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Optimization of the dynamic network data envelopment analysis (DNDEA) model based on the game theory in the capital market

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Abstract

Data envelopment analysis (DEA) is a widely-used method in measuring the relative efficiency of sets of homogeneous decision-making units with the same inputs and outputs. In classical DEA models, the whole system is usually considered as a decision-making unit (DMU) to calculate its efficiency, and the communication of separate processes within the system is ignored. However, the internal communication of different sectors of a decision-making unit can have various structures that cause complexity in evaluating its efficiency. To this end, the Network DEA (NDEA) model is developed using communication variables to communicate the internal structures of the decision-making units, in which the production process has two or more stages and considers according to the communication of the internal sectors and sub-units of a decision-making unit as a network structure, and the efficiency of each internal process and the whole process is calculated independently. Therefore, the present research aimed to evaluate the efficiency and ultimately rank the decision-making units and it thus used the dynamic network data envelopment analysis (DNDEA) model using game theory to examine and solve a problem of capital market investors, which was the correct selection of stocks in the efficient companies compared to the stocks of companies with low-efficiency. After developing the proposed model, the performance of 25 active companies in the petrochemical industry was evaluated for three years of 2016-2018 and the efficiency of each stage was calculated along with their overall efficiency. The results indicated that the proposed model had solved the shortcomings of the previous models and the new approach to the evaluation of efficiency in the stock market could provide a more accurate understanding of the performance and efficiency of active companies in this field.

Keywords: Dynamic Network Data Envelopment Analysis (DNDEA), game theory, stock market, cooperative and competitive strategy, GAMS
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1 Introduction

The main tools for managers and policy-makers to make decisions include the estimation of the efficiency and ranking of active companies and units in different industry sectors and ranking processes and managerial systems. The data envelopment analysis (DEA) model is an important model for estimating the efficiency and ranking of units and managerial systems and it is a widely-used non-parametric method that evaluates the relative efficiency of units in comparison with each other [16]. Data envelopment analysis is a mathematical programming model for estimating the efficient frontier. This method obtains a boundary frontier that covers the whole data, and it is thus called data envelopment analysis [7].

Classical data envelopment analysis models consider the evaluation units as a black box that converts inputs into outputs. These models do not pay any attention to the structure and internal flow of the units. Network models are a generalization of these classical models that also consider the internal structure of units. These models were first introduced by Fare and Grosskopf in two separate studies [11]. Examining the multi-stage production process by Fare and Whittaker (1995), these researchers developed a multi-stage model with intermediate variables which was named Network DEA (NDEA).

In a general review, Network DEA can be performed in a multi-step method that can be divided into three categories: series, parallel, and series-parallel. Multi-stage models are mostly used in evaluating the efficiency of activities such as supply chains consisting of several different organizations [34]. After Fare and Grosskopf, other researchers such as Kao [18, 19] sought to develop network DEA models. Therefore, there was a need for a model that could simultaneously consider the internal structure of organizations as well as time. In addition to Kao, other researchers such as Fukuyama & Weber [13], and Kao & Hwang [18] emphasized the development of such models, especially dynamic network data envelopment analysis (DNDEA) models.

NDEA studies still ignore a very important fact that the processes of DMUs and SDMUs have a time dimension, and failure to examine this dimension can easily lead to unrealistic and distorted efficiency evaluations, and they consider static models and evaluate on this basis [5].

However, the operations of organizations and decision-making units are connected as a chain during their lifetime. Therefore, it is necessary to evaluate the performance of such organizations during several periods and it gives better and more valuable information to the managers. The capital market and companies active in the stock market are the most important sectors of the economy with a multi-stage structure and process of activities and they must be examined from the same perspective. The correct selection of shares of efficient companies compared to shares of low-efficiency companies is an issue and problem for capital market investors. Choosing the optimal portfolio is an important issue in finance and it can be seen that the majority of decisions about this issue are made by evaluating a process and a sector of the company, and there are two main ranking methods based on a variable (usually the sales) or the harmonic mean method for ranking the stock companies and they are less reliable for several reasons [37] because decision-making based on only one sector or process cannot be optimal and desirable, and investors in these companies should evaluate several factors affecting the stock value of a company and consider different processes as a network in evaluating the company's efficiency.

Several researchers have proposed the performance evaluation of stock companies and the selection of optimal portfolios over time, such as Powers and McMullen [33], Feroz et al. [12], and Malhotra et al. [27]. Therefore, it is necessary to use models in the evaluation of stock companies that consider the internal structures of organizations, time, and multiple and separate stages of the overall activity. Therefore, the use of traditional models, and single or network models is insufficient for evaluating the performance of these companies, and they must be combined with the game theory and develop and use a newer hybrid approach. In comparison with the above-mentioned model, these models have advantages such as diversity in the calculation, real optimization capability, taking into account different dimensions of companies' activities, and time dimension. Therefore, the present research developed a dynamic network data envelopment analysis model, which indicated both the efficiency of each stage and the overall efficiency of each period, and finally the dynamic network efficiency of the entire period, considering the two-stage activity structure of companies active in the petrochemical industry. Therefore, the main research question was whether it was possible to provide a network data envelopment analysis model with a game theory approach to calculate efficiency in a space where the capital market member companies are decision-making units in a dynamic space.

2 A theoretical bases and literature review

A wide range of methods has been introduced and used to measure performance, efficiency, and productivity for many years. According to a simple definition of efficiency, it is a simple ratio of output to input, compared to a certain standard. In a general classification, we can divide efficiency measurement methods into three main groups, ratio analysis, parametric methods, and non-parametric methods.

If there is only one input and output in the activity of decision-making units (DMU), the use of the ratio method is the easiest way to compare their performance. The ratio analysis method is widely used because it is a very easy method to measure corporate performance and requires very little information [4]. The simple ratio approach was not enough to measure efficiency in production processes with multiple inputs and outputs, and thus other methods were gradually introduced under a general concept called “frontier analysis”.

