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A method for analyzing the problem of determining the maximum common fragments of temporal directed tree, that do not change with time

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Abstract

In this study two actual types of problems are considered and solved: **1)** determining the maximum common connected fragment of the T-tree (T -directed tree) which does not change with time; **2)** determining all non-isomorphic maximum common connected fragments of the T-tree (T -directed tree) which do not change with time. The choice of the primary study of temporal directed trees and trees is justified by the wide range of their practical applications. Effective method for their solution is proposed. Examples of the solution of the problem for temporal trees and temporal directed trees are given. It is shown that the experimental estimates of the computational complexity of the solution for problems of the temporal directed trees and the temporal trees.

Keywords: maximum common fragments, temporal tree, temporal directed tree, methods of solution, graph-dynamics.

1. Introduction

We can say that the study [1] is one of the first in graph-dynamics, that is, in the analysis of digraphs whose structure changes with time (T -digraphs). The problem of analysis includes the study of changing the global and local characteristics of T -digraphs. The set of methods for describing and studying T -digraphs was called graph-dynamics. The most significant problems of graph-dynamics were:

1) the problem of determining the distance, giving an idea of the stability of changes in the structure of the T -digraph (graph trajectories) with respect to small perturbations and the monotony in the sense of this distance of processes in graph-dynamics; **2)** the problem of determining in the T -digraph the fragment that does not change or little change with time. In connection with the

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expansion of practical applicability spheres in the analysis of temporal networks (computer, social, communication, etc.), since 2006 , the studies on graph-dynamics have been actualized [2]-[15]. In ([9], [11], [13],[15]) the following are distinguished as actual applied problems:
1) determining the maximum subnet of the corporate social network (CSN); which does not change with time. 2) analysis of changes in the time of the CSN structures in particular, communication networks of the company's employees; 3) establishing the direction of changing the structure of the network in order to increase the validity of management decisions, etc. However, the field of research on solving new problems specific to graph-dynamics is not disclosed in the modem scientific references. It should be noted that the paper [6] , in which for the first time two methods were proposed, which allows on the basis of complexity models to study the changes in local and global properties of T -digraphs, determine the similarity of T -digraphs and identify trends of change for the properties of T -digraphs. In all examples, the structure in the simplest and effectively analyzed case is adequately displayed by a tree, directed tree and in the most complex by a network or digraph. In this study, two problems are considered and solved:
1) determining the maximum common connected fragment of the T -tree (T -directed tree) which does not change with time; **2)** determining all non-isomorphic maximum common connected fragments of the T -tree (T -directed tree) which do not change with time.
The choice of the primary study of temporal directed trees and trees is justified by the wide range of their practical applications, for example, in the study of the dynamics of changing: **1)** the nomenclature of goods or products [1]; **2)** the administrative structures of the organization, city, state [1]; **3)** tree-like configuration management structures [16]. In addition, it is established that the following problems can be effectively solved: **1)** determining the maximum common fragment of two trees (directed trees); **2)** determining the orbits of the auto-morphisms group of the tree vertices (directed tree vertices); **3)** disassembling the tree (directed tree) into non-isomorphic subtrees; **4)** determining the isomorphic embedding of one tree (directed tree) in another tree (directed tree) [16].

