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On J-class C_0-semigroups of operators

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Abstract

In this paper, locally topologically transitive (or J-class) C_0-semigroups of operators on Banach spaces are studied. Some similarity and differences of locally transitivity and hypercyclicity of C_0-semigroups are investigated. Next the Kato's limit of a sequence of C_0-semigroups are considered and their locally transitivity relations are studied.

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1. Introduction and preliminaries

A continuous linear operator T on a Banach space X is called hypercyclic if it has a hypercyclic vector $x \in X$, i.e. there is a vector $x \in X$ such that $orb(T,x) := \{T^n x : n \in \mathbb{N}_0\}$ is dense in X. In [13], Kitai, and in [9] Gethner and Shapiro gave independently a sufficient condition for hypercyclicity which is useful in applications. Using Baire's category theorem, it can be shown that a bounded linear operator T on a separable Banach space is hypercyclic if and only if it is topologically transitive, i.e. for every two open, non-empty subsets U, V of X there is a natural number n such that $U \cap T^n(V) \neq \emptyset$.

An operator $T \in B(X)$, the space of all bounded linear operators on X, is called a J-class operator, if there exists $0 \neq x \in X$ such that $J_T(x) = X$, where

$$\begin{aligned} J_T(x) : &= \{y \in X : \text{ there exists a strictly increasing sequence} \\ &\quad \text{of natural numbers } (k_n)_n \text{ and a sequence} \\ &\quad (x_n)_n \text{ in } X \text{ such that } x_n \to x \text{ and } T^{k_n}(x_n) \to y\}. \end{aligned}$$

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The vector x is said to be a J-class vector. It is clear that topologically transitive operators are J-class.
Many facts about hypercyclic and J-class operators are investigated by G. Costakis and A. Manoussos in [4] and [3]. For more properties of J-class operators, one can see [14, 15] and [19].
In the continuous case, a one-parameter family $T = \{T(t)\}_{t\geq 0}$ of continuous linear operators on X, is a strongly continuous semigroup (or C_0-semigroup) of operators, if $T(0) = I$, $T(t)T(s) = T(t+s)$, for all $t, s \geq 0$, and $\lim_{t\to 0} T(t)x = x$ for all $x \in X$. The operator $A : D(A) \subseteq X \to X$ defined by $Ax = \lim_{t\to 0} \frac{T(t)x-x}{t}$ is called the generator of the C_0-semigroups T, where $D(A) = \{x \in X : \lim_{t\to 0} \frac{T(t)x-x}{t}$ exists $\}$. For further information about C_0-semigroups we refer the reader to the books [8, 16].

A C_0-semigroup $T = \{T(t)\}_{t\geq 0}$ is said to be hypercyclic if $orb(T, x) := \{T(t)x : t \geq 0\}$ is dense in X for some $x \in X$. Desch, Schappacher and Webb in [6] initiated the investigation of hypercyclic semigroups. So far, several specific examples of hypercyclic strongly continuous semigroups have been studied, see for example [1, 2, 6, 7, 10, 11, 17]. As in the single operator case, the first example of a hypercyclic C_0-semigroup was given by Rolewicz [18], in 1969. J-class C_0-semigroups of operators, also called topologically transitive C_0-semigroups, where else studied by Nasseri in [14].

Definition 1.1. *A C_0-semigroup $\{T(t)\}_{t\geq 0}$ on a normed space X is called J-class if there exists $0 \neq x \in X$ such that $J_T(x) = X$, where*

$$\begin{aligned} J_T(x) : &= \{y \in X : \text{ there exist a strictly increasing sequence} \\ & (t_n)_n \subseteq [0, \infty) \text{ with } t_n \to \infty \text{ and a sequence} \\ & (x_n)_n \text{ in } X \text{ such that } x_n \to x \text{ and } T(t_n)(x_n) \to y\}. \end{aligned}$$