The frontier analysis measures the level of efficiency, assuming the existence of a hypothetical goal as the efficient frontier and comparing the performance value of each unit with that hypothetical goal and frontier. If the producer acts in a way that exactly matches the desired goal, it is efficient and its efficiency can be considered equal to one (100%). Otherwise, the efficiency is a number between zero and one. Therefore, determining an objective function as the efficient frontier is the most basic step in efficiency measurement. This objective function is usually an efficient frontier function which is defined and obtained by two general parametric and non-parametric methods [35].

In parametric methods, it is assumed that the production function of the measured sector has an analytical structure and they seek to determine the parameters of this function [22]. Regression techniques are usually used to measure efficiency by parametric methods, and they are defined by associating an output of the production function with many inputs. Even though regression analysis is more comprehensive than ratio analysis, this approach also has shortcomings. First, there are problems in expressing different units as a common unit because depending on the definition of output in regression analysis, it is necessary to reduce the outputs to a single value based on a common unit and basis. Second, the regression analysis defines the production function parametrically (by assigning fixed coefficients to inputs or outputs), and does not allow production units to determine technologies or combinations of different objectives, and third, regression analysis measures relative performance compared to average performance instead of the best performance [32].

However, the nonparametric methods measure the difference of the efficiency value obtained from calculation with the effective efficiency limit using linear programming techniques. Relative advantage of these methods is that they do not have to enter into behavioral assumptions about the production unit structure like parametric methods. Another advantage of this method is that it can use more than one descriptive variable. In addition to these advantages, since they do not have a random error term, they transfer the errors, which occur due to data measurement or other reasons, to the model, and can sometimes mistakenly define the efficient frontier (limit). Data envelopment analysis (DEA) is the most widely-used non-parametric method [32].

Based on an innovative method by Charnes et al. [8] introduced the data envelopment analysis model, assuming constant returns to scale (CRS) as a non-parametric technique based on linear programming to measure the efficiency of production units (decision-making units). Later, the BBC model was introduced by Banker, Charnes, and Rhodes [3] to evaluate efficient units with variable scale. DEA assumes that all DMUs are comparable and uniform and consume the same inputs to produce the same outputs, and only the input and output levels are different. In data envelopment analysis, the efficient frontier is considered linear and determined by decision-making units. This efficient frontier is obtained based on the assumptions of constant returns to scale (CRS) or variable returns to scale (VRS) and the decision-making units are measured against this frontier.

Data envelopment analysis models are developed with input or output orientation. In the input orientation, it is sought to achieve the same current level of the observed outputs of the units with less input or minimize the use of inputs. Similarly, it is sought in output orientation to maximize the level of outputs according to the inputs.

Since the CCR and BCC models do not consider the internal processes of decision-making units and the way of converting inputs into outputs and only use inputs and outputs by adopting a “black box” perspective for evaluation, Seiford & Zhu [36], and Luo [26] introduced a two-stage structure according to Figure 1 to evaluate efficiency.

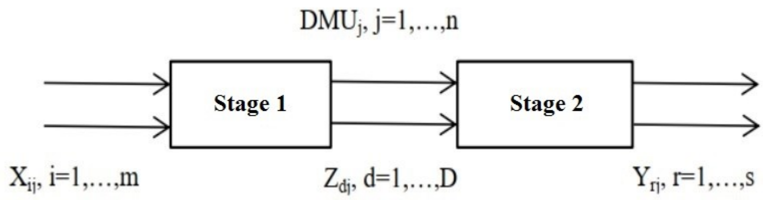

Figure 1: Sequential two-stage structure [36]

The literature review indicates several models which are developed to calculate the efficiency of these systems, each of which is proportional to the internal structure of the systems.

Assume that a system consists of p units where $X_{ij}^{(k)}$ and $Y_{ij}^{(k)}$ are respectively the inputs and outputs of the system, and $X_{ij}^{(k)}$is the ith exogenous input in a way that $i \in I^{(k)}$ in which $I^{(k)}$ is the index of exogenous input set that is used by unit k, and $Y_{rj}^{(k)}$ is the rth output of the system so that $r \in O^{(k)}$ in which $O^{(k)}$ is the index of final products produced by the unit k $(k = 1, 2, \ldots, p)$. If it $Z_{fj}^{(a,k)}$ is considered the fth intermediate product produced by the unit a, then $f \in M^{(k)}$ in which $M^{(k)}$ is an index of the set of intermediate products manufactured by the unit k. Furthermore, $Z_{gj}^{(k,b)}$ is the gth intermediate product manufactured by the unit b so that $g \in N^{(k)}$ in which $N^{(k)}$ is an index of the set of intermediate products manufactured by the unit k. Figure 2 shows the general diagram of this system [18]:

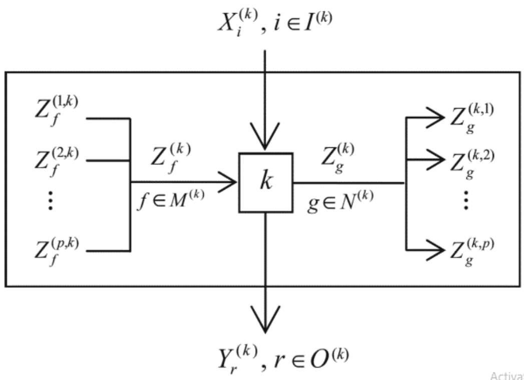

Figure 2: The network data envelopment analysis model for the kth unit

where the following equations are established:

$$\sum_{k=1}^{p} X_{ij}^{(k)} = X_{ij}, \quad \sum_{k=1}^{p} Y_{rj}^{(k)} = Y_{rj}, \quad \sum_{a=1}^{p} Z_{fj}^{(a,k)} = Z_{fj}^{(k)}, \quad \sum_{b=1}^{p} Z_{gj}^{(k,b)} = Z_{gj}^{(k)}. \tag{2.1}$$

Chen et al. [6] also presented a new model for two-stage structures. The researchers used the concept of weighted mean $(w1\theta1 + w2\theta2)$ to explain the relationship between the efficiency of the stages and the total efficiency, which were the weight and importance coefficient of the inputs of each stage compared to the total inputs. The intermediate variable presented by these researchers was considered as both the total input and output of the network. Later, Lozano [25] proposed a new model of network data envelopment analysis in which the current outputs of the stages were compared with the outputs if a central DMU allocated the inputs to the stages. Despotis et al. [10] later introduced models for a variety of series multi-stage structures. An important drawback of two-stage models was that the network models for two-stage structures were static and did not consider time dimension in the evaluation; hence, they were suitable only for single-period evaluation. Therefore, there was a need for models, which considered both the internal structure and the time dimension, and thus these models were gradually introduced.

