



# A New Approach for Finding an Optimal Solution for Grey Transportation Problem

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## Abstract

In ordinary transportation problems, it is always supposed that the mileage from every source to every destination is a definite number. But in special conditions, such as transporting emergency materials when natural calamity occurs or transporting military supplies during wartime, carrying network may be destroyed, mileage from some sources to some destinations are no longer definite. It is uncertain, a grey number. In these conditions, transportation capacity is often poor; the problems of optimization become even more important. In this paper, we proposed a new method to find an optimal solution for grey transportation problems where transportation cost, supply and demand are interval grey numbers. Our method uses the concepts of center and width of grey numbers. One of the advantages of the proposed method compared to other methods that use grey number whitening is that the uncertainty in the input data is taken into account at the output of the method and it consists of five simple steps. The solution procedure is illustrated with a numerical example. Also, the new method can be served as an important tool for decision-makers when they are handling various types of logistic problems having uncertainty parameters such as grey numbers. Further, the proposed method is extended to fuzzy grey transportation problems.

*Keywords:* Center and Width, Grey number, Transportation, Zero point method, Uncertainty.

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## 1. Introduction

The basic transportation problem was originally initiated by Hitchcock [1] and later on by Kantorovich [2], who has given an algorithm for obtaining the solution of the transportation problem. He developed a procedure to solve the transportation problem, which has a close connection with the simplex method for solving TP as the primal simplex transportation method. A large number of

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research works on transportation problems has been done by several researches such as [3, 4, 5] solved a transportation problem with market choice. A study on time-dependent fuzzy random location-scheduling programming for hazardous materials has been conducted Meiyi et al. [6]. Maity and Roy [7] presented a solution procedure for multi-objective transportation problem using a utility function approach. Reisach et al. [8] employed a study on special areas of operational research problems. Transportation models play an important role in logistics and supply-chain management for reducing cost and improving service [9]. In today's competitive global economy, transportation models are powerful tools for finding better ways to meet customer needs. Transportation models ensure the efficient movement and timely availability of raw materials and finished goods [9]. The origin of a transportation problem is the location from which shipments are dispatched. The destination of a transportation problem is the location to which shipments are transported. The unit transportation cost is the cost of transporting one unit of the consignment from an origin to a destination. In the typical problem, a product is transported from  $m$  sources to  $n$  destinations and their supply  $(a_1, a_2, \dots, a_m)$  and demand  $(b_1, b_2, \dots, b_n)$  are respectively. The coefficient  $c_{ij}$  of the objective function could represent the transportation cost, delivery time, number of goods transported, unfulfilled supply and demand and others, are provided with transporting a unit of product from sources  $i$  to destinations  $j$  [10]. The parameters of the normal transport problem are exact values. But in real life, due to lack of information, the parameters of the transportation problem cannot be accurately determined and must be estimated [10]. The imprecision may follow from the inexact environment or may be a consequence of certain flexibility for the given enterprise associated with the cost of objective function. A frequently used means to express the imprecision information generalizes the concepts of multiple states of affairs and the concepts of the inexact parameters [10]. In decision-making theory, uncertain decision-making is a vital branch. There are various approaches to dealing with uncertainty problems [3]. To deal with uncertainties, a number of methods were developed, including interval, fuzzy, Rough and stochastic numbers. The probability and fuzzy approaches are frequently used to describe the uncertain element and treat imprecise present in a decision variable. Maity and Roy [11] studied on a multi-objective transportation problem with nonlinear cost and multi-choice demand. Annie Christi and Kalpana [12] obtained compromise solutions of multi-objective fuzzy transportation problems to solve with non-linear membership functions. Yeola and Jahav [13] have proposed a method to solve multi-objective transportation problem using fuzzy programming technique is used with fuzzy linear membership function for different cost to solve multi-objective transportation problem. Many researchers [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] have proposed different techniques for solving transportation problem in uncertain environment. To deal with such situations, Deng[25] first introduced the concept of grey systems theory. Grey systems theory is another method to deal with uncertain decision-making problems. The concept of grey Numbers emerged as an effective model for systems with partial information only known. Palanci et al. [26] studied on uncertainty under grey goals in a cooperative game. Nasserri et al. [27] proposed a direct approach for solving the grey assignment problem based on grey arithmetic. By combining the grey systems theory with the principle and method of transportation problem, the grey transportation problem is established based on the grey systems theory. Grey transportation does not require distribution information for its parameters since grey numbers are acceptable for the uncertain inputs [28]. Sometimes the values of the transportation parameters are known to us probabilistically and lie between bounds. This is motivated us to consider all the transportation parameters are interval grey number. In general, the existing techniques provide only crisp solutions for the grey transportation problem. In this paper, we propose a new method, for the grey optimal solution of grey transportation problems without converting them to classical transportation problems and the results obtained are discussed. The proposed method is illustrated with the help of numerical example.