Trivially if there exists $t_0 \geq 0$ such that $T(t_0)$ is J-class, then $\{T(t)\}_{t\geq 0}$ is also a J-class C_0-semigroup. Put

$$A_T := \{x \in X : \ J_T(x) = X\}.$$

By Theorem 4.1.9 [14], A_T and $J_T(x)$ are closed subsets of X.
Using proof similar to the proof of Proposition 4.1.8 of [14], one can see that

$$\begin{aligned} J_T(x) &= \{y \in X : \text{for every neighborhood } U \text{ of } x \text{ and neighborhood} \\ & V \text{ of } y \text{ there exists } t > 0 \text{ such that } T(t)U \cap V \neq \emptyset\}. \end{aligned}$$

Remark 1.2. *i) For a C_0-semigroup $\{T(t)\}_{t\geq 0}$, if $\|T(t)\| \leq 1$ for all $t \geq 0$, i.e. T is contraction C_0-semigroup, then T is not J-class. Indeed if in this case, there exists $x \in X$ such that $J_T(x) = X$, then with $M = \|x\| + 1$ and any $y \in X$, there exist a sequence $(x_n) \subseteq X$ and and a sequence $(t_k) \subseteq \mathbb{R}$ such that $x_k \to x$ and $T(t_k)x_k \to y$. For large enough k we know*

$$\|x_k\| \leq \|x_k - x\| + \|x\| < 1 + \|x\| = M.$$

Thus $\|T(t_k)x_k\| \leq \|x_k\| \leq M$, which implies that $\|y\| \leq M$ and this is a contradiction.
ii) If X is a finite dimensional Banach space then one can prove that there is no J-class C_0-semigroup on X. Indeed this follows from the fact that C_0-semigroups on finite dimensional spaces are of the form $exp(tA)$ with A bounded. If the C_0-semigroup $exp(tA)$ is J-class, then the spectrum $\sigma(A)$ of A has to intersect the unit circle. But in finite dimensional case $\sigma(A) = \sigma_p(A)$, where $\sigma_p(A)$ is the point spectrum of A. This implies that $\sigma_p(A^*)$ intersect the unite circle. This together with [14] Proposition 4.1.12 turn to a contradiction (This part is contributed by A. B. Nasseri).

In this paper, we study properties of J-class C_0-semigroups. In Section 2, some elementary properties of J-class C_0-semigroups are studied. In particular, by some examples, it is proved that many properties of hypercyclic C_0-semigroups are not valid for locally topologically transitive C_0-semigroups. In Section 3, the Kato's limit of C_0-semigroups and their locally topologically transitivity properties are studied.

2. J-class C_0-semigroups of operators

The following characterization of J-class C_0-semigroup will be useful in the rest of the paper.

Theorem 2.1. *For a C_0-semigroup $\{T(t)\}_{t\geq 0}$ on a Banach space X, the following assertions are equivalent:*

i) $\{T(t)\}_{t\geq 0}$ is J-class;
ii) There exists a non-zero $x \in X$ such that for every $y \in X$ and $\varepsilon > 0$, there exist $u \in X$ and $t > 0$ with $\|u - x\| < \varepsilon$ and $\|T(t)u - y\| < \varepsilon$.

Proof . Let $\{T(t)\}_{t\geq 0}$ be J-class. So there exists $0 \neq x \in X$ such that $J_T(x) = X$.
For given $y \in X$ and $\varepsilon > 0$, letting $V = N_\varepsilon(y)$ and $U = N_\varepsilon(x)$ ($N_\varepsilon(y)$ is the neighborhood of y with reduce ε), we may find $t > 0$ such that

$$T(t)U \cap V \neq \emptyset.$$

So there exists $u \in U$ such that $\|T(t)u - y\| < \varepsilon$ and $\|u - x\| < \varepsilon$. Conversely, suppose that (ii) holds for some $x \in X$. We shall show that $J_T(x) = X$.
Let $y \in X$ and U be an arbitrary neighborhood of x. There exists ε_0 such that $N_{\varepsilon_0}(x) \subseteq U$. For every neighborhood V of y there exists ε_1 such that $N_{\varepsilon_1}(y) \subseteq V$. Put $\varepsilon = \min\{\varepsilon_0, \varepsilon_1\}$. By (ii) there exists $u \in N_\varepsilon(x) \subseteq U$ and $t > 0$ such that $T(t)u \in N_\varepsilon(y) \subseteq V$, which implies that $T(t)U \cap V \neq \emptyset$. □

