\frac{\beta_1+\beta_2X+\beta_3X^2+\beta_{4\ X^3}}{1+\beta_5X+\beta_6x^2+\beta_{7x^3}}$
16	MGH09	$\frac{\beta1\left(x^2+X\beta_2\right)}{x^2+X\beta_3+\beta_4}$
17	Misrald	$\frac{\beta_1\beta_2X}{1+\beta_2x}$
18	Misrala	$\beta_1\left(1-e^{-\beta_2x}\right)$
19	Chwirut2	$\frac{e^{-\beta_1x}}{\beta_2+\beta_3x}$
20	Rat42	$\frac{\beta_1}{1+e^{(\beta_2-\beta_3x)}}$

3 Classical methods

Lu, Dong and Zhou (2019), the MSE and the LSE are the most frequently used methods for parameter estimations. MSE considered good statistical properties and is preferred by researchers. Furthermore, the LSE approach is simply a data is subjected with the summation of squared residuals as the minimizing goal. Practitioners love LSE because it is pretty convenient. As a result, the parameter of four Non-Linear Regression Models was estimated using Kernel Density estimation Estimation and Least Squares in this research.

4 Gravitational Search Algorithm Technique (GSA)

Rashedi et al. [8] proposed the Gravitational Optimization Technique (GSA) in 2009 to solve efficiency difficulties [14]. The heuristic method devised by the general public is based on mass interactions and gravity law. The solutions in the GSA populations are referred to as agents, and they interact with one another via gravity. The performance of each agent in the population was assessed with its mass. The solution with the higher mass has been the best option.

The gravitational force is a force granules is simply the ratio of their general populace and inversely proportional to the distance between them, according to the law of gravitation: each neutrino attracts any other granules, and the gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the distance between them, R. The present velocity of any mass is equal to the sum of the proportion of its prior velocity and the fluctuation in acceleration, according to the law of motion.

The force acting on the system divided by the amount of inertia equals variation in velocity or acceleration of any material. The objects masses are obeying of gravity low as following:

$$F_{ij} = G\frac{M_{aj}\times M_{pj}}{R^2} \tag{4.1}$$

where G denotes the newtonian constant, M_{PJ} denotes the volume of the moving mirror, F denotes the gravitational force amount, and R denotes the distance between the two elements

$$M_{aj}, M_{PJ}.$$

$$a_i = \frac{F_{ij}}{M_{ii}} \tag{4.2}$$

where M_{ii} is inertia mass of agent i ,

The velocities V_i and position X_i as expressed in equation, will be updated by agents during the inquiry, accordingly 0.

$$V_i^d\,(t{+}1) = rand_i \times V_i^d\,(t) + a_i\,(t) \tag{4.3}$$

$$\mathrm{X}_i^d\,(\mathrm{t}+1) = \mathrm{X}_i^d\,(t) + V_i^d\,(t{+}1) \tag{4.4}$$

where d is the number of variables and $rand_i$ is uniform random variable in the interval [0,1]. The main step of the GSA can sometimes be summed up as follows:

Step 1. Initialize gravitational acceleration equations G_0 , $\propto$, Where G_0 and $\propto$ are initialized in the beginning of the search , and their values will be reduced during the search . T is the total number of iterations.

Step 2. The starting population was generated at arbitrary and comprised of N inhabitants, with each attorney's orientation determined as follows:

$$X_i\,(t) = \left(X_i^1\,(t)\,.X_i^2\,(t)\ldots\ldots\ldots\ldots\ldots X_i^n\,(t)\right), i = \ldots\ldots\ldots\ldots, N \tag{4.5}$$

Step 3. This process is repeated until the shutdown objectives are fulfilled.

A . Awarded the best and worst operatives, and now all agents were rated in the demographic.

B. The gravitational coefficient has been adjusted as above.

C. Calculate the energy using the technique below:

$$F_{ij}^d = G\,(t)\,\frac{M_{aj}\,(t) \times M_{pj}\,(t)}{R_{ij}\,(t) + ?}\left(X_j^d\,(t) - X_i^d\,(t)\right) \tag{4.6}$$

$$R_{\mathrm{ij}}\,(t) = \frac{X_i\,(t)\,.X_j\,(t)}{2} \tag{4.7}$$

where $R_{ij}\,(t)$ is the Euclidian distance between two agents i and j

D. Calculate the total force acting on agent I at iteration of the algorithm as follows:

$$F_i^d\,(t) = \sum_{j \in kbest.j \neq 1} rand_j F_{ij}^d\,(t) \tag{4.8}$$

K best is the set of first K agents with the best fitness value and biggest mass and where $rand_j$ is a random number in the interval [0,1] .