The current paper is in 6 sections. In Section 2, basic knowledge about the grey systems theory and interval grey numbers which are needed in the next sections, we explained. The formulation of the transportation models with the grey number was presented in Section 3. A new proposed algorithm to solve the grey transportation problem based on the center and width of grey numbers and the harmonic mean approach is proposed in Section 4. In Section 5, a numerical example is solved by the proposed procedure. Finally, Section 6 consists of conclusions.

## 2. Preliminaries

In this section, we consider the definitions and concepts needed for the study and analysis of grey system are considered.

### 2.1. Grey System Theory

Grey system is defined as a system that involves non-deterministic information. If the clear information of a system is shown with white color and unknown information is presented as black, then, the information related to the most natural systems is not white (completely known) or black (completely unknown), but a combination of both which means grey. We need the following definitions of the basic arithmetic operators and partial ordering on grey intervals numbers which can be found in [29]. A grey number means a number that is not exactly known. An interval grey number can be treated as continuous and discrete. It may be a discrete number within a certain range or any number within a range of lower and upper limits. In the other words, a number, whose real value cannot be determined with certainty, but whose open interval where this number is located is known, is called a grey number [30].

**Definition 2.1.** Interval grey number is the number with both lower limit  $\underline{x}$  and upper limit  $\bar{x}$ :  
 $\otimes x \in [\underline{x}, \bar{x}]$

**Definition 2.2.** Center ( $\otimes \hat{x}$ ) and width ( $\otimes \acute{x}$ ) of grey number  $\otimes x \in [\underline{x}, \bar{x}]$  are defined as follows:

$$\otimes \hat{x} = \frac{\underline{x} + \bar{x}}{2}, \quad \otimes \acute{x} = \frac{\bar{x} - \underline{x}}{2} \tag{2.1}$$

**Remark 2.3.** For the grey number  $\otimes x \in [\underline{x}, \bar{x}]$ , we have:

$$\underline{x} = \otimes \hat{x} - \otimes \acute{x}, \tag{2.2}$$

**Lemma 2.4.** If  $k \in R - 0$ ,  $\otimes x \in [\underline{x}, \bar{x}]$  be a grey number, then we have:

$$k \cdot (\otimes x) = \otimes(kx) \in [k\underline{x}, k\bar{x}] \quad \text{if} \quad k > 0 \tag{2.3}$$

$$k \cdot (\otimes x) = \otimes(kx) \in [k\bar{x}, k\underline{x}] \quad \text{if} \quad k < 0 \tag{2.4}$$

**Definition 2.5.** [31] Let  $\otimes x_1 \in [\underline{x}_1, \bar{x}_1]$  and  $\otimes x_2 \in [\underline{x}_2, \bar{x}_2]$  be two grey numbers. The following operations can be defined:

$$\begin{aligned} \otimes x_1 + \otimes x_2 &= [\underline{x}_1 + \underline{x}_2, \bar{x}_1 + \bar{x}_2] \\ \otimes x_1 - \otimes x_2 &= \otimes x_1 + (- \otimes x_2) = [\underline{x}_1 - \bar{x}_2, \bar{x}_1 - \underline{x}_2] \\ \otimes x_1 \times \otimes x_2 &= \left[ \min \{ \underline{x}_1 \underline{x}_2, \bar{x}_1 \bar{x}_2, \bar{x}_1 \underline{x}_2, \underline{x}_1 \bar{x}_2 \}, \max \{ \underline{x}_1 \underline{x}_2, \bar{x}_1 \bar{x}_2, \bar{x}_1 \underline{x}_2, \underline{x}_1 \bar{x}_2 \} \right] \\ \frac{\otimes x_1}{\otimes x_2} &= \otimes x_1 \times \otimes x_2^{-1} = \left[ \min \left\{ \frac{\underline{x}_1}{\underline{x}_2}, \frac{\underline{x}_1}{\bar{x}_2}, \frac{\bar{x}_1}{\underline{x}_2}, \frac{\bar{x}_1}{\bar{x}_2} \right\}, \max \left\{ \frac{\underline{x}_1}{\underline{x}_2}, \frac{\underline{x}_1}{\bar{x}_2}, \frac{\bar{x}_1}{\underline{x}_2}, \frac{\bar{x}_1}{\bar{x}_2} \right\} \right], 0 \notin [\underline{x}_2, \bar{x}_2] \\ k \cdot \otimes x &\in \begin{cases} [k\underline{x}, k\bar{x}] & k > 0 \\ [k\bar{x}, k\underline{x}] & k < 0. \end{cases} \end{aligned}$$

2.2. Ranking Grey Numbers by Kernel and Greyness

Ranking of grey numbers plays a very important role in decision-making and grey systems theory applications. Darvishi et al. [32] have referred to more detail in comparing grey numbers.