Theorem 2.2. *Let $T = \{T(t)\}_{t\geq 0}$ and $S = \{S(t)\}_{t\geq 0}$ be two C_0-semigroups on Banach spaces X and Y, respectively and $\phi : X \to Y$ be a continuous function with dense range such that $\phi(A_T \setminus \{0\}) \neq \{0\}$ and $S(t) \circ \phi = \phi \circ T(t)$, for all $t \geq 0$. If T is J-class, then so is S.*

Proof . If T is J-class, then by the fact that $\phi(A_T) \neq \{0\}$ we may choose $0 \neq x \in X$ such that $J_T(x) = X$ and $\phi(x) \neq 0$. We claim that $J_S(\phi(x)) = Y$.
Let $z \in ran\ \phi$, then there exists $y \in X = J_T(x)$ such that $\phi(y) = z$.
So there exists $(x_n) \subseteq X$ and a strictly increasing sequence of positive real numbers $(t_n)_n$ such that $t_n \to \infty$, $x_n \to x$ and $T(t_n)x_n \to y$. By continuity of ϕ, $y_n := \phi(x_n) \to \phi(x)$ and $S(t_n) \circ \phi(x_n) = \phi(T(t_n)x_n) \to \phi(y) = z$.
Thus $J_S(\phi(x)) \supseteq ran\ \phi$. But $J_S(\phi(x))$ is closed and $ran\ \phi$ is dense so $J_S(\phi(x)) = Y$. □ The following example shows that the hypothesis $\phi(A_T) \neq \{0\}$ cannot be removed. Also it shows that if the direct sum of two C_0-semigroups is J-class then its is not necessary that these C_0-semigroups are J-class.

Example 2.3. *Let X, Y be two complex Banach spaces, where X is separable. Let $A \in B(Y)$ with $\sigma(A) \subset \{z \in \mathbb{C} : Rez > 0\}$. If $\{T(t)\}_{t\geq 0}$ is a hypercyclic C_0-semigroup on X, then the system $B(t) := e^{tA} \oplus T(t)$ is a J-class C_0-semigroup on the Banach space $X \oplus Y$ and $A_B = \{0\} \oplus X$ (Theorem 4.1.13, [14]). Now consider $\phi : X \oplus Y \to Y$ defined by $\phi(x \oplus y) = y$. Then with $S(t) := e^{tA}$ we have $\phi \circ B(t) = S(t) \circ \phi$, $B(t)$ is J-class but $S(t)$ is not J-class, since $\sigma(A) \cap i\mathbb{R} = \emptyset$ (see Lemma 4.1.14, [14]). Indeed in this case $\phi(A_B) = \{0\}$.*

This example also shows that if $\{T(t)\}_{t\geq 0}$ is a J-class C_0-semigroup on a Banach space X and M_1, M_2 are two non-trivial invariant closed subspaces of X, where $X = M_1 \oplus M_2$, then $\{T(t)|_{M_i}\}_{t\geq 0}$ is not J-class on M_i, $i = 1, 2$, in general.

The following proposition shows that locally topologically transitivity of the direct sum of a C_0-semigroup with itself, implies that it is also locally topologically transitive.

Proposition 2.4. *Let $\{T(t)\}_{t\geq 0}$ be a C_0-semigroup on a Banach space X. If $\{T(t) \oplus T(t)\}_{t\geq 0}$ is locally topologically transitive C_0-semigroup on $X \oplus X$, then so is $\{T(t)\}_{t\geq 0}$.*

Proof . *Let $J_{T\oplus T}(x \oplus y) = X \oplus X$, for some non-zero $x \oplus y \in X \oplus X$. Without loss of generality let $x \neq 0$. Thus for every $z \in X$, there exist a sequence $(x_n \oplus y_n)_n \in X \oplus X$ and a strictly increasing sequence $(t_n) \in [0, \infty)$ with $t_n \to \infty$ such that $x_n \oplus y_n \to x \oplus y$ and $T(t_n) \oplus T(t_n)(x_n \oplus y_n) \to z \oplus z$. These imply that $x_n \to x$ and $T(t_n)x_n \to z$, i.e. $J_T(x) = X$.* □