E. Calculate the inertial mass as following:

$$\mathrm{m}_i\,(\mathrm{t}) = \frac{\mathrm{fit}_i\,(t) - worst\,(t)}{best\,(t) - worst\,(t)} \tag{4.9}$$

$$M_i\,(t) = \frac{m_i\,(t)}{\sum_{j=1}^{N} m_j\,(t)} \tag{4.10}$$

where $fit_{i(t)}$ symbolize the agent i's transfer function at time t, and worst (t) and best (t) are For something like a minimization issue, it is formulated as having:

$$\text{best}\ \ (t) = \min fit_j\ \ (t) \tag{4.11}$$

$$\text{worst}\ (t) = maxfit_j\ (t) \tag{4.12}$$

F. The accelerated agent was calculated using the following equation:

$$a_i^d\,(t) = \frac{F_i^d\,(t)}{M_{ii}\,(t)} \tag{4.13}$$

G. As illustrated in calculation, the position and velocity of agent I are determined.

H. The number of iterations is incremented until the terminal criteria are met.

Step 4. The quickest and most perfect solution is obtained.

5 Practical Swarm Optimization (PSO)

The PSO metaheuristics was motivated by the coordinate movement of fish schools and bird flocks (Kennedy and Eberhart, 2001)[9]. A potential solution to the problem being solved is represented by a particle, and the PSO is a swarm of particles. Particles "flow" through hyper dimensional search space of the problem, and changes to the position of the particles within the search space are based on the social cognitive tendency of individuals to emulate the success of other individuals. Each individual of a population (in this case, particles) has its own life experience and is able to evaluate the quality of its experience. As social individuals, they also have knowledge about how well their neighbors have behaved. These two kinds of information correspond to the cognitive component (individual learning) and social component (cultural transmission), respectively. Hence, an individual decision is taken considering both the cognitive and social components, thus leading the population (the swarm) to an emergent behavior.

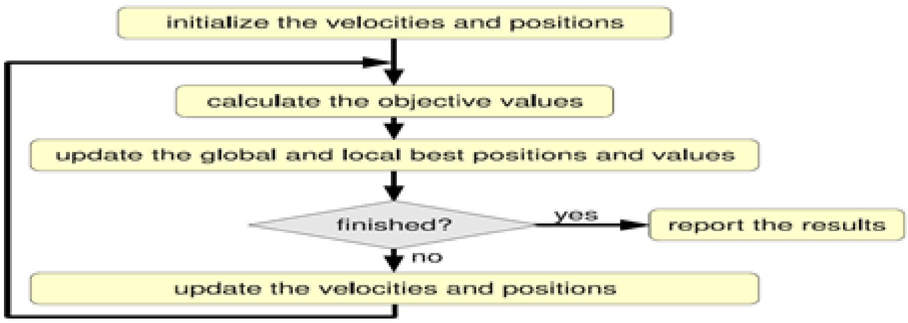

Figure 1: Flowchart of PSO

6 Bat Algorithm

Bat Algorithm(BATA) was always a method biologically inspired that belongs to the SI family. The standard Bat Algorithm was created by Xin - She - Yang in 2010 [16]. BATA focuses on micro bat echolocation which uses echo of bats to explore sound signal called sonar echolocation and use that signals to recognize objects or obstacles around them. Yang concentrated on three rules for bat successful execution:

To begin, bats fly at arbitrary with a specific frequency and velocity against a certain place, even though the loudness and wavelength can change. As a result, bats automatically alter their wavelengths to match their prey. Furthermore, all bats utilize acoustic signals to determine the distance to a certain point.

Finally, the author believed that loudness should be varied from maximum to minimum rather than any other direction. BATA uses automatic zooming to balance exploration during the search phase by simulating the variance in pulse exhaust emissions and brightness experienced by bats during hunting for prey [17].

The steps of BAT Algorithm are introduced as follows:

Step 1. At the first ,f $(x.\beta)$ was used $\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n}$ as a fitness function of the BAT Algorithm,initialize population of BAT x_i,the objective function ,Velocityv_i,Determine pulse frequency f_i at x_i . Loudness A and pulse rate r_i are initialized . $r_i \in${0,1}.

Step 2. By adjusting the frequency, new solution is generated and updating velocities and positions/solution.

Step 3. If (random>r_i)

Form the best solution, select the solution and around the selected the solution and around the selected best solution generate a neighborhood solution.

Step 4. Else fly random to create a new solution.