Nemoto and Goto [30, 31] introduced the dynamic data envelopment analysis (DDEA) model by dividing the inputs of a DMU into two main groups, called semi-constant inputs and variable inputs. Chen & Dalen [9] developed a new model of dynamic data envelopment analysis and used it to evaluate the efficiency of decision-making units by criticizing the traditional and static models of data envelopment analysis. Kao [17] also introduced the DEA model for dynamic structures, but his models considered decision-making units as black boxes in each period and did not consider their internal structures. Later, other researchers such as Fukuyama & Weber [14], Moreno & Lozano [28], Avkiran and Mccrystal [2] developed dynamic network models in their studies.

Evaluating the efficiency in a network structure is not simple despite the positive points of the above-mentioned models. The stages (network nodes) are linked with the criteria of the indices which act as the output of the first stage and the input of the next stage. Therefore, intermediary indicators play contradictory roles in evaluating efficiency. Various methods have been proposed in network DEA studies to overcome this problem and can be divided into two categories: decomposition and combination methods. In addition to these methods, the combination of the DEA approach with other mathematical models such as game theory has also been proposed and used.

Fuzzy hybrid models and DEA were first proposed by Liang et al. [25] for two-stage systems. In centralized models, which use the concept of cooperative games theory, each internal stage is considered as a unit to maximize the efficiency of the entire system. Based on this framework, the efficiency of the whole system can be decomposed into the product of the efficiency multiplication of the subsystems. When the optimal weights are not unique, there is some flexibility in decomposing the efficiency of the whole system into the efficiency of subsystems. Therefore, it is very effective and necessary to determine the fair distribution of the efficiency of the whole system between the subsystems for their cooperation. To solve this problem, Kao & Hwang [20] and Liang et al. [25] both suggested that first the maximum efficiency, which could be obtained from the first stage, should be calculated, and then, the efficiency of the other stage should be calculated according to the efficiency of the whole system and the maximum efficiency that can be obtained from the first stage.

The cooperative (centralized control) and non-cooperative (leader-follower) methods are used in the game theory approach to obtain total efficiency in NDEA models.

In the cooperative approach, Li et al. [23] proposed a centralized approach to analyze the performance of two-stage network structures and defined the total efficiency as the product of the efficiency multiplication in two main stages. Based on Charnes-Cooper-Rhodes (CCR) models, we have:

$$\begin{aligned} &\theta^{\text{cen}} = \max \theta_1^0 \times \theta_2^0 = \max \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \times \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} \\ \text{s.t.}\quad &\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad , \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1, \quad \forall j, \quad j = 1, 2, \ldots, n \\ &v_i,\ u_r,\ w_d,\ Q_h \ge 0 \quad \forall i,\ r,\ d,\ h \end{aligned} \tag{2.2}$$

Due to the presence of additional inputs in the second step, the above model cannot be converted to linear programming. The authors propose the following innovative solution to solve this problem. Consider the following model:

$$\begin{aligned} &\theta_1^{0\max} = \max \ \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.}\quad &\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad , \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1, \quad j = 1, \ldots, n \\ &v_i,\ u_r,\ w_d,\ Q_h \ge 0 \qquad \forall i,\ r,\ d,\ h \end{aligned} \tag{2.3}$$

In this model, the set of constraints is the same as the constraints of the first model, which are added to ensure that the efficiency of the first stage and the second stage do not exceed one. This model is used to obtain the maximum efficiency for the first stage, and the efficiency of the first stage must be between zero and this maximum value; hence, $\theta_1^o \in \left[0,\ \theta_1^{oMax}\right]$ is the third nonlinear model but it can be converted into a linear (parametric) programming using the Charnes-Cooper transformation:

$$\begin{aligned} &\theta_1^{o\max} = \max \sum_{d=1}^{D} w_d z_{do} \\ \text{s.t.}\quad &\sum_{i=1}^{m} v_i x_{io} = 1, \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} - \sum_{h=1}^{H} Q_h x_{hj}^2 \le 0, \quad j = 1,\ 2,\ \ldots,\ n \\ &v_i,\ u_r,\ w_d,\ Q_h \ge 0 \qquad \forall i,\ r,\ d,\ h \end{aligned} \tag{2.4}$$

Therefore, the efficiency of the first stage can be considered as a variable at the interval of $\theta_1^o \in \left[0,\ \theta_1^{oMax}\right]$ and the total efficiency shown by the symbol θ_1^{cen} is a function of θ_1^o; Therefore, it can be written as follows:

$$\begin{aligned} &\theta^{\text{cen}} = \max \theta_1^o \times \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} \\ \text{s.t.}\quad &\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1, \quad j = 1, 2, \ldots, n \\ &\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \le \theta_1^o, \quad \theta_1^o \in [0,\ \theta_1^{o\max}] \\ &v_i,\ u_r,\ w_d,\ Q_h \ge 0 \qquad \forall i,\ r,\ d,\ h \end{aligned} \tag{2.5}$$

This is a non-linear model that is written using the Charnes-Cooper transformation as follows:

$$\begin{aligned} &\theta^{\text{cen}} = \max \theta_1^o \times \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.}\quad &\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2 = 1, \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} - \sum_{h=1}^{H} Q_h x_{hj}^2 \le 0, \quad \forall j, \quad j = 1, 2, \ldots, n \\ &\sum_{d=1}^{D} w_d z_{do} - \theta_1^o \times \sum_{i=1}^{m} v_i x_{io} = 0, \quad \theta_1^o \in \left[0,\ \theta_1^{oMax}\right] \\ &v_i,\ u_r,\ w_d,\ Q_h \ge 0 \quad \forall i,\ r,\ d,\ h \end{aligned} \tag{2.6}$$

Assuming that $\theta_1^o = \theta_1^{oMax} - k\Delta\varepsilon,\ k = 0, 1,\ 3, \dots, [k^{\max}] + 1$ and $\Delta\varepsilon$ is a small value and $[k^{\max}]$ is the largest integer of less than or equal to $\frac{\theta_1^{omax}}{\Delta\varepsilon}$, it is possible to solve the model (87) using linear programming according to the value θ_1^o. To solve this model, we first start with the lowest value of k, i.e. zero, and gradually increase its value, and this model is solved once for each value [23].