As a consequence of this proposition one can see that if X is a real-Banach space, $\widetilde{X}$ is the complexification of X, $\{T(t)\}_{t\geq 0}$ is a C_0-semigroup on X and $\{\widetilde{T}(t)\}_{t\geq 0}$ is the complexification of $\{T(t)\}_{t\geq 0}$, then locally topologically transitivity of $\{\widetilde{T}(t)\}_{t\geq 0}$ implies that $\{T(t)\}_{t\geq 0}$ is locally topologically transitive.

In the following proposition, we show that the direct sum of two J-class C_0-semigroups is not J-class in general. Note that the adjoint of a C_0-semigroup on a Hilbert space is again a C_0-semigroup.

Proposition 2.5. *Let $\{T(t)\}_{t\geq 0}$ be a J-class C_0-semigroup on a Hilbert space H such that $\{T^*(t)\}_{t\geq 0}$ is also J-class. Then $T(t) \oplus T^*(t)$ is not a J-class C_0-semigroup.*

Proof . Assume that $T(t) \oplus T^*(t)$ is a J-class C_0-semigroup. So there exist $x, y \in H$ such that $J_{T\oplus T^*}(x \oplus y) = H \oplus H$ and $x \oplus y \neq 0$.
Case I: Suppose that one of the vectors x, y is zero. Without loss of generality assume $x = 0$. Then there exist a strictly increasing sequence $(t_n)_n \subseteq [0, \infty)$ with $t_n \to \infty$ and sequences $(x_n)_n, (y_n)_n \in H$ such that $x_n \to x = 0, y_n \to y, T(t_n)x_n \to y$ and $T^*(t_n)y_n \to x = 0$. Taking limits in the following equality $\langle T(t_n)x_n, y_n\rangle = \langle x_n, T^*(t_n)y_n\rangle$ we get that $\|x\| = \|y\| = 0$ and hence $y = 0$. Therefore $x \oplus y = 0$, which yields a contradiction.
Case II: Suppose that $x \neq 0$ and $y \neq 0$. Let us show first that $J_{T\oplus T^*}(\lambda x \oplus \mu y) = H \oplus H$, for every $\lambda, \mu \in \mathbb{C}\setminus\{0\}$. Indeed, fix $\lambda, \mu \in \mathbb{C}\setminus\{0\}$. Take any $z, w \in H$. Since $J_{T\oplus T^*}(x\oplus y) = H\oplus H$, there exist a strictly increasing sequence $(t_n)_n \subseteq [0, \infty)$ with $t_n \to \infty$ and sequences $(x_n)_n, (y_n)_n \in H$ such that $x_n \to x, y_n \to y, T(t_n)x_n \to \lambda^{-1}z$ and $T^*(t_n)y_n \to \mu^{-1}w$. This implies that $z \oplus w \in J_{T\oplus T^*}(\lambda x \oplus \mu y)$, hence $J_{T\oplus T^*}(\lambda x \oplus \mu y) = H \oplus H$. With no loss of generality we may assume that $\|x\| \neq \|y\|$ (because if $\|x\| = \|y\|$, by multiplying with a suitable $\lambda \in \mathbb{C} \setminus \{0\}$ we have $\lambda\|x\| \neq \|y\|$ and $J_{T\oplus T^*}(\lambda x \oplus y) = H \oplus H$). Taking limits in the following equality $\langle T(t_n)x_n, y_n\rangle = \langle x_n, T^*(t_n)y_n\rangle$ we get that $\|x\| = \|y\|$, which is a contradiction. □

Proposition 2.6. *Suppose X is a normed space, C_0-semigroup $\{T(t)\}_{t\geq 0}$ is J-class on X, and Y is a Banach space containing X as a dense subspace. Then the extension of T in Y is J-class.*