**Definition 2.6.** Suppose that the background, which makes a grey number  $\otimes x$  come into being, is  $\Omega$  and  $\mu(\Omega)$  is the value of  $\Omega$ , Then  $g^\circ(\otimes x) = \frac{\mu(\otimes x)}{\mu(\Omega)}$  is called the degree of greyness of  $\otimes x$  (denoted as  $g^\circ$  for short).

**Definition 2.7.** [33] Suppose  $\otimes x_1$  and  $\otimes x_2$  are two grey numbers and  $\otimes \hat{x}_1, \otimes \hat{x}_2$  are the center of  $\otimes x_1$  and  $\otimes x_2$  respectively,  $g^\circ(\otimes x_1)$  and  $g^\circ(\otimes x_2)$  are the degree of greyness of  $\otimes x_1$  and  $\otimes x_2$ , respectively.

So, if  $\otimes \hat{x}_1 < \otimes \hat{x}_2 \implies \otimes x_1 <_G \otimes x_2$

$$\text{if } \otimes \hat{x}_1 = \otimes \hat{x}_2 \implies \begin{cases} \text{if } g^\circ(\otimes x_1) = g^\circ(\otimes x_2) \implies \otimes x_1 =_G \otimes x_2 \\ \text{if } g^\circ(\otimes x_1) < g^\circ(\otimes x_2) \implies \otimes x_1 >_G \otimes x_2 \\ \text{if } g^\circ(\otimes x_1) > g^\circ(\otimes x_2) \implies \otimes x_1 <_G \otimes x_2. \end{cases} \quad (2.5)$$

For further study on the grey theory systems see [34].

3. Grey Transportation Problem

The transportation problem is one of the earliest applications of linear programming problems. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly [9]. Generally, a transportation problem is a type of decision making problem in which the main objective is to minimize the transportation cost and it is defined subsequently:

$$\begin{aligned} \text{Minimizez} &= \sum_{j=1}^n \sum_{i=1}^m c_{ij} x_{ij} \\ \text{subject to} & \sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m \\ & \sum_{i=1}^m x_{ij} \leq b_j, j = 1, 2, \dots, n \\ & x_{ij} \geq 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \end{aligned} \quad (3.1)$$

Where  $x_{ij}(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$  is the decision variable and  $c_{ij}(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$  is the transportation cost per unit commodity from the  $i$ th origin to the  $j$ th destination. Here,  $a_i(i = 1, 2, \dots, m)$  and  $b_j(j = 1, 2, \dots, n)$  are availability and demand at the  $i$ th origin and the  $j$ th destination, respectively, and the feasibility condition is  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

In order to reduce information costs and also to construct a real model, the use of uncertain parameter transportation problems is more appropriate. Impreciseness in the parameters means that the information for these parameters is not complete. However, even with incomplete information, the model used is normally able to give a grey value for the parameters. Therefore, the use of grey transportation problems is more appropriate to model and solve the real world problems [9]. Grey transportation problem is a generalization of the transportation problem in which input data are

expressed as interval grey numbers instead of fixed values. This problem can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as interval grey numbers. The grey transportation problem is represented in Table1.

Table 1: Matrix of the grey transportation problem

sources \ destinations	1	2	...	$j$	...	$n$	Supply
1	$\otimes c_{11}$	$\otimes c_{12}$	...	$\otimes c_{1j}$	...	$\otimes c_{1n}$	$\otimes a_1$
2	$\otimes c_{21}$	$\otimes c_{22}$	...	$\otimes c_{2j}$	...	$\otimes c_{2n}$	$\otimes a_2$
...	...	...	...	...	...	...	...
$i$	$\otimes c_{i1}$	$\otimes c_{i2}$	...	$\otimes c_{ij}$	...	$\otimes c_{in}$	$\otimes a_i$
...	...	...	...	...	...	...	...
$m$	$\otimes c_{m1}$	$\otimes c_{m2}$	...	$\otimes c_{mj}$	...	$\otimes c_{mn}$	$\otimes a_m$
Demand	$\otimes b_1$	$\otimes b_2$	...	$\otimes b_j$	...	$\otimes b_n$	