2. Basic Definitions

Directed tree is a connected acyclic digraph. Below we give all definitions for the temporal directed tree.
The triple $\vec{G}=\left\langle V^{(t)}, E^{(t)}, T\right\rangle$ is called the temporal directed tree, where $V^{(t)}$ is the vertex set of $\vec{G}$ at time t with the number of vertices $\left|V^{(t)}\right|=p^{(t)}, T=\{t_1, \ldots, t_i, \ldots, t_n\}$ is the set of natural numbers that define (discrete) time, $E^{(t)}-$ the family of correspondences or mappings $\Gamma_t \in E^{(t)}$ of the set of vertices $V^{(t)}$ into itself at time $t \in T$. By $t\vec{G}$, we denote $\vec{G}$ -directed tree at time t. If we remove one or some of vertices and incident edges (directed edges) in $t\vec{G}$, we obtain its fragment $\vec{f}$.
The directed tree $t_1\vec{G}=\left\langle V^{(t_1)}, E^{(t_1)}, T\right\rangle$ is isomorphic to the directed tree $t_2\vec{G}=\left\langle V^{(t_2)}, E^{(t_2)}, T\right\rangle$ (denoted by $\left. t_1\vec{G} \approx t_2\vec{G}\right)$, if there exists a mapping $\varphi: V^{(t_1)} \rightarrow V^{(t_2)}$,such that,
$\left(\forall v_i, v_j \in V^{(t_1)}\right)\left(\langle v_i, v_j\rangle \in E^{(t_1)} \leftrightarrow\langle\varphi\left(v_i\right), \varphi\left(v_j\right)\rangle \in E^{(t_2)}\right)$, where $\varphi\left(v_i\right), \varphi\left(v_j\right) \in V^{(t_2)}$.
The directed tree $t_1\vec{G}=\left\langle V^{(t_1)}, E^{(t_1)}, T\right\rangle$ is isomorphically embedded in the directed tree $t_2\vec{G}=\left\langle V^{(t_2)}, E^{(t_2)}, T\right\rangle$, If there exists in $t_2\vec{G}$ a fragment $\vec{f} \approx t_1\vec{G}$. For an isomorphic embedding, we denote by $t_1\vec{G}\overrightarrow{\subseteq} t_2\vec{G}$

The set of all isomorphisms $t\vec{G}$ onto itself forms the group $Aut(t\vec{G})$ by multiplication of the permutations φ, we denote the order of the group by $|\operatorname{Aut}(t\vec{G})|$. By the number of canonical isomorphic embedding $\overrightarrow{tG_*}$ in $t\vec{G}$ we will understand the quantity defined as follows:
$w\left(t\vec{G}_*, t\vec{G}\right)=W\left(t\vec{G}_*, t\vec{G}\right) /\left|\operatorname{Aut}\left(t\vec{G}_*\right)\right|$, where $W\left(t\vec{G}_*, t\vec{G}\right)$ is the number of all isomorphic em-

beddings of $t\vec{G}_*$ in $t\vec{G}$.

3. Problems of determining the maximum common fragments of temporal tree or temporal directed tree.

Problem 1.1 . Given the temporal tree $G = \left\langle V^{(t)}, E^{(t)}, T \right\rangle, T = \{t_1, \ldots, t_n\}$.
It is necessary to find the maximum common connected fragment, i.e. find a connected $MCF\,(t_1G, \ldots, t_iG, \ldots, t_nG)$
Problem 1.2. Given the temporal tree $G = \left\langle V^{(t)}, E^{(t)}, T \right\rangle, T = \{t_1, \ldots, t_n\}$.
It is necessary to find all non-isomorphic maximum common connected fragments, i.e. find all non-isomorphic connected $MCF\,(t_1G, \ldots, t_iG, \ldots, t_nG)$.
Problem 2.1. Given the temporal directed tree $\vec{G} = \left\langle V^{(t)}, E^{(t)}, T \right\rangle, T = \{t_1, \ldots, t_n\}$.
It is necessary to find the maximum common connected fragment, i.e. find a connected $\overrightarrow{MCF}\left(t_1\vec{G}, \ldots, t_i\vec{G}, \ldots, t_n\vec{G}\right)$.
Problem 2.2. Given the temporal directed tree $\vec{G} = \left\langle V^{(t)}, E^{(t)}, T \right\rangle, T = \{t_1, \ldots, t_n\}$.
It is necessary to find all non-isomorphic maximum common connected fragments, i.e. find all non-isomorphic connected $\overrightarrow{MCF}\left(t_1\vec{G}, \ldots, t_i\vec{G}, \ldots, t_n\vec{G}\right)$.
Figure 1 shows an example of a T -tree G consisting of all trees with the number of vertices $p = 6$

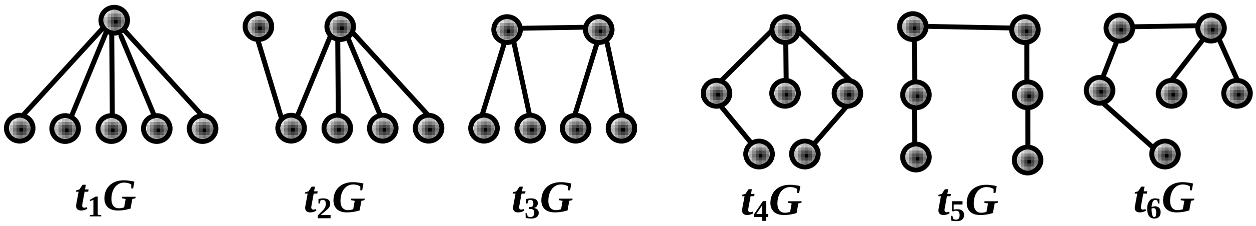


Figure 1: T-tree G consisting of all trees with the number of vertices $p = 6$

The result of solving problems 1.1 and 1.2 for a T -tree G (fig.1) is easy to obtain on the basis of the definition of $MCF\,(t_1G, t_5G)$.