Step 5. If (random $<A_i$ and f(x_i) < f(x_0)) , whereas f(x), x=$(x_1.x_2 \ldots .x_d)^t$objective function .Accept the new solution , increase r_i and reduce A_i .

Step 6. Find the current best (x_0) by ranking the bats.

Step 7. While (t<maximum number of iteration)

Post procedure outcomes and representation. The algorithm terminates with the best aggregate solution.

7 The Proposed Method (PSOGSA)

First, we'll go over the MOP terminology in this piece. The framework of the suggested hybrid approach is then shown. Following that, we'll go over the performance assignment procedure. Finally, the tactics for mating and also the processes of ambient adjustment are discussed.

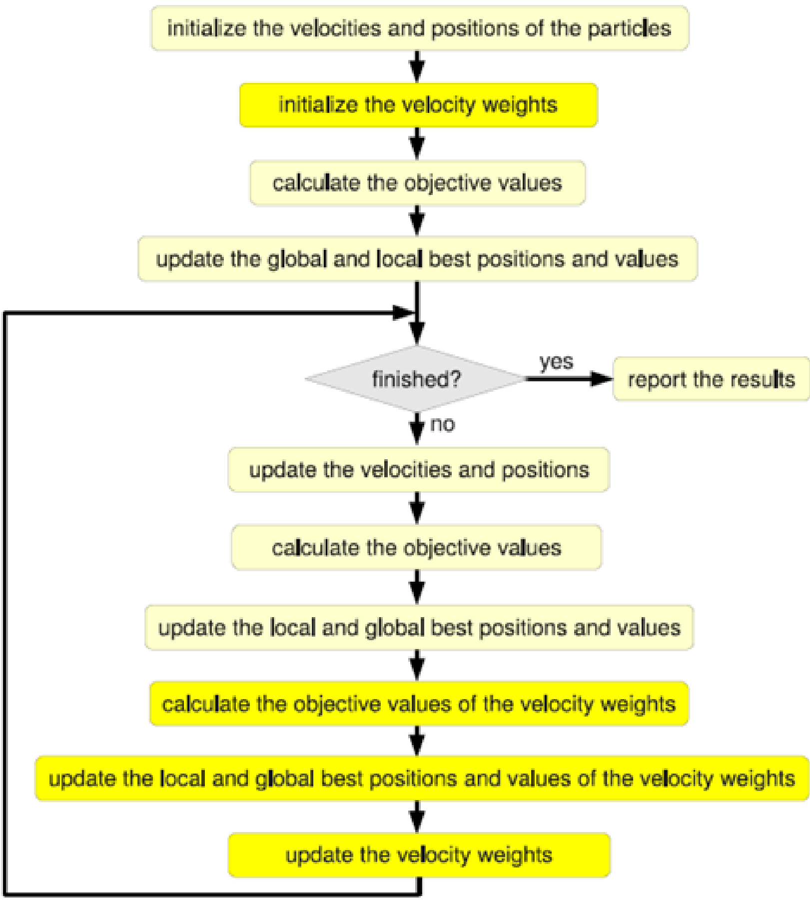

Figure 2: Flowchart of PSOGSA

Furthermore, no formal study has been done to link the characteristics to convergence rates. The scientist proposes a hybrid technique called PSOGSA by incorporating two additional aspects, repository and leader, as discovered in the MOPSO algorithm proposed by [7] to produce improved optimal technique estimate Non-Linear Regression Models. The archive is in charge of archiving and recovering the most notable non-dominated and non-controllable Pareto optimum solutions discovered so far. The archive also has a main unit, which is the archive's control centre.