The mathematical model of the grey transportation problem is presented as follows:

$$\begin{aligned}
 &\text{Minimize } \otimes Z =_G \sum_{j=1}^n \sum_{i=1}^m \otimes c_{ij} \otimes x_{ij} \\
 &\text{subject to} \quad \sum_{j=1}^n \otimes x_{ij} =_G \otimes a_i, i = 1, 2, \dots, m \\
 &\quad \quad \quad \sum_{i=1}^m \otimes x_{ij} =_G \otimes b_j, j = 1, 2, \dots, n \\
 &\quad \quad \quad \otimes x_{ij} \geq_G \otimes 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n
 \end{aligned} \tag{3.2}$$

where  $\otimes c_{ij}$ ,  $a_i$  and  $b_j$  are grey interval numbers for all  $i$  and  $j$ . We assume that  $\otimes c_{ij}$  are the cost coefficients in the objective function,  $a_i (i = 1, 2, \dots, m)$  and  $b_j (j = 1, 2, \dots, n)$  may be random variables in transportation problem. Here  $\otimes Z$  represents the minimum value of objective function and it is assumed that  $a_i > 0$ ,  $b_j > 0$ , and  $\otimes c_{ij} > 0$ .

Therefore:

$$\begin{aligned}
 &\text{Minimize } \otimes Z =_G \sum_{j=1}^n \sum_{i=1}^m \otimes [c_{ij}, \bar{c}_{ij}] \times \otimes [x_{ij}, \bar{x}_{ij}] \\
 &\text{subject to} \quad \sum_{j=1}^n \otimes [x_{ij}, \bar{x}_{ij}] =_G \otimes [a_i, \bar{a}_i], i = 1, 2, \dots, m \\
 &\quad \quad \quad \sum_{i=1}^m \otimes [x_{ij}, \bar{x}_{ij}] =_G \otimes [b_j, \bar{b}_j], j = 1, 2, \dots, n \\
 &\quad \quad \quad \underline{x}_{ij} > 0, \bar{x}_{ij} > 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n
 \end{aligned} \tag{3.3}$$

where  $\underline{c}_{ij}$  and  $\bar{c}_{ij}$  are positive real numbers for all  $i$  and  $j$ ,  $\underline{a}_i$  and  $\bar{a}_i$  are positive real numbers for all  $i$ , and  $\underline{b}_j$  and  $\bar{b}_j$  are positive real numbers for all  $j$ .

**Definition 3.1.** The  $[\underline{x}_{ij}, \bar{x}_{ij}]$ , for all  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  is said to be a feasible solution of grey transportation problems if they satisfy the equations (3.3).

**Definition 3.2.** A feasible solution  $[\underline{x}_{ij}, \bar{x}_{ij}]$ , for all  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ , for the grey transportation problems is said to be an optimal solution of grey transportation problems if

$$\sum_{j=1}^n \sum_{i=1}^m [\underline{c}_{ij}, \bar{c}_{ij}] \times [\underline{x}_{ij}, \bar{x}_{ij}] \leq \sum_{j=1}^n \sum_{i=1}^m [\underline{c}_{ij}, \bar{c}_{ij}] \times [\underline{y}_{ij}, \bar{y}_{ij}] \quad (3.4)$$

for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  and for all feasible solution  $[\underline{y}_{ij}, \bar{y}_{ij}]$ , for all  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

**Definition 3.3.** A grey feasible solution to a grey transportation problem with  $m$  sources and  $n$  destinations is said to be a grey basic feasible solution if the number of positive allocations are  $(m + n - 1)$ .

If the number allocations in a grey basic solution is less than  $(m + n - 1)$ , it is called degenerate grey basic feasible solution.

**Definition 3.4.** A grey feasible solution is said to be a grey optimal solution if it minimizes the total grey transportation cost.

## 4. Proposed Separation Method

### 4.1. Central Transportation Problem

Consider the mathematical model of the grey transportation problem 3.1.