Proof . Let for $0 \neq x \in X$, $J_T(x) = X$. For every $\varepsilon > 0$ and $y \in Y = \bar{X}$ there exists $y_1 \in X$ such that $\|y_1 - y\| < \frac{\varepsilon}{2}$. For $y_1 \in X$ there exist $u \in X$ and $t_1 > 0$ such that $\|u - x\| < \frac{\varepsilon}{2}$, $\|y_1 - T(t)u\| < \frac{\varepsilon}{2}$. So

$$\|y - T(t)u\| \leq \|y - y_1\| + \|y_1 - T(t)v\| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

□

3. Limit of C_0-semigroups in the sense of Kato

A sequence $\{(X_n, \|\cdot\|_n) : n \in \mathbb{N}\}$ of Banach spaces is said to be convergent to a Banach space $(X, \|\cdot\|)$ in the sense of Kato (see [12], Chap. IX, Sect. 4) and is denoted by $X_n \xrightarrow{K} X$, if for any n there is a linear operator $P_n \in B(X, X_n)$ (called an approximating operator) satisfying the following two conditions:

(K_1) $\lim_{n\to\infty} \|P_n f\|_n = \|f\|$ for any $f \in X$;
(K_2) for any $f_n \in X_n$, there exists $f^{(n)} \in X$ such that $f_n = P_n f^{(n)}$ with $\|f^{(n)}\| \leq C\|f_n\|_n$ (C is independent of n).

Let $X_n \xrightarrow{K} X$ and $B_n \in B(X_n)$. The sequence $(B_n)_{n\in\mathbb{N}}$ is said to be convergent to B in the sense of Kato if $\lim_{n\to\infty} \|B_n P_n f - P_n B f\|_n = 0$, for any $f \in X$. In this case we write $B_n \xrightarrow{K} B$.

Theorem 3.1. *Let $\{(X_n, \|\cdot\|_n) : n \in \mathbb{N}\}$ be a sequence of Banach spaces convergeing to a Banach space $(X, \|\cdot\|)$ in the sense of Kato. Suppose that $T = \{T(t)\}_{t\geq 0}$ is a J-class C_0-semigroup on X for which $P_n(A_T) \neq \{0\}$ and $\{T_n(t)\}_{t\geq 0}$ is a C_0-semigroup on $(X_n, \|\cdot\|_n)$. If for some $n \in \mathbb{N}$ one has*

$$P_n T(t) f = T_n(t) P_n f, \quad (f \in X,\ t \geq 0), \tag{3.1}$$

then $\{T_n(t)\}_{t\geq 0}$ is also J-class.

Proof . Let $\{T(t)\}_{t\geq 0}$ be J-class. So there exists a non-zero $f^* \in X$ such that $J_T(f^*) = X$. By our hypothesis, we may choose f^* such that $P_n f^* \neq 0$. We shall prove that $J_{T_n}(P_n f^*) = X_n$. For any $g_n \in X_n$ from (K_2), there exists $g^{(n)} \in X$ such that $g_n = P_n g^{(n)}$ and $\|g^{(n)}\| \leq C\|g_n\|_n$. For arbitrary $\varepsilon > 0$, there exist $u \in X$ and $t > 0$ such that $\|u - f^*\| < \varepsilon$ and $\|g^{(n)} - T(t)u\| < \varepsilon$. Put $f_n^* := P_n f^*$, $u_n := P_n u$ and $t_n := t$. The assumption (K_1) implies the uniform boundedness of $\{P_n\}$. Therefore

$$\|u_n - f_n^*\|_n = \|P_n u - P_n f^*\|_n \leq \|P_n\|\ \|u - f^*\| \leq \|P_n\|\varepsilon$$

and

$$\begin{aligned} \|g_n - T_n(t_n)u_n\| &= \|P_n g^{(n)} - T_n(t) P_n u\|_n \\ &= \|P_n g^{(n)} - P_n T(t) u\|_n \\ &\leq \|P_n\|\|g^{(n)} - T(t)u\| \leq \|P_n\|\varepsilon. \end{aligned}$$