Pseudo-code of the PSOGSA
Set $k := 0$ and velocity $=0$ μ=0.1, $r0 = 0.5$, $A = 0.6$.
Randomly initialize point P_i $for\ n.population$;
Calculate the fitness values of initial population: $f\ (P\ )$;
Find the non-dominated solutions and initialized the archive with them
WHILE (the termination conditions are not met)
PSO Steps
$\mathrm{Q} = \mathrm{Qmin} + (\mathrm{Qmin} - \mathrm{Qmax}) * \mathrm{rand}$
$\mathrm{V}_{\mathrm{ij}\ (\mathrm{t}+1)}^{\mathrm{t}+1} = \mathrm{V}_{(ij)}^{\mathrm{t}} + \mathrm{C}_1 R_1 \left(\mathrm{B}_{BEST_{ij}} - X_{ij}^{t}\right) + \mathrm{C}_2\mathrm{R}_2(\mathrm{G}_{\mathrm{BEST}_{\mathrm{ij}}} - \mathrm{X}_{\mathrm{ij}}^{\mathrm{t}})$
$X_i^{t+1} = X_i^t + \mathrm{V}^{\mathrm{t}+1}{}_{\mathrm{I}}$
If rand $>$ r
$\mathrm{P}_{\mathrm{leader2}}$= Select Leader(archive)
$\mathrm{X}_{\mathrm{new}} = \mathrm{X}_{(\mathrm{t})} + \mathrm{rand} * \left(\mathrm{X}_{\mathrm{leader2}} - X_{(\mathrm{t})}\right)$
End
$if\ X_{new}\ dominated\ on\ X_{(t)}$ & $(rand < A)\mathrm{X}_{(\mathrm{t})} = \mathrm{X}_{\mathrm{new}}$
End
If rand $<(\frac{1-(k-1)}{Max\ iteration-1})^{1/\mu}$
S =Mutation($\mathrm{P}_{(\mathrm{t})}$)
$if\ X_{new}\ dominated\ on\ X_{(t)}$ & $(rand < A)\ \mathrm{P}_{(\mathrm{t})} = \mathrm{S}$
End
End
Find the non-dominated solutions
Update the archive concerning to the obtained non-dominated solutions
If the archive is full
Run the grid mechanism to omit one of the current archive members
Add the new solution to the archive
end if
If any of the new added solutions to the archive is located outside the hypercube
Update the grids to cover the new solution(s)
end if
Inc
$rease$ r $and\ reduce\ A$
$Set\ k := k + 1;;$
End While

8 Simulation Experiment and Analysis

8.1 Performance Measures

To endorse, both quantitative information associations are used the PSOGSA calculation against other algorithms. The graphs of final Pareto occasions were used for meaningful comparing are presented. As for the quantitative comparison, convergence metric gravitational distance (GD), Inverted gravitational distance (IGD) and hyper volume (HV) [1] are used, which is Mathematics featured in (8.1), (8.2) and (8.3). gravitational distance (GD). In deciding if arrangements of Q can be incorporated with the arrangement of P^* or don't, the utilization of the (GD) It is OK to use the term metric to describe it, because it evaluates the normal separations of the arrangement sets of Q as proceeds with P :

$$GD = \frac{\sqrt[p]{\sum_{n=1}^{R} (d_i^p)}}{R} \tag{8.1}$$

$$IGD = \frac{1}{R}\,(\sum_{n=1}^{R}\min?(\sqrt[p]{\sum_{n=1}^{R}(d_i^p)}\,)) \tag{8.2}$$

where R is the number of solutions in Pareto front (PF) P^* established, also known as the IGD measurement, which quantifies the cohesiveness of circulation of purchased structures in terms of scattering and augmentation, i.e. inverted generational distance is the average distance between every way to solve in the testing dataset and the proximity solution in the approximate solution set; it thus reflects consolidation of alternatives.

The volume of the objective space that is pitifully commanded by a PF estimation is quantified by the hyper volume identifier (Hyper volume) 4.1. A point of comparison is used in hyper quantity v^* that also denotes an overarching goal with a cap v^* Is defined as perhaps the most egregiously bad objective esteems found in A (for instance) v^* Is described as one of A's highest outrageously low outward image and self (for instance) (Λ), the hyper volume is characterized as:

$$HV(A) \;= \Lambda(\cup\{x\;|a\;?\;x\;?\;v^*, a \in A\}). \tag{8.3}$$

In addition, Using Multi-Objective Bat Algorithm for Solving Multi-Objective Non-linear Programming Problem [15] are introduced and a metaheuristic hybrid algorithm for solving multi-objective optimization problems proposed and using the same indicators that used in this paper [2]. The lingering effects of IGD are shown in Table 1 The results of using hyper volume HV are shown in Table 2. The p-estimation is shown in the last paragraph. of two followed matched t-test between the (PSOGSA) and different techniques where strong text style shows a factually huge distinction. Figure 1 Determine the difference between PF valid and PF approximated for each of the five estimates under consideration.

8.2 Multi-Objective Test Functions

To show the proficiency of the proposed hybrid PSOGSA algorithm six benchmark functions are used to comparative and validate the proposed hybrid algorithm. Figures (3.1) -(3.30) shows the details of the benchmarking functions [58] for the test capacities of Non-Linear functions.

8.3 Decision space

The choice spectrum represents the range of options that are available to us. The weights of the criterion will just be determined by the decisions we make. As a result, a comparable problem in the decision space can be defined. For example, while creating products, we choose design specifications (respect to price), each of which has an impact on the performance measures (criteria) that we use to analyze it. True Pareto front: is a quantitative strategy for selecting a subset of possibilities from a vast number of alternatives, based on their relative importance.