If we determine the grey parameters center of the grey transportation problem, we have:

$$\otimes \hat{Z} = \frac{\underline{z} + \bar{z}}{2}, \quad \otimes \hat{c}_{ij} = \frac{\underline{c}_{ij} + \bar{c}_{ij}}{2}, \quad \otimes \hat{x}_{ij} = \frac{\underline{x}_{ij} + \bar{x}_{ij}}{2}, \quad \otimes \hat{a}_i = \frac{\underline{a}_i + \bar{a}_i}{2}, \quad \otimes \hat{b}_j = \frac{\underline{b}_j + \bar{b}_j}{2} \quad (4.1)$$

then, we can present the mathematical model of the central transport problem as follows,

$$\begin{aligned} \text{Minimize } \otimes \hat{Z} &= \sum_{j=1}^n \sum_{i=1}^m \otimes \hat{c}_{ij} \times \otimes \hat{x}_{ij} \\ \text{subject to } & \sum_{j=1}^n \otimes \hat{x}_{ij} = \otimes \hat{a}_i, \quad i = 1, 2, \dots, m \\ & \sum_{i=1}^m \otimes \hat{x}_{ij} = \otimes \hat{b}_j, \quad j = 1, 2, \dots, n \\ & \otimes \hat{x}_{ij} \geq 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \end{aligned} \quad (4.2)$$

4.2. *Width Transportation Problem*

According to model 3.1, if we determine the grey parameters width of the grey transportation problem, we have:

$$\otimes Z' = \frac{z + \bar{z}}{2}, \quad \otimes c'_{ij} = \frac{c_{ij} + \bar{c}_{ij}}{2}, \quad \otimes x'_{ij} = \frac{x_{ij} + \bar{x}_{ij}}{2}, \quad \otimes a'_i = \frac{a_i + \bar{a}_i}{2}, \quad \otimes b'_{ij} = \frac{b_{ij} + \bar{b}_{ij}}{2} \quad (4.3)$$

then, we can present the mathematical model of the central transport problem as follows.

$$\begin{aligned} \text{Minimize } \otimes Z' &= \sum_{j=1}^n \sum_{i=1}^m \otimes c'_{ij} \times \otimes x'_{ij} \\ \text{subject to } &\sum_{j=1}^n \otimes x'_{ij} = \otimes a'_i, \quad i = 1, 2, \dots, m \\ &\sum_{i=1}^m \otimes x'_{ij} = \otimes b'_j, \quad j = 1, 2, \dots, n \\ &\otimes x'_{ij} \geq 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \end{aligned} \quad (4.4)$$

Now, we introduce a new algorithm namely, the separation method using the center and width of grey parameters for finding an optimal solution for a grey transportation problem.

4.3. *Proposed Algorithm (Separation method)*

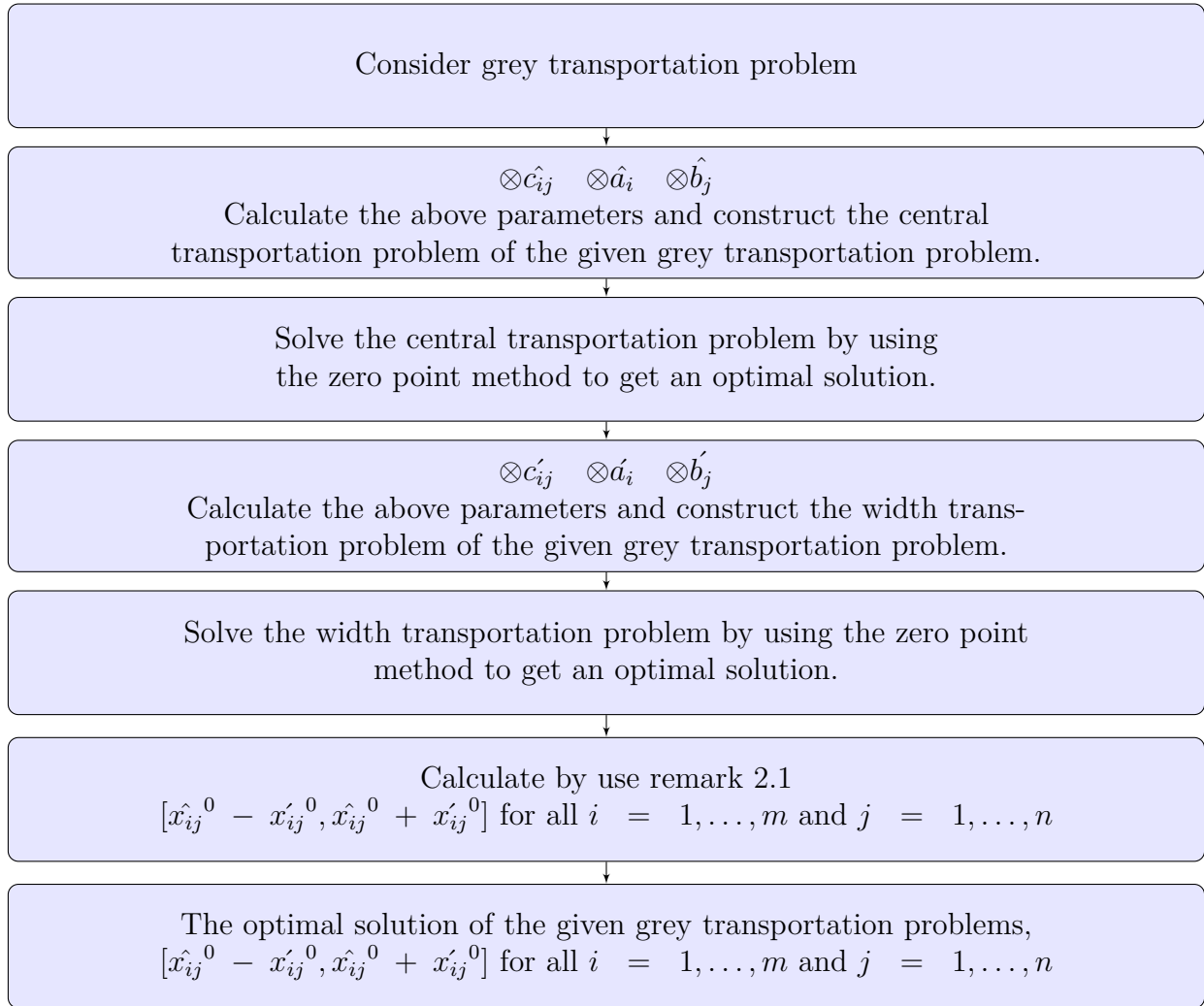
**Step1:** Construct the central transportation problem of the given grey transportation problem.