4. The basis of the method for solving the problems of determining the maximum common fragments of temporal tree or temporal directed tree.

The method of the solving includes the following four basic steps: **1)** choosing the smallest order among $t_iG \in G\left(t_i\vec{G} \in \vec{G}\right)$ of tree (directed tree) or $t_1G\left(t_1\vec{G}\right)$ if the orders of all structures in $G(\vec{G})$ are the same ; **2)** the selection of the set of non-automorphic hanging vertices in the chosen $t_iG \in G\left(t_i\vec{G} \in \vec{G}\right)$; **3)** sequential removal (from $t_iG \in G\ (t_i\vec{G} \in \vec{G})$) of vertices from the set of hanging non-automorphic in order to identify a set of connected non-automorphic fragments of maximum order in $t_iG \in G\left(t_i\vec{G} \in \vec{G}\right)$; **4)** for each selected fragment from the constructed set of fragments, determining the possibility of its isomorphic embedding (of all canonical isomorphic embeddings) in each of the remaining structures of the T -tree (T -directed tree).
Calculations were performed based on the author's software package [17].

5. Experimental estimates of the computational complexity of solving the problems of searching the maximum common fragments of temporal tree (temporal directed tree).

Below, an approach is implemented to determine the experimental estimation of the computational complexity ($EECC$) of the method on the average based on the use of tG ($t\vec{G}$) structures with average values of complexity indices in the basis of chains (paths).

The methods of constructive enumeration of all $tG(t\vec{G})$ with a given number of vertices, the generation of medium-complexity $tG(t\vec{G})$ and random $tG(t\vec{G})$ are programmed in the "Graph-models Workshop" [18] , and were used for research.

Figure 2 shows an example of a medium-complexity diagram in the basis of a tree chains with vertex degrees at most four (from the left) and a random tree (from the right) with a number of vertices 100. Figure 3 shows an example of a T -directed tree $\vec{G}$ and its maximum common connected fragment $\overrightarrow{MCF}(\vec{G})$.