In the second method, which is also called the decentralized control method or Stackelberg competition, first we measure the efficiency of the leader stage (the stage that is more important to us, for example, the first stage), and then we have to keep the first step constant and add it to the model as a constraint to get the efficiency of the follower stage (here, the second stage). In other words, we obtain the efficiency of the second stage (follower) in a way that the efficiency of the leader stage (first) does not change. The efficiency of the first stage (leader) is obtained using the CCR model.

$$\begin{array}{ll} & e_1^o = \max \sum_{d=1}^{D} w_d z_{do} \\ \text{s.t.} & \sum_{i=1}^{m} v_i x_{io} = 1\ , \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0\ , \quad j = 1, 2, \dots, n \\ & v_i,\ u_r,\ w_d \ge 0 \quad \forall i,\ r,\ d \end{array} \tag{2.7}$$

If e_1^{o*} , v_i^* , u_r^*, w_d^* are the optimal values of the first stage, the efficiency of the second stage (follower) is obtained using the following model according to the values of v_i^* , w_d^* (which may not be unique) because the two stages are correlated by intermediate criteria:

$$\begin{array}{ll} & e_2^0 = \max \ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} \\ \text{s.t.} & \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1, \quad \forall j, \quad j = 1, 2, \dots, n \\ & \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} = e_1^{o*} \\ & v_i,\ u_r,\ w_d,\ Q_h \ge 0 \qquad \forall i,\ r,\ d,\ h \end{array} \tag{2.8}$$

In this model, the efficiency of the second stage is optimized by keeping the efficiency of the first stage constant. This model can be converted into the following form:

$$\begin{array}{ll} & e_2^0 = \max \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.} & \sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2 = 1, \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} - \sum_{h=1}^{H} Q_h x_{hj}^2 \le 0\ , \quad \forall j, \quad j = 1, 2, \dots, n \\ & \sum_{d=1}^{D} w_d z_{do} - e_1^{o*} \times \sum_{i=1}^{m} v_i x_{io} = 0 \\ & v_i,\ u_r,\ w_d,\ Q_h \ge 0 \qquad \forall i,\ r,\ d,\ h \end{array} \tag{2.9}$$

The total efficiency of the system is obtained from Equation (2.9)[23]:

$$e^{non*} = e_1^{o*} \times e_2^{o*}. \tag{2.10}$$

Few but useful studies have been conducted on combining game theory and data envelopment analysis. Nakabayashi and Tone [29] investigated the combination of game theory and data envelopment analysis in research titled "Egoist's dilemma, a DEA game", which was relatively comprehensive research at the National Institute of University Education in Japan. Lozano et al. [25] utilized a linear programming model to consider a situation where decision-makers intended to combine their resources and share them in a cooperative game. In this method, the maximum level of production was achieved by obtaining the results of various coalitions and using the core solution in multi-objective linear programming. Anisi [1] combined the game theory and data envelopment analysis and investigated the issue of agreement between individuals or organizations when there were several criteria to evaluate their performance. Sajjadi et al. (2016) used data envelopment analysis models of a general dynamic network to investigate the efficiency of two-level suppliers under cooperative and non-cooperative strategies in which each level had inputs and outputs, and some outputs of the first level could be restored to the second level. Since the proposed models were non-linear, a heuristic method was presented as an alternative solution that could be used instead of the existing methods such as the parametric linear programming approach.

3 Research methodology

A- Research method:

The research has an analytical method and is a subgroup of mathematical analysis research to develop relationships between the concepts by developing mathematical relations. After presenting the target models in this research, they are used to evaluate the efficiency and ranking stock companies. Furthermore, the present research is applied in terms of objective. The purpose of conducting applied research is to gain the necessary understanding or knowledge to determine a tool that can fulfill a specific and known need. The present research is also causal or post-event in terms of the data collection method.

B- Data collection methods and tools:

Data collection is performed using a documentary and desk method, and they are collected by taking notes. Therefore, the necessary information and statistics for the variables are collected from the Tehran Stock Exchange to implement the model. The Kodal website is the main available source. Rahavard Novin software is used to collect the financial data of the companies, and the collected data are summarized using the Excel software, and then the desired variables are calculated and used in the proposed models.

C- Data analysis method:

According to the method of implementing network data envelopment analysis and the game theory, first, the network data envelopment analysis is modeled by stating the necessary hypotheses and variables for each process and the inputs and outputs of each process are defined and specified. Thereafter, game modeling of the network data envelopment analysis model is done with the definition of players and..., and finally, the efficiency of the model is calculated with mathematical techniques. In this research, statistical and research software is used in MATLAB and GEMS operations after coding the models.

D- Proposed research model:

The research model has a dynamic two-stage type in the present study. (Figure 3) The network-related phase of the research has two sections, and its first process consisted of two sub-sections. The model is a developed form of the models by Liang et al. [25], Yongjun Li et al. [23], and Han-Ying Kao (2017). Each DMU of the model consists of three Sub-DMUs. (Sub-DMU1j, Sub-DMU2j, and Sub-DMU3j).

The number of DMUs is equal to $n(j = 1, 2, \ldots, n)$. The number of sub-DMU1j inputs is $I1(i_1 = 1, 2, \ldots, I_1)$, the number of sub-DMU2j inputs is $I2(i_2 = 1, 2, \ldots, I_2)$, and the number of sub-DMU3j inputs is $I3(i_3 = 1, 2, \ldots, I_3)$. Here we have the intermediary variable Z_{mj} $(m = 1, 2, \ldots, M)$ which is the product of the first stage and the data of the second stage. Y_{r_2j} $(r_2 = 1, 2, \ldots, S_2)$ is the Carry-Over variable for the dynamics of this model. Y_{r_2j} and Y_{r_1j} $(r_1 = 1, 2, \ldots, S_1)$ are the final products of this model which originates from Sub-DMU3j. This structure is usually used for different stages of a company.