8.4 Results and Discussion

This article is devoted to proving the suggested algorithm's accuracy. The proposed Hybrid algorithm (PSOGSA) is carried out in Matlab, and registering time is inside a couple of moments to not exactly a moment, going to depend on the topic of discussion. We put it to the test and to use a variety of variables, including total population (n=20, 40, 80, 160 and 200), Number of targets M, dimensions D.

The experimental results have verified the effectiveness of the proposed strategy in balancing proximity and diversity. The extensive experiments provided a conceptual policy's effectiveness in balancing proximity and diversity. Researchers, on the other hand, have created a variety of statistic estimation methods of Non-Linear Regression Based on Hybrid Algorithms.

In this area, we first notification the exhibition of MSE with the new calculation PSOGSA and some different calculations GSA, PSO and BAT.

Tables 3.1 and 3.2 presents the mean square error of the comparing estimation esteems got by various calculations in comparative conditions for the algorithms (BAT, PSO and GSA) depended on the first model (Meyer (7)) of Non-Linear Regression, with the set parameters for this model as (β_1**,** β_2 **and** β_3) = (500,1,1) and (700,2,1). Specifically, the best-estimated qualities are in blue color.

These tables indicated that PSOGSA hybrid algorithm shows the best results for all samples in terms of accuracy as compared to other methods since has the less Mean Square Error.

The validity of the hybrid PSOGSA algorithm is being investigated using different design criteria. The true value of an algorithm appears when compared to other algorithms that solve the same problem. We take the MSE criteria to compare the new hybrid PSOGSA algorithm and the GSA, PSO and BAT algorithms. Employing deviation values, all algorithms are solved using the same standard functions.

In Tables (1, and 2) we observe the performance of the MSE indicator with the functions of (Meyer (7), Meyer (4), Militky (4), Militky (2), Misra 1d and MGH09) compared to the new PSOGSA algorithm with the GSA, PSO and BAT algorithms where the new PSOGSA algorithm has average mean performance in the calculation of the median and arithmetic mean, while in The rank and the better are the other algorithms compared to it in the criteria.

Table 2: Contrasting Findings of PSOGSA, GSA,PSO, BAT and MSE for the Model one When $\beta1$=500, $\beta2$=1 and $\beta3$=1

			500	1	1	
N	Methods	Statics	$\beta\mathbf{1}$	$\beta\mathbf{2}$	$\beta\mathbf{3}$	MSE
20	PSOGSA	Estimated	518.3291709	0.910730555	1.05E+00	2.24E-12
		MSE	3.02E+03	2.05E-01	2.34E-02	
	GSA	Estimated	515.4825876	1.25E+00	9.89E-01	2.39E+03
		MSE	2.90E+03	0.358705136	0.005827518	
	PSO	Estimated	497.6370965	1.05679442	0.995633956	5.96E-06
		MSE	37.67715537	0.013324019	0.000122612	
	BAT	Estimated	531.4655455	1.400174082	1.037335817	13829.76061
		MSE	22289.82665	1.069844882	0.034858133	
40	PSOGSA	Estimated	498.5497421	1.002126615	1.00E+00	1.79E+02
		MSE	2.00E+01	1.88E-05	1.68E-07	
	GSA	Estimated	492.1928976	9.23E-01	1.02E+00	1.50E+04
		MSE	1.38E+03	0.139679747	0.011074036	
	PSO	Estimated	500.2296406	1.010216035	0.998925962	60.62771928
		MSE	4.530248954	0.000564709	5.62E-06	
	BAT	Estimated	455.5381546	1.532121643	1.002800611	18493.81266
		MSE	12260.97835	2.232083626	0.019262127	
80	PSOGSA	Estimated	509.6823808	0.938666734	1.01E+00	2.14E+03
		MSE	2.91E+02	8.66E-02	1.29E-03	
	GSA	Estimated	521.0421176	1.02E+00	1.01E+00	6.57E+03
		MSE	1724.788461	0.27331325	0.002382886	
	PSO	Estimated	510.1979776	1.016459186	0.999510806	108.5979041
		MSE	998.2189473	0.034823198	0.000173046	
	BAT	Estimated	559.7793919	0.935522935	1.024702154	285600.0237
		MSE	91881.31271	0.292519058	0.004938972	
160	PSOGSA	Estimated	4.68E+02	1.004616382	1.00E+00	1.00E-12
		MSE	1.02E+04	3.24E-03	1.72E-05	
	GSA	Estimated	5.03E+02	9.90E-01	1.00E+00	4.32E+02
		MSE	7.23E+02	0.039640993	0.000358727	
	PSO	Estimated	502.6107137	0.946248144	1.004684861	2.40E-07
		MSE	107.984559	0.014718695	0.000115536	
	BAT	Estimated	470.0163907	1.284715396	0.994172704	6.64E+03
		MSE	15603.57524	0.572973707	0.004286772	
200	PSOGSA	Estimated	480.8337505	1.048992482	1.00E+00	5.81E-08
		MSE	3.06E+04	1.36E-01	4.62E-04	
	GSA	Estimated	539.8993542	1.05E+00	9.97E-01	3.48E+03
		MSE	7.05E+03	0.06833948	0.000358611	
	PSO	Estimated	460.3350117	1.113483939	0.994344839	1.68E-10
		MSE	8595.18961	0.047798726	0.000113817	
	BAT	Estimated	518.0323702	1.117883772	1.017977949	1.26E+04
		MSE	43905.41592	0.877613304	0.006934813	