**Step2:** Solve the central transportation problem by using the zero point method to get optimal solution. Let  $\{\widehat{x}_{ij}^{\circ}\}$  for all  $i$  and  $j$  be an optimal solution for the central transportation problem.

**Step3:** Construct the width transportation problem of the given grey transportation problem.

**Step4:** Solve the width transportation problem by using the zero point method to get optimal solution. Let  $\{x'_{ij}\}$  for all  $i$  and  $j$  be an optimal solution of the width transportation problem.

**Step5:** The optimal solution of the given grey transportation problem, by use remark 2.3, is,  $[\widehat{x}_{ij}^{\circ} - x'_{ij}, \widehat{x}_{ij}^{\circ} + x'_{ij}]$  for all  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, n$ .



Now, we prove the following theorem which finds a relation between optimal solutions of a grey transportation problem and a pair of corresponding transportation problems and also, is used in the proposed method.

**Theorem 4.1.** *If the set  $\{\otimes\hat{x}_{ij}^0 \mid \text{for all } i=1,2,\dots,n \text{ and } j=1,2,\dots,m\}$  is an optimal solution of the center transportation problem of grey transportation problem where*

$$\begin{aligned}
 & \text{Minimize } \otimes \hat{Z} = \sum_{j=1}^n \sum_{i=1}^m \otimes \hat{c}_{ij} \times \otimes \hat{x}_{ij} \\
 & \text{subject to} \quad \sum_{j=1}^n \otimes \hat{x}_{ij} = \otimes \hat{a}_i, \quad i = 1, 2, \dots, m \\
 & \quad \quad \quad \sum_{j=1}^n \otimes \hat{x}_{ij} = \otimes \hat{b}_j, \quad j = 1, 2, \dots, n \\
 & \quad \quad \quad \otimes \hat{x}_{ij} \geq 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n
 \end{aligned} \tag{4.5}$$

and the set  $\{\otimes x'_{ij}{}^0 \mid \text{for all } i=1,2,\dots,n \text{ and } j=1,2,\dots,m\}$  is an optimal solution of the width transporta-



tion problem of grey transportation problem where

$$\begin{aligned}
 \text{Minimize } \otimes Z' &= \sum_{j=1}^n \sum_{i=1}^m \otimes c'_{ij} \times \otimes x'_{ij} \\
 \text{subject to} \quad &\sum_{j=1}^n \otimes x'_{ij} = \otimes a'_i, i = 1, 2, \dots, m \\
 &\sum_{i=1}^n \otimes x'_{ij} = \otimes b'_j, j = 1, 2, \dots, n \\
 &\otimes x'_{ij} \geq 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n.
 \end{aligned} \tag{4.6}$$

then, the set  $\{[\hat{x}_{ij}^{\circ} - x'_{ij}, \hat{x}_{ij}^{\circ} + x'_{ij}]\}$ , for all  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$  is an optimal solution of the grey transportation problem.