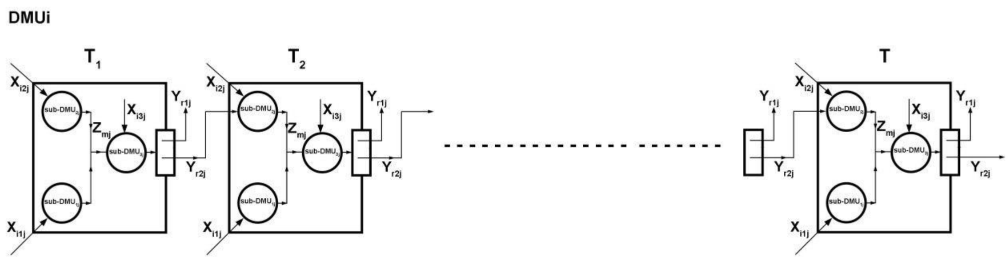

Figure 3: The two-stage model of the research

This model is considered to rank the stock of 25 petrochemical companies listed on the Tehran Stock Exchange. The condition of homogeneity is observed for selecting the companies from an industry group. Therefore, the statistical population consists of petrochemical companies active in the stock market. The study investigates a period of 3 years from 2016 to 2018 and the relevant statistics are often extracted from the Kodal website.
According to the studies, the two main processes of every listed company active in the petrochemical industry can be divided into the following two main groups:

A: Financing for the necessary resources of production and survival in the company

B: Production

There are two ways of financing in every company:

- Stockholders' equity which is mainly provided by shareholders' reserves in the form of a capital increase or retained earnings (which is shown with S-E in this model).

- Bank Loan which is performed by obtaining a loan or facility (which is shown with B-E in this model).

In this stage and process, two inputs (the stock risk rate and the beta coefficient) are jointly entered into each subsection above, and the material inventory index is the output of this process. The efficiency of financing is obtained in this process.
It is possible to produce and then sell after financing for the company. In other words, the company can reach the main goal of its establishment if its financial resources are provided. Therefore, the production process (which is shown by p in this model) is considered after the financing process. In the second process, the production efficiency is obtained and its input is the output of the first process, and another input is added to the production process, called the credit sales index, which finally has three outputs, namely the stock price index, the rate of return on investment, and the capital acquired from this process. In the continuation of the production and sales in the company, it needs financing as it is developed in the DNDEA model with a game theory approach.
A cooperative model is presented to fully calculate the efficiency, and then the efficiency is measured under non-cooperative conditions in which the dynamic model has the first stage of leader (financing) and the second stage of follower (production), and once again the second stage of leader (production) and the first stage of follower (financing), and the results are evaluated. All models are estimated with output orientation and the assumption of variable returns to scale.

C-1- Cooperative model

In this case, we use the following formula to obtain the total efficiency:

$$
\begin{array}{ll}
 & E_0^{overall} = \frac{\sum_{t_1=1}^{T}\prod_{k=1}^{K} E_0^{k,t}}{T} \\
\text{s.t.} & E_0^{1,t} = \max \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 o}^{1,t}}{T} \\
 & E_0^{2,t} = \max \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1 o}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2 o}^{2,t}}{T} \\
 & \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i,j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 j}^{1,t}}{T} \leq 1 \\
 & \quad t = 1,2,\ldots,T,\ j = 1,2,\ldots,n \\
 & (\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 o}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 o}^{1,t})^2 = 1 \\
 & \sum_{i_1=1}^{I_1} V_{i_1}^{2,t} X_{i_1 o}^{2,t} + \sum_{i_2=1}^{s_2} u_m^{2,t} Z_{mo}^t = 1 \\
 & \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1 j}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2 j}^{2,t}}{T} \leq 1 \quad t = 1,2,\ldots,T, j = 1,2,\ldots,n \\
 & u_m^{2,t}, V_{i_3}^{2,t}, u_{r_1}^{2,t}, u_{r_2}^{2,t}, u_m^{1,t}, V_{i_1}^{1,t}, V_{i_2}^{1,t}, u_{r_2}^{1,t} \geq 0 \quad m = 1,2,\ldots,M, i_3 = 1,2,\ldots,I_3, r_1 = 1,2,\ldots,s_1 \\
 & r_2 = 1,2,\ldots,s_2, \quad i_1 = 1,2,\ldots,I_1, i_2 = 1,2,\ldots,I_2
\end{array}
\tag{3.11}
$$

We calculate $E_0^{1,t-\max}$ to calculate $E_0^{overall}$ in centralized mode (full cooperation) and put the values obtained from model (3.12) in Model (3.13) to obtain the overall efficiency value at times t.

$$
\begin{array}{ll}
 & E_0^{1,t-\max} = \max \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 o}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 o}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 o}^{1,t})^2} \\
\text{s.t.} & \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 j}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1 j}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2 j}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2 j}^{1,t})^2} \leq 1 \\
 & \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
 & \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1 j}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2 j}^{2,t}}{\sum_{i_3=1}^{I_3} V_{i_3}^{2,t} X_{i_3 j}^{2,t} + \sum_{m=1}^{M} u_m^{2,t} Z_{mj}^t} \leq 1 \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
 & u_m^{2,t}, V_{i_3}^{2,t}, u_{r_1}^{2,t}, u_{r_2}^{2,t}, u_m^{1,t}, V_{i_1}^{1,t}, V_{i_2}^{1,t}, u_{r_2}^{1,t} \geq 0, \quad m = 1,2,\ldots,M, \quad i_3 = 1,2,\ldots,I_3, \quad r_1 = 1,2,\ldots,s_1 \\
 & r_2 = 1,2,\ldots,s_2, \quad i_1 = 1,2,\ldots,I_1,\ i_2 = 1,2,\ldots,I_2
\end{array}
\tag{3.12}
$$