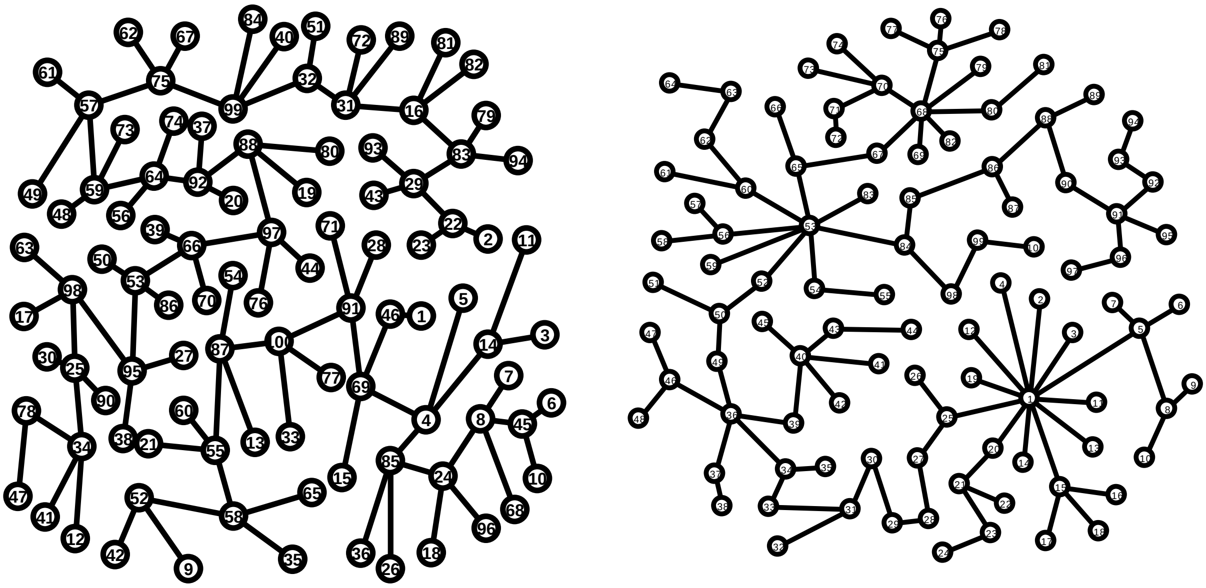


Figure 2: Medium-complexity diagram in the basis of a tree chains with vertex degrees at most four (from the left) and a random tree (from the right) with a number of vertices 100

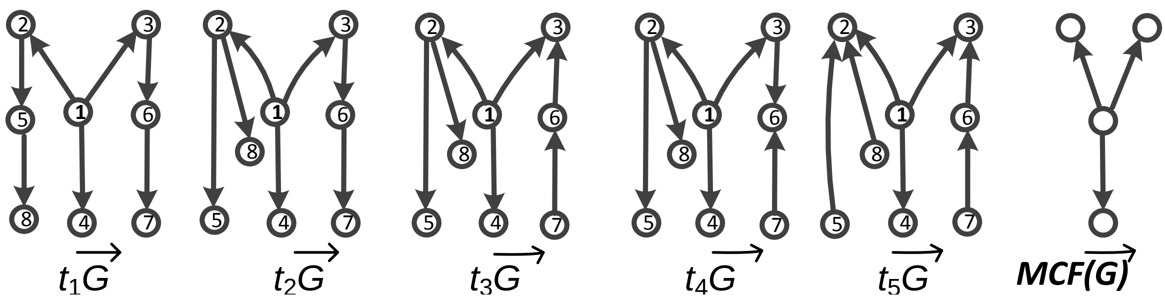


Figure 3: T-directed tree $\vec{G}$ and its maximum common connected fragment $\overrightarrow{MCF}(\vec{G})$

Figure 4 shows an example of the existence of different types of connected $\overrightarrow{MCF}$ for $t_1\vec{G} \in \vec{G}, t_2\vec{G} \in \vec{G}$, where $\vec{G} = \left\langle V^{(t)}, E^{(t)}, T \right\rangle, T = \{t_1, t_2\}$

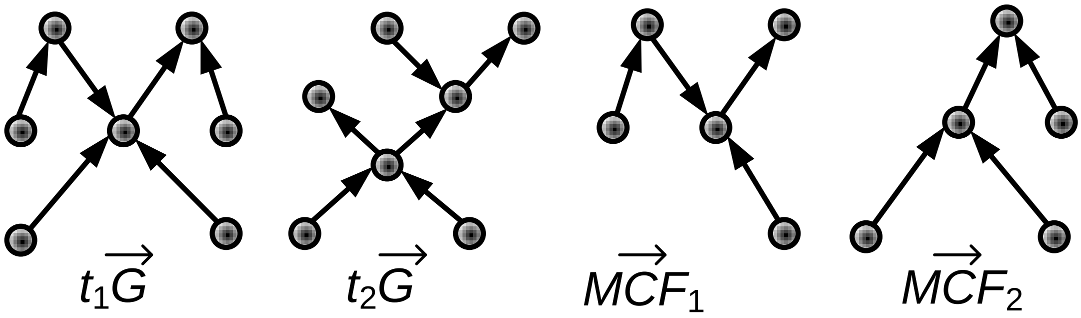
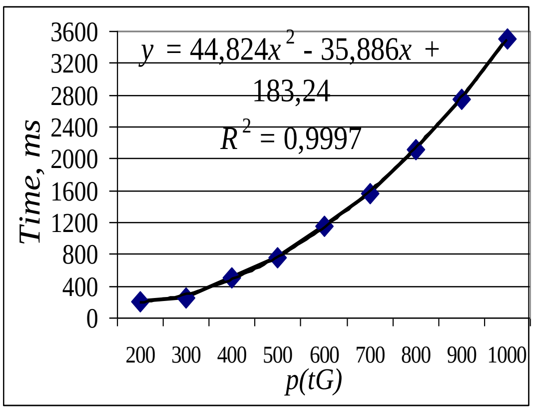


Figure 4: Different types of connected $\overrightarrow{MCF}$ for $t_1\vec{G} \in \vec{G}, t_2\vec{G} \in \vec{G}$

Figure 5 (figure 6) shows a diagram of the computational complexity function of the method for solving the problem 1.1(1.2) for the medium-complexity ($tG \in G$) in the basis of chains and the degree of vertex at most four (random $tG \in G$) with the numbers of vertices from 200 to 1000 (from 50 up to 500). Each T-tree G included 10 non-isomorphic structures $tG \in G$ with the same number of vertices.