**Proof .** Let  $\{[\underline{x}_{ij}, \bar{x}_{ij}]\}$ , for all  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$  be a feasible solution of the grey transportation problems. Convert the grey transportation problem to the interval transportation problem.

$$\begin{aligned}
 \text{Minimize}[Z_1, Z_2] &= \sum_{j=1}^n \sum_{i=1}^m [c_{ij}, \bar{c}_{ij}] \times [\underline{x}_{ij}, \bar{x}_{ij}] \\
 \text{subject to} \quad &\sum_{j=1}^n [\underline{x}_{ij}, \bar{x}_{ij}] = [a_i, \bar{a}_i], i = 1, 2, \dots, m \\
 &\sum_{i=1}^n [\underline{x}_{ij}, \bar{x}_{ij}] = [b_j, \bar{b}_j], j = 1, 2, \dots, n \\
 &\underline{x}_{ij} > 0, \bar{x}_{ij} > 0, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n
 \end{aligned} \tag{4.7}$$

Set  $\{[\hat{x}_{ij}^{\circ} + x'_{ij}]\}$ , for all  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$  is an optimal solution of the upper bound transportation problem. And set  $\{[\hat{x}_{ij}^{\circ} - x'_{ij}]\}$ , for all  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$  is an optimal solution of the lower bound transportation problem.

$$\begin{aligned}
 \sum_{j=1}^n \sum_{i=1}^m c_{ij}(\hat{x}_{ij}^{\circ} - x'_{ij}) &\leq \sum_{j=1}^n \sum_{i=1}^m c_{ij} \underline{x}_{ij} \\
 , \\
 \sum_{j=1}^n \sum_{i=1}^m \bar{c}_{ij}(\hat{x}_{ij}^{\circ} + x'_{ij}) &\leq \sum_{j=1}^n \sum_{i=1}^m \bar{c}_{ij} \bar{x}_{ij}
 \end{aligned} \tag{4.8}$$

and

$$(\hat{x}_{ij}^{\circ} - x'_{ij}) \leq (\hat{x}_{ij}^{\circ} + x'_{ij}) \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m \tag{4.9}$$

This implies that,

$$\left[ \sum_{j=1}^n \sum_{i=1}^m c_{ij}(\hat{x}_{ij}^{\circ} - x'_{ij}), \sum_{j=1}^n \sum_{i=1}^m \bar{c}_{ij}(\hat{x}_{ij}^{\circ} + x'_{ij}) \right] \leq \left[ \sum_{j=1}^n \sum_{i=1}^m c_{ij} \underline{x}_{ij}, \sum_{j=1}^n \sum_{i=1}^m \bar{c}_{ij} \bar{x}_{ij} \right] \tag{4.10}$$

That is

$$\sum_{j=1}^n \sum_{i=1}^m [\underline{c}_{ij}, \bar{c}_{ij}] [\hat{x}_{ij}^{\circ} - x'_{ij}, \hat{x}_{ij}^{\circ} + x'_{ij}] \leq \sum_{j=1}^n \sum_{i=1}^m [\underline{c}_{ij}, \bar{c}_{ij}] [\underline{x}_{ij}, \bar{x}_{ij}] \quad (4.11)$$

Therefore, the set  $\{\hat{x}_{ij}^{\circ} - x'_{ij}, \hat{x}_{ij}^{\circ} + x'_{ij}\}$ , for all  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$  is an optimal solution of the grey transportation problem  $\square$

## 5. Numerical Example

In this section, for an illustration of the above approach, a numerical example of the grey transportation problem will be solved based on the proposed method.

**Example 5.1.** *A transportation company wants to deliver the basic food needs of people in four cities trapped in floods from three neighboring cities. Shipping costs per unit of commodity, the amount of food supply and demand for each city cannot be precisely determined, but given the flood damage, their value is presented in gray numbers in Table2 Solve the problem so that while meeting the needs of cities, transportation costs are minimized.*

Table 2: Matrix of the grey transportation problem

		sources				Supply
		1	2	3	4	
destinations	1	$\otimes[1, 3]$	$\otimes[2, 4]$	$\otimes[4, 14]$	$\otimes[4, 12]$	$\otimes[2, 16]$
	2	$\otimes[1, 3]$	$\otimes[3, 17]$	$\otimes[4, 8]$	$\otimes[2, 8]$	$\otimes[4, 38]$
	3	$\otimes[2, 16]$	$\otimes[4, 18]$	$\otimes[2, 8]$	$\otimes[2, 12]$	$\otimes[2, 34]$
Demand		$\otimes[2, 22]$	$\otimes[2, 6]$	$\otimes[2, 28]$	$\otimes[2, 32]$	$\otimes[8, 88]$