We know that: $E_0^{1,t} \in [0E_0^{1,t-max}]$

Therefore, the total efficiency in cooperation mode is as follows:

$$
\begin{aligned}
& E_0^{overall} = \frac{\sum_{t_1=1}^{T}\prod_{k=1}^{K} E_o^{k,t}}{T} \\
\text{s.t.}\quad & E_0^{1,t} = \max \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{l,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t})^2} \\
& E_0^{1,t} = [0, E_0^{1,t-\max}] \\
& E_0^{2,t} = \max \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1o}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2o}^{2,t}}{\sum_{i_1=1}^{I_1} V_{i_3}^{2,t} X_{i_3o}^{2,t} + \sum_{r_2=1}^{s_2} u_m^{2,t} Z_{mo}^t} \\
& \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \dot{\sum}_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \dot{\sum}_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1j}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t})^2} \le 1 \\
& \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
& \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1j}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2j}^{2,t}}{\sum_{i_3=1}^{I_3} V_{i_3}^{2,t} X_{i_3j}^{2,t} + \sum_{m=1}^{M} u_m^{2,t} Z_{mj}^t} \le 1, \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
& u_m^{2,t}, V_{i_3}^{2,t}, u_{r_1}^{2,t}, u_{r_2}^{2,t}, u_m^{1,t}, V_{i_1}^{1,t}, V_{i_2}^{1,t}, u_{r_2}^{1,t} \ge 0 \quad m = 1,2,\ldots,M, i_3 = 1,2,\ldots,I_3, r_1 = 1,2,\ldots,s_1 \\
& r_2 = 1,2,\ldots,s_2, \quad i_1 = 1,2,\ldots,I_1, \quad i_2 = 1,2,\ldots,I_2
\end{aligned}
\tag{3.13}
$$

C-2- Non-cooperative model

When the first stage is the leader and the second stage is the follower, we have:

$$
\begin{aligned}
& e_0^{1,t,\max} = \max \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t})^2} \\
\text{s.t.}\quad & \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \dot{\sum}_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \dot{\sum}_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1j}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t})^2} \le 1 \\
& \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
& u_m^{1,t}, V_{i_1}^{1,t}, V_{i_2}^{1,t}, u_{r_2}^{1,t} \ge 0, \quad m = 1,2,\ldots,M, \quad i_1 = 1,2,\ldots,I_1, \quad i_2 = 1,2,\ldots,I_2
\end{aligned}
\tag{3.14}
$$

Now, we get the efficiency of the second stage, which is the follower, as follows:

$$
\begin{aligned}
& e_0^{2,t,\max} = \max \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1o}^{2,t} + \sum_{i_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2o}^{2,t}}{\sum_{i_1=1}^{I_1} V_{i_3}^{2,t} X_{i_3o}^{2,t} + \sum_{r_2=1}^{s_2} u_m^{2,t} Z_{mo}^t} \\
\text{s.t.}\quad & \frac{\sum_{r_1=1}^{s_1} u_{r_1}^{2,t} Y_{r_1j}^{2,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{2,t} Y_{r_2j}^{2,t}}{\sum_{i_3=1}^{I_3} V_{i_3}^{2,t} X_{i_3j}^{2,t} + \sum_{m=1}^{M} u_m^{2,t} Z_{mj}^t} \le 1 \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
& \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \dot{\sum}_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mo}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2o}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2o}^{1,t})^2} = e_0^{1,t,\max} \\
\text{s.t.}\quad & \frac{\sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{m=1}^{M} u_m^{1,t} Z_{mj}^t \cdot \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t}}{(\sum_{i_1=1}^{I_1} V_{i_1}^{1,t} X_{i_1o}^{1,t})^2 + (\sum_{i_2=1}^{I_2} V_{i_2}^{1,t} X_{i_2j}^{1,t} + \sum_{r_2=1}^{s_2} u_{r_2}^{1,t} Y_{r_2j}^{1,t})^2} \le 1 \\
& \quad t = 1,2,\ldots,T, \quad j = 1,2,\ldots,n \\
& u_m^{2,t}, V_{i_3}^{2,t}, u_{r_1}^{2,t}, u_{r_2}^{2,t}, u_m^{1,t}, V_{i_1}^{1,t}, V_{i_2}^{1,t}, u_{r_2}^{1,t} \ge 0, \quad m = 1,2,\ldots,M, \quad i_3 = 1,2,\ldots,I_3, \quad r_1 = 1,2,\ldots,s_1 \\
& r_2 = 1,2,\ldots,s_2, \quad i_1 = 1,2,\ldots,I_1, \quad i_2 = 1,2,\ldots,I_2
\end{aligned}
\tag{3.15}
$$

We can also choose the second stage as the leader and the first stage as the follower and compare the results.

4 Research variables and measurement scale

In the present study, the condition of $N \ge 3(M+S)$ is met.
Where N is the number of decision-making units; M is the number of inputs, and S is the number of outputs. Therefore, table variables were selected as the input, intermediate, and output variables in the combined model of network data envelopment analysis and game theory to increase the separability of the model and use the largest number of effective variables. Table 1 presents the variables.

5 Research findings

According to the above-mentioned, problem modeling was done using the leader-follower approach of game theory in the form of suggested patterns. Almost all the experts and managers of stock companies emphasize the role of leadership in the first stage and believe that survival and success in this expensive industry is dependent on financing, which in turn depends on the continuation of production and ultimately sales and profit.

Table 1: Research variables

Row	Nature of the variable or stage	Name of variable	Abbreviation	Definition
1	Linking	Material inventory index	Z1 (output-input)	Inventory to cost ratio
2	Input	Stock risk rate	X1	Standard deviation of stock returns
		Beta coefficient	X2	The ratio of covariance of securities return and market return to the variance of market return in a certain period
		Credit sales index	X3	The ratio of the company's receivables to the rate of sales
3	Output	Acquired capital	Y1	The ratio of dividend retained earnings to assets
		Stock price index	Y2	The price at which stocks are traded on the stock exchange
		Return on investment	Y3	The ratio of profit to the amount of cost and investment
4	Financing stage	Shareholder Equity (SE)	SE (the first sub-stage)	The sum of common and preferred stock of the company to retained earnings
		Bank Loan	BL (the second sub-stage)	Sum of loans and credits in foreign currency and Rials received from the banking network
5	The production stage	Petrochemical production	p	The total value of the product

The following tables present the results of calculating the efficiency with the proposed models during which the efficiency of the units is obtained according to whether they are leaders or followers.