Computational experiments have shown that the $EECC$ of the method for solving problem 2.1 of medium -complexity and random $t\vec{G} \in \vec{G}$ has polynomials of second degree and third, but with different coefficients. We note that the time for solving Problem 1.2 essentially depends on the



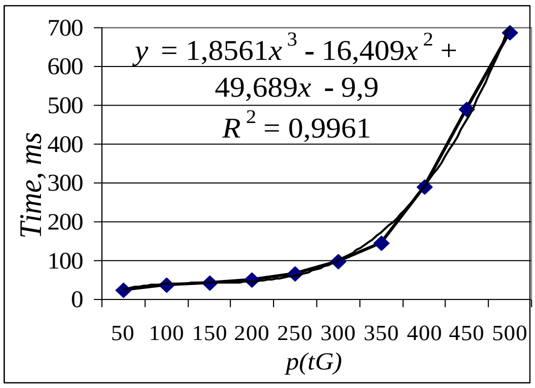


Figure 5: Diagram of the computational complexity function of the method for solving the problem 1.1

Figure 6: Diagram of the computational complexity function of the method for solving the problem 1.2

symmetry of the analyzed trees tG, which are difficult examples when determining all canonical isomorphic embeddings of a fragment in tG.

Increasing classes of automorphic hanging vertices and vertices in each class means increasing the time spent in solving problem 1.2. By NIE we denote the number of isomorphic embeddings of MCF, into one structure tG, and by $NCIE$ - the number of canonical isomorphic embeddings of MCF.

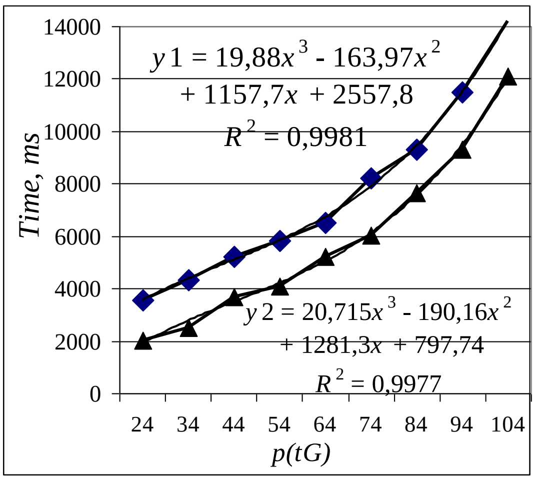
In Table 1, shows the results of solving the problem 1.2(2.2) of medium-complexity structures $tG(t\vec{G})$ of T -trees (T -directed trees) with numbers $p(tG)(p(t\vec{G}))$ from 24 to 104 inclusive, containing for each number of vertices 10 non-isomorphic structures tG $(t\vec{G})$

Table 1: The results of solving the problem 1.2 (2.2)

$p(tG)$	T -trees				T -directed trees			
	$p(MCF)$	NIE	$NCIE$	Time $Ms.$	$p(\overline{MCF})$	NIE	$NCIE$	Time $Ms.$
24	24	4	1	3578	24	4	1	2012
34	34	32	1	4323	34	16	1	2509
44	44	288	1	5210	44	62	1	3678
54	54	12288	1	5812	54	384	1	4099
64	64	6144	1	6500	64	190	1	5201
74	74	2304	1	8211	74	264	1	6012
84	84	3456	1	9308	84	108	1	7655
94	94	73728	1	11499	94	192	1	9324
104	104	73728	1	14201	104	192	1	12109

Figure 7 shows the $EECC$ of method for solving problem 1.2($y1$) and problem 2.2($y2$) using the results given in table 1 .