**Solution:** *Let us formulate the example, as follows:*

$$\begin{aligned} \text{Minimize } \otimes Z = & \otimes_G \otimes[1, 3] \times \otimes x_{11} + \otimes[2, 4] \times \otimes x_{12} + \otimes[4, 14] \times \otimes x_{13} + \otimes[4, 12] \times \otimes x_{14} \\ & + \otimes[1, 3] \times \otimes x_{21} + \otimes[3, 17] \times \otimes x_{22} + \otimes[4, 8] \times \otimes x_{23} + \otimes[2, 8] \times \otimes x_{24} \\ & + \otimes[2, 16] \times \otimes x_{31} + \otimes[4, 18] \times \otimes x_{32} + \otimes[2, 8] \times \otimes x_{33} + \otimes[2, 12] \times \otimes x_{34} \\ \text{subject to } & \sum_{j=1}^4 \otimes x_{1j} =_G \otimes[2, 16] \\ & \sum_{j=1}^4 \otimes x_{2j} =_G \otimes[4, 38] \\ & \sum_{j=1}^4 \otimes x_{3j} =_G \otimes[2, 34] \\ & \sum_{i=1}^3 \otimes x_{i1} =_G \otimes[2, 22] \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^3 \otimes x_{i2} &= {}_G \otimes [2, 6] \\ \sum_{i=1}^3 \otimes x_{i3} &= {}_G \otimes [2, 28] \\ \sum_{i=1}^3 \otimes x_{i4} &= {}_G \otimes [2, 32] \\ \otimes \hat{x}_{ij} &\geq 0, \quad i = 1, 2, 3; j = 1, 2, 3, 4 \end{aligned}$$

Now, the central transportation problem of the given problem is given in table3.

Table 3: Matrix of the central transportation problem

destinations \ sources	sources				Supply
	1	2	3	4	
1	2	3	9	8	9
2	2	10	6	5	21
3	9	11	5	7	18
Demand	12	4	15	17	48

By using the zero point method, the optimal solution to the central transportation problem is

$$\hat{x}_{11}^{\circ} = 5, \hat{x}_{12}^{\circ} = 4, \hat{x}_{21}^{\circ} = 7, \hat{x}_{24}^{\circ} = 14, \hat{x}_{33}^{\circ} = 15, \hat{x}_{34}^{\circ} = 3$$

Now, the width transportation problem of the given problem is given in table4.

Table 4: Matrix of the width transportation problem

destinations \ sources	sources				Supply
	1	2	3	4	
1	1	1	5	4	7
2	1	7	2	3	17
3	7	7	3	5	16
Demand	10	2	13	15	40

Using the zero point method, the optimal solution to the width transportation problem is

$$x'_{11}{}^{\circ} = 5, x'_{12}{}^{\circ} = 2, x'_{21}{}^{\circ} = 5, x'_{24}{}^{\circ} = 12, x'_{33}{}^{\circ} = 13, x'_{34}{}^{\circ} = 3$$

Thus, an optimal solution to the given grey transportation problem, by Remark2.3 is;

$$\begin{aligned} \otimes x_{11} &\in \otimes [0, 10], \quad \otimes x_{12} \in \otimes [2, 6], \quad \otimes x_{21} \in \otimes [2, 12], \\ \otimes x_{24} &\in \otimes [2, 26], \quad \otimes x_{33} \in \otimes [2, 28], \quad \otimes x_{34} \in \otimes [0, 6] \end{aligned}$$

Also, the minimum transportation cost is,

$$\begin{aligned} \otimes Z &= {}_G \otimes [1, 3] \times \otimes [0, 10] + \otimes [2, 4] \times \otimes [2, 6] + \otimes [1, 3] \times \otimes [2, 12] \\ &+ \otimes [2, 8] \times \otimes [2, 26] + \otimes [2, 8] \times \otimes [2, 28] + \otimes [2, 12] \times \otimes [0, 6] \\ &= {}_G \otimes [14, 594] \end{aligned}$$

## 6. Conclusion

Transportation problem has wide practical applications in logistic systems, man power planning, personnel allocation, inventory control, production planning, etc. and aims to find the best way to fulfill the demand of  $n$  demand points using the capacities of  $m$  supply points. Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter and the penalty factors. In real life problems, these conditions may not be satisfied always. In this paper, we formulate and solve transportation problem under grey uncertainty i.e., we consider all the transportation parameters are interval grey numbers. Grey number theory is used to solve the uncertain decision making problem where input data are lies between lower and upper bounds with the probability. The separation method based on the zero point method provides appropriate solution for the grey transportation problem. This method is a systematic procedure, both easy to understand and to apply.

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