Table 2 presents the efficiency of all units for 3 consecutive years separately. As shown, the efficiency of 3 units (3, 12, and 20) is considered in all stages 1, 2, and overall stages and is equal to 100 in all years and it has the highest value compared to the rest of the units. When the first stage (financing) is considered as the leader, and the second stage (production) as the follower, the three values of efficiency in these columns show that only 7 units (3, 10, 12, 16, 20, 21, and 25) in 2015 achieved 100% efficiency in the first stage. Therefore, when financing is the leader and the main stage of the activities of these units, optimal performance is observed in the continuation of the activities of these units. In this same year, the number of efficient units of the second stage is equal to the same 3 units (3, 12, and 20) which can be introduced as units with stable efficiency. Therefore, the same three units are achieved with full efficiency in this year.
Similarly, the fully functional units of the first stage are also 7 units (3, 7, 10, 12, 16, 20, and 23) in 2017. In the same year, the number of efficient units of the second stage is equal to the same 3 units (3, 12, and 20) which have the overall efficiency and are stable in their efficiency values.
Six units (3, 12, 20, 21, 23, and 25) also have full efficiency again in the first stage in 2018 according to the results. In the same year, the number of efficient units of the second stage is the same 3 units (3, 12, and 20) which have overall efficiency.
Assuming that the second stage is the leader and the first stage is the follower, the results obtained for the efficiency of the units are somewhat different except for the stable units.

According to the results of this table and the mentioned assumption, there are only three fully efficient units (3, 12, and 20) in 20016. In the same year, the same units along with units 7 and 25 have the efficiency of the second stage. From the perspective of overall efficiency, only stable units in efficiency achieve 100% efficiency.
Three units (20, 12, and 3) are again efficient in the first stage in 2017. In the same year and the second stage, there are stable units in efficiency along with new units 8 and 14 with 100% efficiency. In terms of overall efficiency, the same units 20, 12, and 3 have the best performance as the last year. The efficient units can be identified similarly for 2018.

Table 3 also presents the overall efficiency of these 3 years at once. According to the results of the table, the overall efficiency of all 25 units indicates that unit 3 has the highest efficiency and performance with an overall efficiency of 0.0989 and it thus has the first rank. The rest of the units are also ranked according to the value of efficiency. According to this table, unit 19 has the worst performance and is ranked 25 with an overall efficiency of 0.464. An important point of the table is that the units with the best efficiency and performance have performed best in the first stage of financing (through equity or loans). Therefore, companies that are successful in one of their financing

Table 2: The results of the efficiency of the non-cooperative model

	The first stage as the leader and the second stage as the follower									The second stage as the leader and the first stage as the follower								
Year	2016			2017			2018			2016			2017			2018		
Stages	1	2	Total	1	2	Total	1	2	Total	1	2	Total	1	2	Total	1	2	Total
Companies																		
1	0.418	0.581	0.456	0.498	0.661	0.616	0.552	0.789	0.714	0.639	0.452	0.531	0.752	0.639	0.691	0.798	0.610	0.675
2	0.789	0.556	0.556	0.889	0.736	0.801	0.569	0.712	0.638	0.459	0.593	0.553	0.411	0.698	0.539	0.596	0.798	0.701
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	0.536	0.666	0.609	0.498	0.559	0.559	0.359	0.759	0.697	0.569	0.879	0.663	0.663	0.777	0.603	0.602	0.536	0.492
5	0.921	0.801	0.832	0.863	0.702	0.773	0.890	0.763	0.789	0.521	0.663	0.561	0.441	0.598	0.493	0.553	0.663	0.592
6	0.632	0.552	0.593	0.550	0.985	0.759	0.442	0.886	0.669	0.598	0.669	0.609	0.689	0.796	0.729	0.690	0.896	0.821
7	0.892	0.751	0.779	1	0.694	0.753	0.863	0.741	0.863	0.637	1	0.911	0.531	0.712	0.669	0.631	0.812	0.702
8	0.401	0.501	0.466	0.498	0.463	0.409	0.528	0.596	0.555	0.789	0.996	0.836	0.899	1	0.901	0.701	0.986	0.863
9	0.752	0.496	0.501	0.889	0.450	0.639	0.912	0.669	0.769	0.369	0.896	0.896	0.456	0.956	0.822	0.553	0.980	0.719
10	1	0.569	0.772	1	0.714	0.869	0.923	0.669	0.719	0.691	0.896	0.725	0.569	0.775	0.602	0.426	0.694	0.701
11	0.469	0.669	0.669	0.433	0.789	0.663	0.553	0.869	0.711	0.631	0.445	0.553	0.736	0.401	0.493	0.796	0.520	0.520
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	0.778	0.897	0.815	0.669	0.948	0.756	0.569	0.956	0.705	0.396	0.569	0.497	0.498	0.603	0.509	0.596	0.669	0.607
14	0.559	0.777	0.609	0.796	0.964	0.863	0.896	0.806	0.832	0.539	0.774	0.693	0.501	1	0.796	0.460	0.869	0.771
15	0.696	0.889	0.889	0.896	0.991	0.921	0.968	0.869	0.900	0.401	0.639	0.569	0.536	0.769	0.639	0.613	0.889	0.776
16	1	0.634	0.792	1	0.721	0.863	0.861	0.701	0.767	0.531	0.418	0.469	0.636	0.752	0.692	0.714	0.529	0.613
17	0.798	0.569	0.701	0.968	0.663	0.721	0.901	0.725	0.863	0.639	0.852	0.768	0.598	0.963	0.739	0.456	0.836	0.553
18	0.898	0.469	0.596	0.963	0.559	0.691	0.901	0.789	0.771	0.769	0.668	0.502	0.413	0.552	0.456	0.638	0.593	0.601
19	0.537	0.889	0.663	0.590	0.759	0.612	0.698	0.701	0.759	0.528	0.986	0.885	0.450	0.889	0.759	0.697	0.956	0.712
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	0.715	0.896	0.963	0.668	0.715	1	0.778	0.896	0.694	0.568	0.612	0.714	0.601	0.659	0.663	0.569	0.601
22	0.952	0.539	0.693	0.869	0.667	0.778	0.899	0.596	0.628	0.439	0.967	0.814	0.520	0.901	0.762	0.550	0.821	0.638
23	0.936	0.891	0.901	1	0.698	0.771	1	0.559	0.559	0.691	0.775	0.701	0.553	0.793	0.738	0.493	0.669	0.559
24	0.802	0.568	0.669	0.896	0.663	0.663	0.989	0.701	0.839	0.639	0.969	0.741	0.569	0.980	0.774	0.501	0.982	0.763
25	1	0.936	0.950	0.896	0.886	0.871	1	0.812	0.791	0.912	1	0.951	0.741	0.826	0.706	0.568	0.714	0.666