$EECC$ of method for solving problem 2.2 of random root growing structures $t\vec{G} \in \vec{G}$ with numbers of vertices from 50 to 500 including 10 non-isomorphic $t\vec{G}$ for each number of vertices is shown in figure 8 . Note that the root growing regular structures $t\vec{G}$ are the most laborious for calculations, since they have a high degree of symmetry.



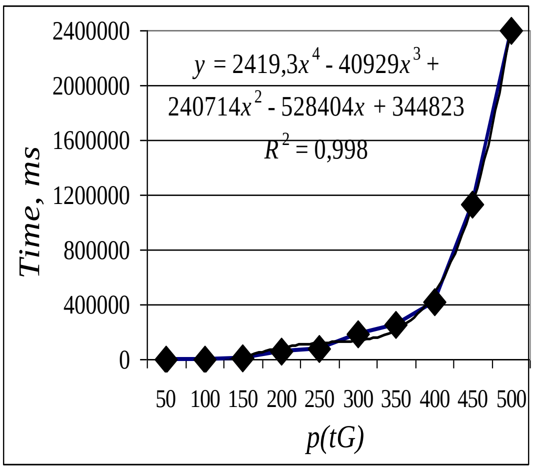


Figure 7: $EECC$ of method for solving problem 1.2(y1) and problem 2.2(y2)

Figure 8: $EECC$ of method for solving problem 2.2 of random root growing structures $t\vec{G} \in \vec{G}$ with numbers of vertices from 50 to 500 including 10 non-isomorphic $t\vec{G}$ for each number of vertices

The analysis of the obtained results shows the high efficiency of the method when solving the problems 1.1, 1.2, 2.1, and 2.2 of medium-complexity and random T -trees and T -directed trees.

6. Conclusion:

Currently, the most relevant area is the development of methods for analyzing graphs with a changeable structure with time (temporal digraphs). The study included two types of graph-dynamics problems related to determining the maximum common connected fragment of temporal directed trees (T-directed trees) (2.1,2.2) and two types of graph-dynamics problems related to determining the maximum common connected fragment of temporal trees (T-trees)$(1.1, 1.2)$, which does not change with time. The solution method included four basic steps related to the global and local characteristics of (T-trees) and T-directed trees. The analysis of the obtained results using Experimental estimates of the computational complexity, shows the high efficiency of the method when solving the problems $1.1, 1.2, 2.1$, and 2.2 of medium-complexity and random T-trees and T-directed trees. In general, it is clear that the study of the characteristics of the graphs enables the researcher to develop the solution methods used to solve these problems and therefore the difference of the solution methods depends mainly on the nature of the characteristics used and this is evident in the time spent on the solution. In future work, it is possible to suggest generalizing the results of solving problems 1.1 and 1.2 for a T -tree G (figure 1) to consider the validity of the following propositions.

Proposition 1 . If a T -tree G with a number of vertices $p > 3$ consists of an N-set of all non-isomorphic tG trees with a number of vertices p, then $MCF\left(t_1G, \ldots, t_iG, \ldots, t_nG\right) \approx P_k$, where P_k- is a chain with length k and $k = 2$

Proposition 2 . If a T -tree G with a number of vertices $p > 3$ consists of an N-set of all non-isomorphic tG trees with a number of vertices p, then the number of all non-isomorphic $MCF\left(t_1G, \ldots, t_iG, \ldots, t_nG\right)$ is equal to 1 .

A similar analysis leads to the validity of the following propositions.

Proposition 3 . If a T-directed tree $\vec{G}$ with a number of vertices $p > 2$ consists of the M-set of all non-isomorphic $t\vec{G}$ directed trees with a number of vertices p, then $\overrightarrow{MCF}\left(t_1\vec{G}, \ldots, t_i\vec{G}, \ldots, t_n\vec{G}\right) \approx \overrightarrow{P_k}$, where $\overrightarrow{P_k}-$ is a path with length k and $k = 1$

Proposition 4 . If a T -directed tree $\vec{G}$ with a number of vertices $p > 2$ consists of the M-set of all non-isomorphic $t\vec{G}$ directed trees with a number of vertices p, then the number of all non-isomorphic $\overrightarrow{MCF}\left(t_1\vec{G}, \ldots, t_i\vec{G}, \ldots, t_n\vec{G}\right)$ is equal to 1 .

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