Table 3: Contrasting Findings of PSOGSA, GSA,PSO, BAT and MSE for the Model one When $\beta 1$=700, $\beta 2$=2 and $\beta 3$=1

			700	2	1	
N	Methods	Statics	B1	B2	B3	MSE
20	PSOGSA	Estimated	703.9904304	1.864084059	1.01E+00	1.30E-12
		MSE	1.03E+02	9.33E-02	2.17E-04	
	GSA	Estimated	586.3725632	1.10E+00	1.11E+00	1.80E+03
		MSE	2.34E+04	1.03915246	0.027710749	
	PSO	Estimated	635.2227859	1.576677818	1.040009342	5.69E-05
		MSE	49145.35846	0.671402495	0.00684904	
	BAT	Estimated	724.198291	1.607755894	1.040191683	5493.084317
		MSE	62253.89049	1.06374377	0.006128501	
40	PSOGSA	Estimated	631.9552125	1.884436722	1.01E+00	1.24E-12
		MSE	4.90E+04	1.16E-01	3.35E-04	
	GSA	Estimated	629.8379786	1.05E+00	1.09E+00	3.19E+02
		MSE	1.14E+04	1.052759282	0.01047274	
	PSO	Estimated	700.8884558	1.932221546	1.003110728	5.72E-10
		MSE	7.89232288	0.045936859	9.68E-05	
	BAT	Estimated	650.9819254	4.876559547	1.007771297	1074.122974
		MSE	35142.48365	94.57285581	0.009524098	
80	PSOGSA	Estimated	713.9606908	1.81088193	1.01E+00	1.28E-12
		MSE	1.95E+03	3.58E-01	2.05E-03	
	GSA	Estimated	606.9944184	1.45E+00	1.04E+00	1.39E+03
		MSE	18911.37891	0.568493155	0.004845854	
	PSO	Estimated	728.174662	1.999545506	1.000011216	4.78E-09
		MSE	7938.114346	2.06E-06	1.25E-09	
	BAT	Estimated	753.6403571	1.974549382	1.002685229	295.7799145
		MSE	90555.28186	0.497655844	0.001535721	
160	PSOGSA	Estimated	6.86E+02	1.928703	1.00E+00	1.21E+07
		MSE	2.80E+04	2.53E-02	4.57E-05	
	GSA	Estimated	5.94E+02	1.29E+00	1.05E+00	1.45E+10
		MSE	2.25E+04	0.696924646	0.003672247	
	PSO	Estimated	640.9004828	1.938465389	1.002147911	6268560.804
		MSE	39636.47069	0.019418117	2.48E-05	
	BAT	Estimated	705.3181901	1.604488495	1.02252595	6.04E+08
		MSE	101310.3441	0.532550035	0.001763688	
200	PSOGSA	Estimated	540.2090806	1.729504441	1.01E+00	1.03E-12
		MSE	1.16E+05	2.92E-01	8.02E-04	
	GSA	Estimated	546.3144824	1.72E+00	1.01E+00	3.99E+07
		MSE	3.11E+04	0.259405223	0.000704578	
	PSO	Estimated	720.7870626	1.684650089	1.019423862	118.6986807
		MSE	2232.371458	0.415247309	0.002021409	
	BAT	Estimated	422.9555071	1.596857927	1.01846512	1.73E+06
		MSE	182340.8134	0.25715727	0.000531839	

9 Convergence Graphs

For something like the samples, consolidation graphs were created to show how quickly the standard evaluation converges with the number of iterations. For all datasets, 100000 iterations have been run. Following graphs demonstrate the effectiveness of our suggested hybrid approach in obtaining the optimal value more quickly.