Table 3: Results of the overall efficiency of the non-cooperative model

Companies	**The overall efficiency of the non-cooperative model**	**Efficiency rank**
1	0.675	10
2	0.646	14
3	0.989	1
4	0.726	7
5	0.670	11
6	0.640	15
7	0.701	8
8	0.661	12
9	0.612	16
10	0.652	13
11	0.601	17
12	0.965	2
13	0.530	23
14	0.769	6
15	0.548	22
16	0.689	9
17	0.561	20
18	0.495	24
19	0.464	25
20	0.946	3
21	0.810	5
22	0.550	21
23	0.598	18
24	0.573	19
25	0.912	4
Mean efficiency	0.679	-

methods in the best way and have better output in the first stage are among the units that have obtained the best overall efficiency ranks. The mean efficiency of all 25 companies is equal to 0.679 for three years.

The achievements of the models of this research can be examined from two very important aspects according to the data analysis. Performance evaluation is the first debatable aspect which is determined relatively and will have values between zero and one. The advantage of performance evaluation is that these evaluations consider the internal structure and time and specify details of each company's performance separately for each stage of activity. For example, company 1 has obtained an overall efficiency score of 0.675 according to Table 3, indicating its inefficiency and low performance for almost the entire three-year period, and it is clear that the source of this inefficiency is related to what year and what stage. According to Table 2 and both assumptions, this company can observe its performance annually and in stages and compare its situation with other homogeneous companies.

Other advantages of this model are modeling and the way to achieve efficiency for inefficient units. The models presented for each stage and each period, whether each stage is the leader or the follower, introduce models for efficiency according to the output-oriented or input-oriented nature so that the inefficient company can achieve its full efficiency. Since the model of this research is output-oriented and the goal is to maximize the outputs, the efficiency scores in each stage and period show the difference between the current outputs and the outputs of the model; hence, we can create a model for each inefficient company to improve efficiency. For example, company 1 has an efficiency of 0.418 in stage 1 in 2016 (when financing is the leader and production is the follower), that is, this company should increase the outputs of this stage by 2.39 times (1 divided by 0.418) to be efficient. The efficiency of the 2nd stage of this company is equal to 0.581 in the same year; that is, if this company wants to optimize only its second stage independent of the first stage and the network structure and with the current level of its inputs, it should increase its output by 1.72 times (1 divided by 0.581). For this company, the results of 2017 can also be interpreted according to the efficiency scores of both stages and the overall efficiency. The overall efficiency of this company is equal to 0.616 in 2016; that is, stages 1 and 2 of the previous year (2016) and the current year of 2017 should be efficient respectively to achieve efficiency, and thus the geometric mean ratio of the increase in the output of this company in these two years should increase by 1.62 times (1 divided by 0.616) in which both stages of 2016 and 2017 play fundamental roles. The modeling of 2018 is explainable and similar to 2016 and 2017. The advantage of this type of modeling is that if the evaluation is done with traditional models, the rate of increase in the output of each company can be only specified, and thus it is not possible to separate the increase of each stage according to whether it is a leader or a follower, and the contribution of each stage cannot be specified for every year.

6 Conclusion

Given the importance of the subject and the network structure and the use of game theory in the evaluation of companies active in the petrochemical industry, a dynamic network data envelopment analysis model was developed in the present, during which the overall efficiency and the efficiency of each stage was obtained, assuming that each of the stages was a leader or a follower.

The results of calculating the efficiency of each stage of the activities indicated that the performance and efficiency of each active company in the petrochemical industry could be affected by the performance of each separate stage of their activities. Since this industry is expensive and requires significant financing and capital to start and continue the production activity, the method of financing national resources from Shareholders' Equity (SE) or Bank Loan (BL) can be effective in the level of efficiency and performance of these companies and can play a great role in this regard. Another result of the analysis of the models indicated that changing the positions of the first and second stages of financing and production as a leader or follower can affect the efficiency values of the companies and change their performance.

The studies indicated that how the overall efficiency in each year was separately dependent on the efficiency of each of the first and second stages and how we should increase the outputs of each stage to be efficient. According to the models, the improvement in the overall efficiency of each specific year depended on the improvement of the efficiency of each stage in the same year and the stages of the previous years. According to the findings of this research, the units (3, 12, and 20) respectively had the highest performance and were stable in their efficiency, but unit 19 had the lowest efficiency.

The possibility of separate and overall performance evaluation in each stage was an important advantage for the developed models of the present research. Therefore, the efficiency of stages and courses was first calculated, and then the total efficiency was calculated in this study. According to the output orientation of the models, the output of each stage increased according to the same period and stage, not in an equal value, to increase the efficiency, and thus it was possible to provide a model for the efficiency of all inefficient units at every stage.

Based on the results of the proposed models in the present research, the shortcomings of the previous models were resolved and this new approach to the evaluation of the efficiency of decision-making units in the stock market could provide a more accurate understanding of the performance of components of the efficiency units in active companies for stakeholders.

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