□ For any constant C and $f_n \in X_n$, define

$$l_C(f_n) := \{f^{(n)} \in X : P_n f^{(n)} = f_n \ \textit{with}\ \|f^{(n)}\| \leq C\|f_n\|\}.$$

Theorem 3.2. *Suppose that* (3.1) *holds for some $n \in \mathbb{N}$ and $\{T_n(t)\}_{t\geq 0}$ is J-class. If there exists a constant C such that for every $f \in X$ and $\varepsilon > 0$ there is an $f^{(n)} \in l_C(P_n f)$ with $\|f - f^{(n)}\| < \varepsilon$, then $\{T(t)\}_{t\geq 0}$ is also J-class.*

Proof . Let $\{T_n(t)\}_{t\geq 0}$ be J-class on X_n. So there exists a non-zero $f_n^* \in X_n$ such that $J_{T_n}(f_n^*) = X_n$. From (K_2), there exists $f_*^{(n)} \in X$ such that $f_n^* = P_n f_*^{(n)}$. By the linearity of P_n, $f_*^{(n)} \neq 0$. We shall show that $J_T(f_*^{(n)}) = X$.

Let $g \in X$ and $\varepsilon > 0$ be given. Put $g_n := P_n g$. So there exist $t > 0$ and $u_n^* \in X_n$ such that $\|u_n^* - f_n^*\| < \varepsilon$ and $\|g_n - T_n(t)u_n^*\|_n < \varepsilon$. From (K_2), there exists $u_*^{(n)} \in X$ such that $u_n^* = P_n u_*^{(n)}$. Now for $h = g - T(t)u_*^{(n)}$, there exists $h^{(n)} \in P_n h$, with $\|h - h^{(n)}\| < \varepsilon$ and

$$P_n h^{(n)} = P_n h = g_n - P_n T(t) u_*^{(n)}.$$

As a consequence of (3.1), we obtain that

$$g_n - P_n T(t) u_*^{(n)} = g_n - T_n(t) P_n u_*^(n).$$

So

$$\begin{aligned} \|g - T(t)u_*^{(n)}\| &\leq \|h - h^{(n)}\| + \|h^{(n)}\| \\ &\leq \varepsilon + C\|g_n - T_n(t)P_n u_*^{(n)}\|_n \\ &\leq (1+C)\varepsilon \end{aligned}$$

and

$$\|u_*^{(n)} - f_*^{(n)}\| \leq C\|P_n u_*^(n) - P_n f_*^{(n)}\|_n \leq C\|P_n\|\varepsilon.$$

□

Remark 3.3. *Let $T_n = \{T_n(t)\}_{t\geq 0}$ and $T = \{T(t)\}_{t\geq 0}$ be C_0-semigroups on the Banach spaces $(X_n, \|\cdot\|)$ and $(X, \|\cdot\|)$, respectively, $n \in \mathbb{N}$. The sequence (T_n) is said to be convergent to T in the sense of Kato if for any $\tau > 0$,*

$$\lim_{n\to\infty} \sup_{t\in[0,\tau]} \|T_n(t)P_n(f) - P_n T(t) f\|_n = 0, \quad (f \in X).$$

If T_n is J-class then it is not true that T is also J-class, in general. For showing this, we apply Theorem 3.3 of [5]. Let $X_n = X := l^1$, B is the backward shift on l^1 and $A = \alpha(B - I)$, for some $\alpha > 0$. If $T = \{T(t)\}_{t\geq 0}$ is the C_0-semigroup generated by A then

$$\|T(t)\| = \|e^{\alpha t(B-I)}\| = e^{-\alpha t}\|e^{t\alpha B}\| \leq e^{-\alpha t} e^{\|t\alpha B\|} = 1.$$

This, by Remark 1.2, implies that T is not J-class. Now, by [5] Theorem 3.3, the C_0-semigroup $\{T_n(t)\}_{t\geq 0}$ generated by $A_n := -\alpha I + \beta_n B$ is hypercyclic and so is J-class, where $\beta_n > \alpha > 0$ and $\beta_n \to \alpha$. Also the sequence $(T_n)_{n\in\mathbb{N}}$ converges to T, in the sense of Kato (see [5] Theorem 3.3).

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