For better illustration, all the comparative analysis on different Non-Linear Regression models with different sample sets for n=20, 40, 80, 160 and 200 are depicted in Figures (1)- (3). It can be seen from these figures that Meta-heuristic algorithms give lowest MSE values amongst different models than the others. However, comparing GSA, BAT and PSO was better in terms of accuracy.

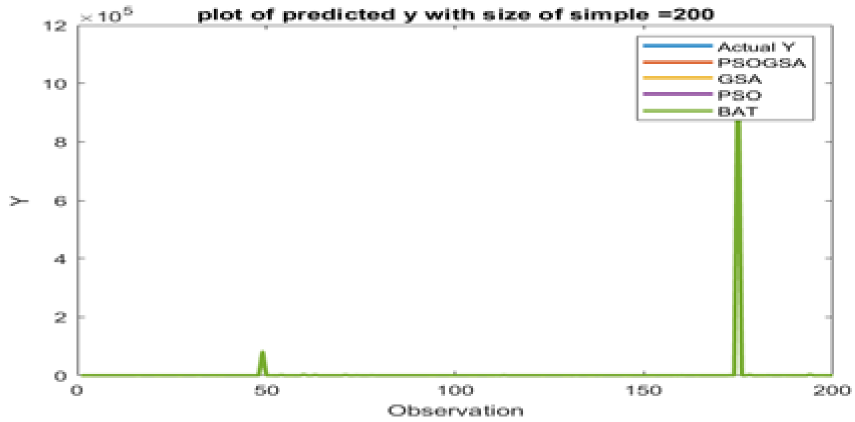

Figure 3: Comparative PSOGSA,BAT,PSO and GSA algorithms for the Modle one when of sample size=200

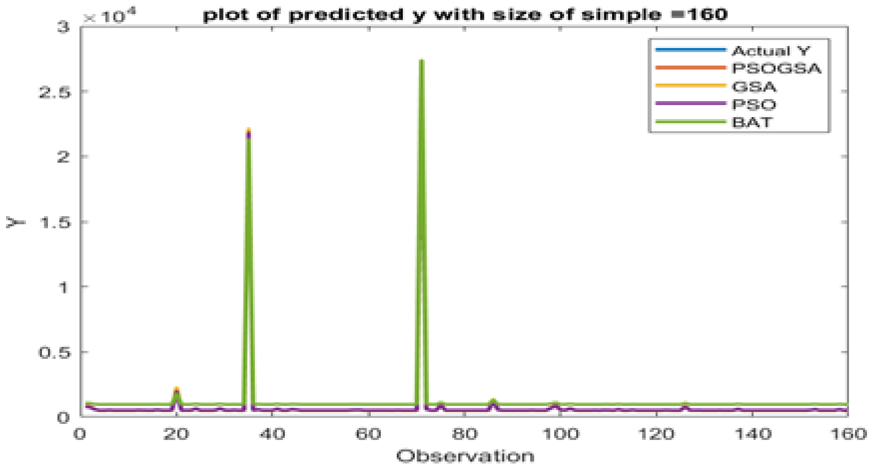

Figure 4: Comparative PSOGSA,BAT,PSO and GSA algorithms for the Modle one when of sample size=160

10 Conclusion and Future Work

The work in this thesis has been devoted for solving Non-Linear Regression in the field of the combinatorial optimization problem. First, we summarize the research conducted within each chapter in sections. In doing so, we explain how we deal with the Non-Linear Regression challenges mentioned at the beginning of the thesis. Then, we present some directions deal with our hybrid algorithm (PSOGSA) in details. Second, three Meta-heuristic algorithms were utilized as an alternative approach for estimation of N0n-Linear Regression models. Six type of Non-Linear Regression models () were used, which have a different number of parameters. In addition, different sample (20, 40, 160, 200) were used. Lastly, more experiments and comparisons of the proposals are critical. We may conclude from the findings of this research that hybrid algorithms can be built in a variety of methods. Multi-purpose problems, segmentation solutions, and the PSOGSA algorithm are among them. Further investigation could focus on measuring the results of this algorithm to other common optimization technique.

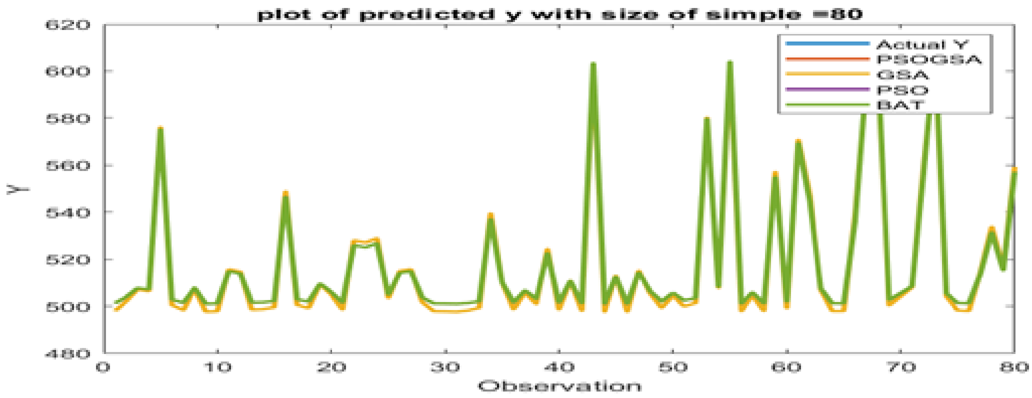

Figure 5: Comparative PSOGSA,BAT,PSO and GSA algorithms for the Modle one when of sample size=80

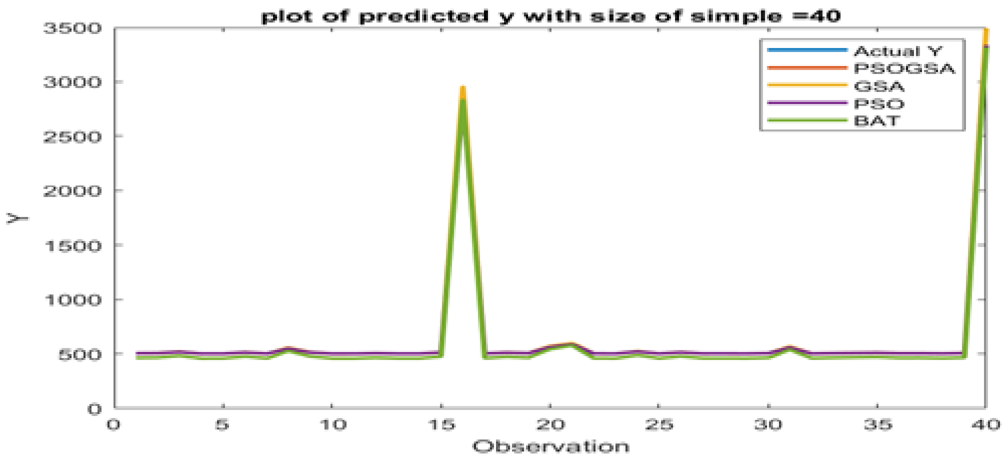

Figure 6: Comparative PSOGSA,BAT,PSO and GSA algorithms for the Modle one when of sample size=40

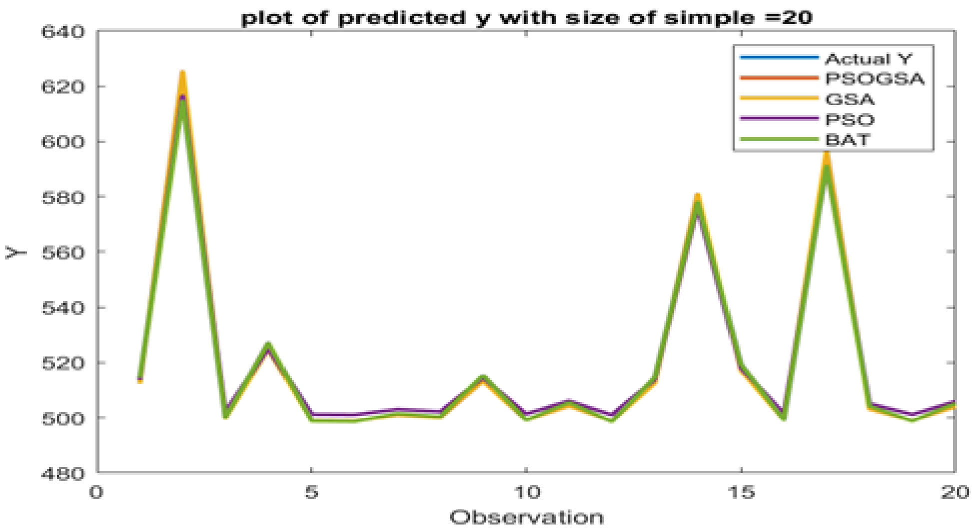

Figure 7: Comparative PSOGSA,BAT,PSO and GSA algorithms for the Modle one when of sample size=20

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